Computer algebra independent integration tests

5-Inverse-trig-functions/5.2-Inverse-cosine/5.2.2-d-x-^m-a+b-arccos-c-x-^n

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [227]. This is test number [145].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (227)	% 0. (0)
Mathematica	% 100. (227)	% 0. (0)
Maple	% 94.71 (215)	% 5.29 (12)
Maxima	% 28.19 (64)	% 71.81 (163)
Fricas	% 34.8 (79)	% 65.2 (148)
Sympy	% 40.53 (92)	% 59.47 (135)
Giac	% 74.01 (168)	% 25.99 (59)

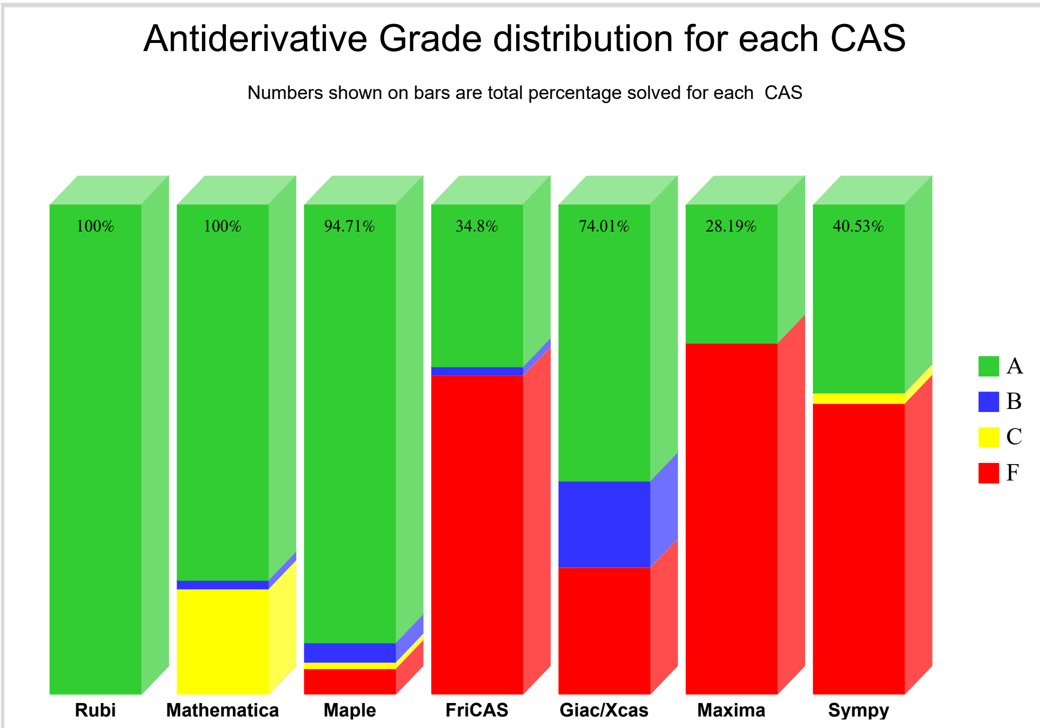
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

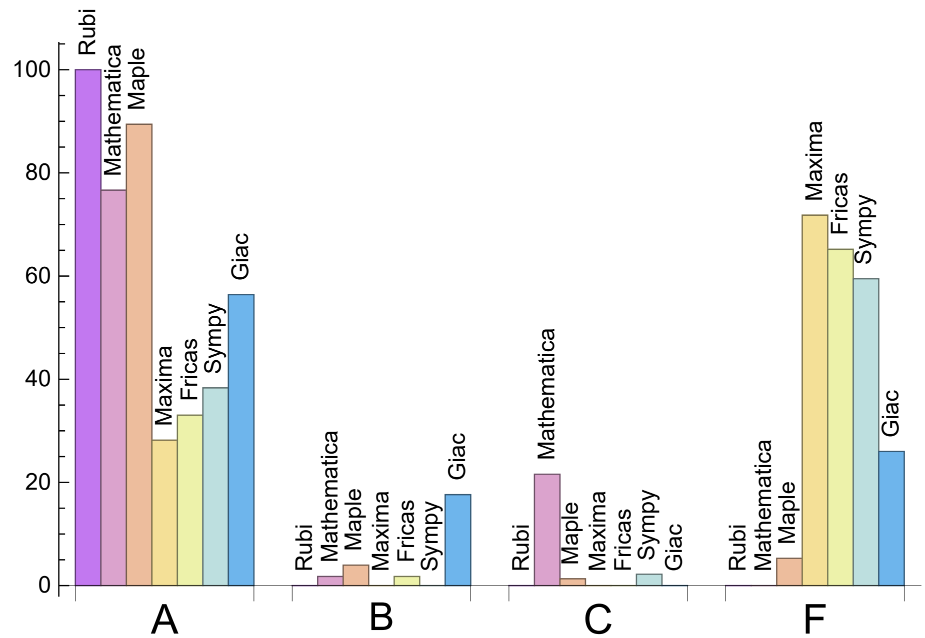
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	76.65	1.76	21.59	0.
Maple	89.43	3.96	1.32	5.29
Maxima	28.19	0.	0.	71.81
Fricas	33.04	1.76	0.	65.2
Sympy	38.33	0.	2.2	59.47
Giac	56.39	17.62	0.	25.99

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.16	82.33	0.73	75.	1.
Mathematica	2.79	98.51	0.82	69.	0.88
Maple	0.12	92.53	0.79	66.	0.87
Maxima	0.73	55.89	0.72	17.5	0.56
Fricas	1.21	97.19	1.25	65.	1.36
Sympy	2.57	50.58	0.56	0.	0.
Giac	0.85	204.43	1.82	75.	1.18

1.4 list of integrals that has no closed form antiderivative

{49, 50, 58, 59, 65, 66, 72, 73, 79, 85, 91, 97, 98, 106, 112, 118, 119, 120, 123, 124, 125, 126, 127, 128, 129, 134, 135, 136, 137, 138, 139, 161, 162, 166, 167, 171, 172, 176, 177, 181, 182, 186, 187, 191, 192, 196, 197, 201, 202, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {18, 20, 28, 39, 41, 74, 75, 76, 78, 80, 81, 82, 84, 86, 87, 88, 90, 92, 93, 94, 96, 99, 100, 101, 102, 103, 105, 107, 108, 109, 111, 113, 114, 115, 117, 121, 144, 151, 152, 156, 157, 173, 175, 178, 180, 183, 185, 188, 190, 193, 195, 198, 200, 209, 210, 211, 212, 213, 214}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

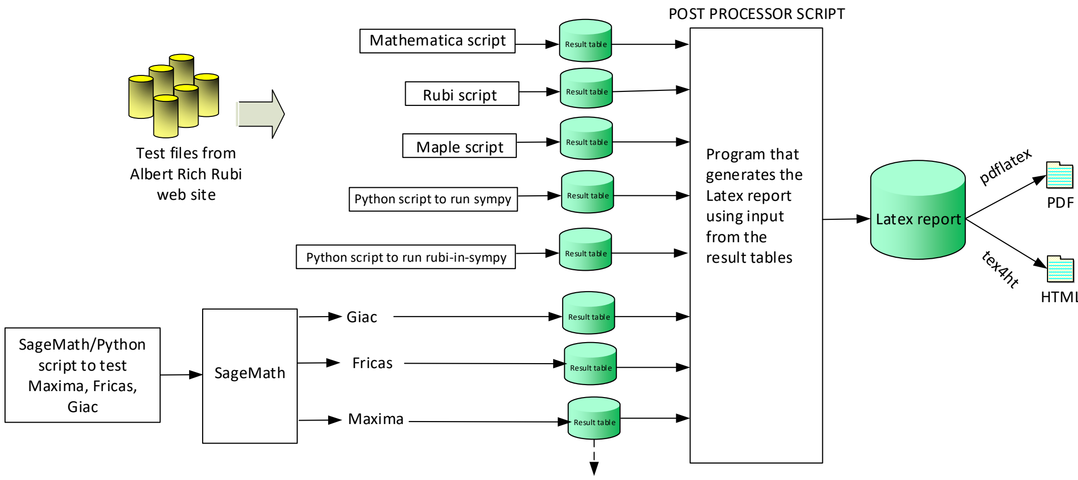
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 77, 79, 83, 85, 89, 91, 95, 97, 98, 104, 106, 110, 112, 116, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 174, 176, 177, 179, 181, 182, 184, 186, 187, 189, 191, 192, 194, 196, 197, 199, 201, 202, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227 }

B grade: { 39, 41, 157, 209 }

C grade: { 74, 75, 76, 78, 80, 81, 82, 84, 86, 87, 88, 90, 92, 93, 94, 96, 99, 100, 101, 102, 103, 105, 107, 108, 109, 111, 113, 114, 115, 117, 121, 173, 175, 178, 180, 183, 185, 188, 190, 193, 195, 198, 200, 203, 204, 205, 206, 207, 208 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 123, 124, 125, 126, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 179, 181, 182, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 201, 202, 203, 204, 205, 206, 207, 208, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227 }

B grade: { 156, 178, 180, 183, 184, 185, 198, 199, 200 }

C grade: { 130, 132, 133 }

F grade: { 28, 39, 121, 122, 131, 157, 209, 210, 211, 212, 213, 214 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 14, 16, 19, 21, 22, 24, 26, 33, 35, 37, 49, 50, 58, 65, 66, 72, 73, 123, 140, 141, 142, 143, 145, 146, 147, 148, 150, 153, 155, 161, 162, 171, 172, 176, 177, 181, 182, 186, 187, 191, 192, 196, 197, 201, 202, 220, 221, 222, 223, 224, 225, 226, 227

B grade: { }

C grade: { }

F grade: { 6, 13, 15, 17, 18, 20, 23, 25, 27, 28, 29, 30, 31, 32, 34, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 144, 149, 151, 152, 154, 156, 157, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 173, 174, 175, 178, 179, 180, 183, 184, 185, 188, 189, 190, 193, 194, 195, 198, 199, 200, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 8, 10, 11, 12, 13, 14, 15, 16, 19, 21, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 49, 50, 58, 59, 65, 66, 72, 73, 119, 120, 123, 124, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 146, 148, 149, 150, 153, 154, 155, 161, 162, 166, 167, 171, 172, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227 }

B grade: { 7, 9, 145, 147 }

C grade: { }

F grade: { 6, 17, 18, 20, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 121, 122, 125, 126, 127, 128, 130, 131, 132, 133, 144, 151, 152, 156, 157, 158, 159, 160, 163, 164, 165, 168, 169, 170, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 12, 13, 14, 15, 16, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 49, 50, 58, 59, 65, 66, 72, 73, 79, 85, 97, 98, 106, 112, 119, 120, 123, 124, 126, 127, 128, 129, 134, 135, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 153, 154, 155, 161, 162, 166, 167, 171, 172, 176, 177, 181, 182, 191, 192, 196, 197, 201, 204, 205, 216, 220, 221, 222, 223, 224, 225, 226, 227 }

B grade: { }

C grade: { 7, 8, 9, 10, 11 }

F grade: { 6, 17, 18, 19, 20, 21, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 118, 121, 122, 125, 130, 131, 132, 133, 136, 144, 151, 152, 156, 157, 158, 159, 160, 163, 164, 165, 168, 169, 170, 173, 174, 175, 178, 179, 180, 183, 184, 185, 186, 187, 188, 189, 190, 193, 194, 195, 198, 199, 200, 202, 203, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 7, 9, 11, 12, 13, 14, 15, 16, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 79, 83, 85, 89, 91, 97, 98, 106, 112, 118, 119, 120, 123, 124, 125, 126, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 148, 149, 150, 153, 155, 158, 159, 160, 161, 162, 166, 167, 171, 172, 174, 176, 177, 181, 182, 186, 187, 188, 189, 190, 191, 192, 196, 197, 201, 202, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227 }

B grade: { 8, 10, 19, 21, 74, 75, 76, 77, 78, 80, 81, 82, 84, 86, 87, 88, 90, 92, 93, 94, 95, 96, 145, 146, 147, 154, 163, 164, 165, 168, 169, 170, 173, 175, 178, 179, 180, 183, 184, 185 }

C grade: { }

F grade: { 6, 17, 18, 20, 27, 28, 29, 30, 31, 38, 39, 40, 41, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 121, 122, 130, 131, 132, 133, 144, 151, 152, 156, 157, 193, 194, 195, 198, 199, 200, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	51	72	96	113	75	90
normalized size	1	1.	0.68	0.96	1.28	1.51	1.	1.2
time (sec)	N/A	0.048	0.031	0.003	1.486	2.012	2.185	1.147

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	54	60	99	109	66	77
normalized size	1	1.	0.78	0.87	1.43	1.58	0.96	1.12
time (sec)	N/A	0.031	0.032	0.004	1.462	1.958	1.087	1.177

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	42	52	68	92	53	63
normalized size	1	1.	0.78	0.96	1.26	1.7	0.98	1.17
time (sec)	N/A	0.035	0.025	0.003	1.457	1.896	0.587	1.163

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	42	40	70	88	42	50
normalized size	1	1.	0.93	0.89	1.56	1.96	0.93	1.11
time (sec)	N/A	0.016	0.016	0.002	1.456	1.966	0.241	1.138

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	35	57	24	35
normalized size	1	1.	1.	1.04	1.35	2.19	0.92	1.35
time (sec)	N/A	0.008	0.007	0.002	1.441	1.916	0.15	1.16

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	68	0	0	0	0
normalized size	1	1.	1.	1.33	0.	0.	0.	0.
time (sec)	N/A	0.057	0.017	0.063	0.	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	34	29	51	203	34	65
normalized size	1	1.	1.26	1.07	1.89	7.52	1.26	2.41
time (sec)	N/A	0.022	0.011	0.003	1.455	2.478	1.615	1.132

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	31	38	38	65	53	92
normalized size	1	1.	0.91	1.12	1.12	1.91	1.56	2.71
time (sec)	N/A	0.016	0.014	0.003	1.463	2.333	1.388	1.114

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	67	53	81	259	110	95
normalized size	1	1.	1.2	0.95	1.45	4.62	1.96	1.7
time (sec)	N/A	0.035	0.024	0.004	1.453	2.491	3.007	1.138

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	41	58	68	88	102	176
normalized size	1	1.	0.71	1.	1.17	1.52	1.76	3.03
time (sec)	N/A	0.022	0.022	0.003	1.439	2.029	2.673	1.136

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	72	73	111	289	184	120
normalized size	1	1.	0.9	0.91	1.39	3.61	2.3	1.5
time (sec)	N/A	0.047	0.062	0.003	1.451	2.197	6.438	1.157

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	82	76	138	189	121	135
normalized size	1	1.	0.68	0.63	1.15	1.58	1.01	1.12
time (sec)	N/A	0.196	0.056	0.05	1.496	2.002	4.149	1.173

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	74	93	0	162	97	117
normalized size	1	1.	0.76	0.95	0.	1.65	0.99	1.19
time (sec)	N/A	0.17	0.038	0.054	0.	1.954	2.381	1.18

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	63	59	97	143	83	92
normalized size	1	1.	0.77	0.72	1.18	1.74	1.01	1.12
time (sec)	N/A	0.126	0.046	0.048	1.466	1.976	1.16	1.166

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	57	63	0	123	58	74
normalized size	1	1.	0.95	1.05	0.	2.05	0.97	1.23
time (sec)	N/A	0.097	0.025	0.048	0.	1.927	0.59	1.137

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	37	45	89	37	45
normalized size	1	1.	1.	1.06	1.29	2.54	1.06	1.29
time (sec)	N/A	0.047	0.016	0.046	1.434	1.874	0.239	1.139

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	101	0	0	0	0
normalized size	1	1.	1.	1.38	0.	0.	0.	0.
time (sec)	N/A	0.093	0.02	0.054	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	74	74	98	135	0	0	0	0
normalized size	1	1.	1.32	1.82	0.	0.	0.	0.
time (sec)	N/A	0.107	0.132	0.105	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	42	53	112	0	117
normalized size	1	1.	1.	0.98	1.23	2.6	0.	2.72
time (sec)	N/A	0.078	0.028	0.051	1.524	2.396	0.	1.19

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	152	173	0	0	0	0
normalized size	1	1.	1.23	1.4	0.	0.	0.	0.
time (sec)	N/A	0.177	0.571	0.19	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	69	76	100	149	0	223
normalized size	1	1.	0.79	0.87	1.15	1.71	0.	2.56
time (sec)	N/A	0.152	0.041	0.053	1.496	2.382	0.	1.318

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	122	159	231	265	202	236
normalized size	1	1.	0.61	0.79	1.15	1.32	1.	1.17
time (sec)	N/A	0.403	0.068	0.056	1.501	2.373	7.922	1.144

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	115	151	0	234	167	190
normalized size	1	1.	0.69	0.9	0.	1.4	1.	1.14
time (sec)	N/A	0.315	0.076	0.056	0.	2.432	4.587	1.219

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	95	106	162	189	134	158
normalized size	1	1.	0.7	0.78	1.19	1.39	0.99	1.16
time (sec)	N/A	0.241	0.05	0.051	1.514	2.256	2.458	1.156

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	85	90	0	170	99	112
normalized size	1	1.	0.86	0.91	0.	1.72	1.	1.13
time (sec)	N/A	0.156	0.041	0.05	0.	2.426	1.196	1.141

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	57	80	116	60	76
normalized size	1	1.	1.	0.95	1.33	1.93	1.	1.27
time (sec)	N/A	0.081	0.019	0.049	1.449	2.268	0.573	1.119

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	101	135	0	0	0	0
normalized size	1	1.	1.	1.34	0.	0.	0.	0.
time (sec)	N/A	0.111	0.02	0.07	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	122	139	0	0	0	0	0
normalized size	1	1.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.175	0.11	0.207	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	92	113	0	0	0	0
normalized size	1	1.	0.9	1.11	0.	0.	0.	0.
time (sec)	N/A	0.175	0.192	0.139	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	165	272	0	0	0	0
normalized size	1	1.	0.86	1.42	0.	0.	0.	0.
time (sec)	N/A	0.304	0.739	0.175	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	149	176	0	0	0	0
normalized size	1	1.	0.88	1.04	0.	0.	0.	0.
time (sec)	N/A	0.29	0.471	0.188	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	167	318	0	383	275	331
normalized size	1	1.	0.59	1.13	0.	1.36	0.98	1.17
time (sec)	N/A	0.874	0.088	0.09	0.	2.456	24.165	1.188

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	150	197	278	352	248	286
normalized size	1	1.	0.6	0.79	1.11	1.41	0.99	1.14
time (sec)	N/A	0.669	0.084	0.058	1.525	2.337	13.673	1.169

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	135	207	0	292	197	234
normalized size	1	1.	0.68	1.05	0.	1.47	0.99	1.18
time (sec)	N/A	0.52	0.067	0.06	0.	2.329	8.284	1.205

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	114	130	197	247	165	189
normalized size	1	1.	0.69	0.78	1.19	1.49	0.99	1.14
time (sec)	N/A	0.366	0.072	0.052	1.529	2.427	4.373	1.178

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	96	113	0	205	110	136
normalized size	1	1.	0.86	1.01	0.	1.83	0.98	1.21
time (sec)	N/A	0.237	0.038	0.053	0.	2.328	2.343	1.161

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	67	100	149	70	88
normalized size	1	1.	1.	0.97	1.45	2.16	1.01	1.28
time (sec)	N/A	0.12	0.022	0.047	1.485	2.382	1.104	1.142

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	119	168	0	0	0	0
normalized size	1	1.	1.	1.41	0.	0.	0.	0.
time (sec)	N/A	0.125	0.021	0.069	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	176	176	549	0	0	0	0	0
normalized size	1	1.	3.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.204	1.081	0.206	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	115	149	0	0	0	0
normalized size	1	1.	0.95	1.23	0.	0.	0.	0.
time (sec)	N/A	0.213	0.358	0.124	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	304	304	1475	451	0	0	0	0
normalized size	1	1.	4.85	1.48	0.	0.	0.	0.
time (sec)	N/A	0.421	12.067	0.184	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	40	40	0	0	0	63
normalized size	1	1.	0.73	0.73	0.	0.	0.	1.15
time (sec)	N/A	0.091	0.103	0.058	0.	0.	0.	1.159

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	33	33	0	0	0	50
normalized size	1	1.	0.77	0.77	0.	0.	0.	1.16
time (sec)	N/A	0.079	0.087	0.055	0.	0.	0.	1.156

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	31	31	0	0	0	47
normalized size	1	1.	0.76	0.76	0.	0.	0.	1.15
time (sec)	N/A	0.075	0.074	0.047	0.	0.	0.	1.122

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	24	24	0	0	0	34
normalized size	1	1.	0.83	0.83	0.	0.	0.	1.17
time (sec)	N/A	0.064	0.062	0.046	0.	0.	0.	1.177

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	20	22	0	0	0	31
normalized size	1	1.	0.74	0.81	0.	0.	0.	1.15
time (sec)	N/A	0.06	0.053	0.045	0.	0.	0.	1.172

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	0	0	0	16
normalized size	1	1.	1.	0.93	0.	0.	0.	1.14
time (sec)	N/A	0.035	0.019	0.043	0.	0.	0.	1.161

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	0	0	0	14
normalized size	1	1.	1.	1.1	0.	0.	0.	1.4
time (sec)	N/A	0.015	0.024	0.041	0.	0.	0.	1.118

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.23	0.145	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.855	0.188	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	86	105	0	0	0	97
normalized size	1	1.	1.05	1.28	0.	0.	0.	1.18
time (sec)	N/A	0.079	0.158	0.062	0.	0.	0.	1.186

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	63	78	0	0	0	84
normalized size	1	1.	0.9	1.11	0.	0.	0.	1.2
time (sec)	N/A	0.064	0.163	0.056	0.	0.	0.	1.168

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	61	81	0	0	0	81
normalized size	1	1.	0.9	1.19	0.	0.	0.	1.19
time (sec)	N/A	0.063	0.16	0.05	0.	0.	0.	1.175

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	50	54	0	0	0	68
normalized size	1	1.	0.89	0.96	0.	0.	0.	1.21
time (sec)	N/A	0.05	0.139	0.045	0.	0.	0.	1.188

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	50	57	0	0	0	65
normalized size	1	1.	0.93	1.06	0.	0.	0.	1.2
time (sec)	N/A	0.046	0.123	0.047	0.	0.	0.	1.176

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	37	30	0	0	0	49
normalized size	1	1.	0.97	0.79	0.	0.	0.	1.29
time (sec)	N/A	0.024	0.085	0.045	0.	0.	0.	1.219

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	0	0	0	45
normalized size	1	1.	1.	0.91	0.	0.	0.	1.29
time (sec)	N/A	0.081	0.041	0.046	0.	0.	0.	1.142

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.96	0.146	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	16.643	0.206	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	103	121	0	0	0	116
normalized size	1	1.	1.05	1.23	0.	0.	0.	1.18
time (sec)	N/A	0.344	0.13	0.059	0.	0.	0.	1.231

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	70	82	0	0	0	101
normalized size	1	1.	0.84	0.99	0.	0.	0.	1.22
time (sec)	N/A	0.305	0.152	0.055	0.	0.	0.	1.201

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	65	82	0	0	0	97
normalized size	1	1.	0.79	1.	0.	0.	0.	1.18
time (sec)	N/A	0.243	0.132	0.051	0.	0.	0.	1.219

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	43	0	0	0	77
normalized size	1	1.	1.	0.68	0.	0.	0.	1.22
time (sec)	N/A	0.165	0.039	0.044	0.	0.	0.	1.175

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	47	43	0	0	0	58
normalized size	1	1.	0.92	0.84	0.	0.	0.	1.14
time (sec)	N/A	0.083	0.031	0.045	0.	0.	0.	1.158

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.708	0.149	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	9.133	0.207	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	159	171	0	0	0	186
normalized size	1	1.	1.01	1.08	0.	0.	0.	1.18
time (sec)	N/A	0.338	0.17	0.059	0.	0.	0.	1.237

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	107	114	0	0	0	169
normalized size	1	1.	0.75	0.8	0.	0.	0.	1.18
time (sec)	N/A	0.291	0.255	0.056	0.	0.	0.	1.202

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	129	117	0	0	0	163
normalized size	1	1.	0.91	0.83	0.	0.	0.	1.16
time (sec)	N/A	0.312	0.12	0.056	0.	0.	0.	1.193

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	86	60	0	0	0	112
normalized size	1	1.	0.89	0.62	0.	0.	0.	1.15
time (sec)	N/A	0.166	0.103	0.046	0.	0.	0.	1.156

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	71	63	0	0	0	89
normalized size	1	1.	0.91	0.81	0.	0.	0.	1.14
time (sec)	N/A	0.156	0.044	0.045	0.	0.	0.	1.152

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	2.79	0.153	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	27.256	0.214	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	212	143	0	0	0	425
normalized size	1	1.	1.75	1.18	0.	0.	0.	3.51
time (sec)	N/A	0.281	0.266	0.096	0.	0.	0.	1.329

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	125	91	0	0	0	255
normalized size	1	1.	1.32	0.96	0.	0.	0.	2.68
time (sec)	N/A	0.195	0.11	0.08	0.	0.	0.	1.288

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	122	96	0	0	0	284
normalized size	1	1.	1.42	1.12	0.	0.	0.	3.3
time (sec)	N/A	0.184	0.185	0.079	0.	0.	0.	1.274

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	49	42	0	0	0	120
normalized size	1	1.	0.83	0.71	0.	0.	0.	2.03
time (sec)	N/A	0.151	0.036	0.066	0.	0.	0.	1.248

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	44	44	76	49	0	0	0	142
normalized size	1	1.	1.73	1.11	0.	0.	0.	3.23
time (sec)	N/A	0.09	0.034	0.065	0.	0.	0.	1.245

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.388	0.096	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	282	282	185	193	0	0	0	586
normalized size	1	1.	0.66	0.68	0.	0.	0.	2.08
time (sec)	N/A	0.515	0.124	0.104	0.	0.	0.	1.348

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	128	121	0	0	0	363
normalized size	1	1.	0.82	0.77	0.	0.	0.	2.31
time (sec)	N/A	0.368	0.072	0.089	0.	0.	0.	1.327

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	125	130	0	0	0	390
normalized size	1	1.	0.85	0.88	0.	0.	0.	2.65
time (sec)	N/A	0.302	0.086	0.087	0.	0.	0.	1.297

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	64	64	0	0	0	174
normalized size	1	1.	0.72	0.72	0.	0.	0.	1.96
time (sec)	N/A	0.179	0.066	0.076	0.	0.	0.	1.281

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	66	72	0	0	0	194
normalized size	1	1.	0.88	0.96	0.	0.	0.	2.59
time (sec)	N/A	0.101	0.026	0.071	0.	0.	0.	1.295

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.399	0.095	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	298	298	212	233	0	0	0	738
normalized size	1	1.	0.71	0.78	0.	0.	0.	2.48
time (sec)	N/A	0.79	0.192	0.109	0.	0.	0.	1.453

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	205	205	140	154	0	0	0	466
normalized size	1	1.	0.68	0.75	0.	0.	0.	2.27
time (sec)	N/A	0.589	0.127	0.094	0.	0.	0.	1.323

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	178	178	122	156	0	0	0	491
normalized size	1	1.	0.69	0.88	0.	0.	0.	2.76
time (sec)	N/A	0.457	0.119	0.09	0.	0.	0.	1.315

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	73	79	0	0	0	225
normalized size	1	1.	0.61	0.66	0.	0.	0.	1.89
time (sec)	N/A	0.287	0.089	0.078	0.	0.	0.	1.234

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	76	88	0	0	0	244
normalized size	1	1.	0.86	1.	0.	0.	0.	2.77
time (sec)	N/A	0.16	0.037	0.073	0.	0.	0.	1.32

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.391	0.097	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	106	192	72	0	0	0	273
normalized size	1	1.	1.81	0.68	0.	0.	0.	2.58
time (sec)	N/A	0.105	0.1	0.078	0.	0.	0.	1.411

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	130	43	0	0	0	153
normalized size	1	1.	2.	0.66	0.	0.	0.	2.35
time (sec)	N/A	0.078	0.068	0.07	0.	0.	0.	1.358

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	71	71	126	50	0	0	0	182
normalized size	1	1.	1.77	0.7	0.	0.	0.	2.56
time (sec)	N/A	0.079	0.082	0.069	0.	0.	0.	1.351

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	21	0	0	0	69
normalized size	1	1.	1.	0.75	0.	0.	0.	2.46
time (sec)	N/A	0.04	0.023	0.054	0.	0.	0.	1.312

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	31	31	68	26	0	0	0	92
normalized size	1	1.	2.19	0.84	0.	0.	0.	2.97
time (sec)	N/A	0.023	0.024	0.049	0.	0.	0.	1.246

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.309	0.08	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	3.469	0.154	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	171	171	306	182	0	0	0	0
normalized size	1	1.	1.79	1.06	0.	0.	0.	0.
time (sec)	N/A	0.146	0.287	0.109	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	226	121	0	0	0	0
normalized size	1	1.	1.78	0.95	0.	0.	0.	0.
time (sec)	N/A	0.104	0.483	0.092	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	136	136	233	140	0	0	0	0
normalized size	1	1.	1.71	1.03	0.	0.	0.	0.
time (sec)	N/A	0.095	0.2	0.089	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	154	81	0	0	0	0
normalized size	1	1.	1.69	0.89	0.	0.	0.	0.
time (sec)	N/A	0.065	0.396	0.074	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	159	96	0	0	0	0
normalized size	1	1.	1.64	0.99	0.	0.	0.	0.
time (sec)	N/A	0.069	0.109	0.079	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	44	42	0	0	0	0
normalized size	1	1.	0.8	0.76	0.	0.	0.	0.
time (sec)	N/A	0.031	0.048	0.066	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	86	66	0	0	0	0
normalized size	1	1.	1.46	1.12	0.	0.	0.	0.
time (sec)	N/A	0.087	0.035	0.07	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.446	0.095	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	235	235	322	173	0	0	0	0
normalized size	1	1.	1.37	0.74	0.	0.	0.	0.
time (sec)	N/A	0.41	1.634	0.102	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	126	126	203	107	0	0	0	0
normalized size	1	1.	1.61	0.85	0.	0.	0.	0.
time (sec)	N/A	0.325	0.891	0.085	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	125	125	220	115	0	0	0	0
normalized size	1	1.	1.76	0.92	0.	0.	0.	0.
time (sec)	N/A	0.286	0.821	0.085	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	61	56	0	0	0	0
normalized size	1	1.	0.69	0.63	0.	0.	0.	0.
time (sec)	N/A	0.173	0.083	0.069	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	122	83	0	0	0	0
normalized size	1	1.	1.61	1.09	0.	0.	0.	0.
time (sec)	N/A	0.093	0.239	0.072	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.466	0.098	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	264	264	418	225	0	0	0	0
normalized size	1	1.	1.58	0.85	0.	0.	0.	0.
time (sec)	N/A	0.383	7.733	0.11	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	190	190	264	139	0	0	0	0
normalized size	1	1.	1.39	0.73	0.	0.	0.	0.
time (sec)	N/A	0.323	4.24	0.091	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	191	191	281	154	0	0	0	0
normalized size	1	1.	1.47	0.81	0.	0.	0.	0.
time (sec)	N/A	0.341	2.726	0.091	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	75	73	0	0	0	0
normalized size	1	1.	0.63	0.61	0.	0.	0.	0.
time (sec)	N/A	0.172	0.099	0.075	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	151	110	0	0	0	0
normalized size	1	1.	1.44	1.05	0.	0.	0.	0.
time (sec)	N/A	0.166	1.045	0.075	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.461	0.102	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	64	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	1.179	0.842	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	64	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	1.063	0.614	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	150	132	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	2.18	0.808	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	54	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.035	0.866	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.65	0.536	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.662	0.693	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	5.825	0.108	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	6.052	0.1	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	5.737	0.099	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	3.402	0.098	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.987	0.839	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	130	287	0	0	0	0
normalized size	1	1.	0.79	1.74	0.	0.	0.	0.
time (sec)	N/A	0.176	0.101	0.215	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	152	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.149	0.194	0.152	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	74	138	0	0	0	0
normalized size	1	1.	0.89	1.66	0.	0.	0.	0.
time (sec)	N/A	0.083	0.055	0.117	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	70	148	0	0	0	0
normalized size	1	1.	0.93	1.97	0.	0.	0.	0.
time (sec)	N/A	0.049	0.03	0.095	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.328	0.121	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.609	0.102	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	4.308	0.169	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	5.454	0.162	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	1.744	0.153	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	1.808	0.148	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	68	72	112	139	85	90
normalized size	1	1.	0.89	0.95	1.47	1.83	1.12	1.18
time (sec)	N/A	0.036	0.047	0.006	1.448	2.454	1.289	1.133

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	55	64	81	119	70	76
normalized size	1	1.	0.92	1.07	1.35	1.98	1.17	1.27
time (sec)	N/A	0.042	0.042	0.005	1.443	2.597	0.643	1.133

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	56	52	84	111	60	62
normalized size	1	1.	1.1	1.02	1.65	2.18	1.18	1.22
time (sec)	N/A	0.019	0.036	0.005	1.439	2.578	0.331	1.155

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	32	42	73	29	42
normalized size	1	1.	1.	1.03	1.35	2.35	0.94	1.35
time (sec)	N/A	0.019	0.012	0.002	1.436	2.495	0.154	1.123

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	58	77	0	0	0	0
normalized size	1	1.	0.92	1.22	0.	0.	0.	0.
time (sec)	N/A	0.072	0.018	0.06	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	43	41	63	221	41	468
normalized size	1	1.	1.34	1.28	1.97	6.91	1.28	14.62
time (sec)	N/A	0.028	0.012	0.006	1.424	2.743	1.955	1.52

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	44	50	50	86	63	664
normalized size	1	1.	1.13	1.28	1.28	2.21	1.62	17.03
time (sec)	N/A	0.019	0.021	0.005	1.456	2.848	1.563	1.181

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	79	65	93	281	121	2206
normalized size	1	1.	1.27	1.05	1.5	4.53	1.95	35.58
time (sec)	N/A	0.037	0.025	0.004	1.435	3.187	3.263	2.428

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	121	126	192	255	175	193
normalized size	1	1.	1.19	1.24	1.88	2.5	1.72	1.89
time (sec)	N/A	0.152	0.137	0.052	1.451	2.8	1.487	1.162

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	104	118	0	221	131	161
normalized size	1	1.	1.37	1.55	0.	2.91	1.72	2.12
time (sec)	N/A	0.118	0.111	0.051	0.	2.643	0.756	1.175

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	76	74	99	159	87	101
normalized size	1	1.	1.62	1.57	2.11	3.38	1.85	2.15
time (sec)	N/A	0.058	0.062	0.046	1.438	2.419	0.332	1.145

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	128	194	0	0	0	0
normalized size	1	1.	1.39	2.11	0.	0.	0.	0.
time (sec)	N/A	0.118	0.117	0.058	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	134	187	0	0	0	0
normalized size	1	1.	1.51	2.1	0.	0.	0.	0.
time (sec)	N/A	0.128	0.207	0.053	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	218	235	369	441	333	390
normalized size	1	1.	1.22	1.32	2.07	2.48	1.87	2.19
time (sec)	N/A	0.295	0.198	0.05	1.492	2.424	3.298	1.213

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	185	211	0	378	269	312
normalized size	1	1.	1.48	1.69	0.	3.02	2.15	2.5
time (sec)	N/A	0.208	0.168	0.052	0.	2.293	1.627	1.208

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	128	134	194	262	165	203
normalized size	1	1.	1.56	1.63	2.37	3.2	2.01	2.48
time (sec)	N/A	0.108	0.114	0.047	1.441	2.294	0.77	1.148

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	204	353	0	0	0	0
normalized size	1	1.	1.61	2.78	0.	0.	0.	0.
time (sec)	N/A	0.141	0.171	0.063	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	308	0	0	0	0	0
normalized size	1	1.	2.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.214	0.292	0.161	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	117	91	102	0	0	0	232
normalized size	1	0.97	0.75	0.84	0.	0.	0.	1.92
time (sec)	N/A	0.216	0.161	0.05	0.	0.	0.	1.207

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	56	58	0	0	0	116
normalized size	1	1.	0.89	0.92	0.	0.	0.	1.84
time (sec)	N/A	0.121	0.068	0.046	0.	0.	0.	1.173

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	46	49	0	0	0	68
normalized size	1	1.	0.85	0.91	0.	0.	0.	1.26
time (sec)	N/A	0.062	0.062	0.049	0.	0.	0.	1.147

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.302	0.263	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	3.59	0.18	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	151	124	147	0	0	0	830
normalized size	1	0.97	0.8	0.95	0.	0.	0.	5.35
time (sec)	N/A	0.183	0.565	0.056	0.	0.	0.	1.311

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	80	78	0	0	0	436
normalized size	1	1.	0.88	0.86	0.	0.	0.	4.79
time (sec)	N/A	0.097	0.258	0.052	0.	0.	0.	1.216

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	82	72	74	0	0	0	261
normalized size	1	0.95	0.84	0.86	0.	0.	0.	3.03
time (sec)	N/A	0.168	0.142	0.052	0.	0.	0.	1.17

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	6.415	0.247	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	51.281	0.198	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	246	169	291	0	0	0	1997
normalized size	1	1.25	0.86	1.48	0.	0.	0.	10.14
time (sec)	N/A	0.535	0.455	0.06	0.	0.	0.	1.45

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	107	157	0	0	0	1161
normalized size	1	1.	0.82	1.21	0.	0.	0.	8.93
time (sec)	N/A	0.311	0.278	0.053	0.	0.	0.	1.336

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	89	139	0	0	0	649
normalized size	1	1.	0.8	1.25	0.	0.	0.	5.85
time (sec)	N/A	0.166	0.233	0.058	0.	0.	0.	1.177

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	3.374	0.317	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	31.544	0.51	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	242	242	243	357	0	0	0	549
normalized size	1	1.	1.	1.48	0.	0.	0.	2.27
time (sec)	N/A	0.702	0.516	0.134	0.	0.	0.	2.007

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	123	173	0	0	0	243
normalized size	1	1.	0.9	1.26	0.	0.	0.	1.77
time (sec)	N/A	0.4	0.201	0.102	0.	0.	0.	1.576

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	120	179	0	0	0	274
normalized size	1	1.	0.99	1.48	0.	0.	0.	2.26
time (sec)	N/A	0.278	0.183	0.099	0.	0.	0.	1.407

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	3.589	0.209	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	9.94	0.274	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	313	313	589	541	0	0	0	1800
normalized size	1	1.	1.88	1.73	0.	0.	0.	5.75
time (sec)	N/A	0.949	9.595	0.148	0.	0.	0.	3.148

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	155	267	0	0	0	791
normalized size	1	1.	0.9	1.55	0.	0.	0.	4.6
time (sec)	N/A	0.461	0.825	0.114	0.	0.	0.	1.971

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	159	159	295	270	0	0	0	905
normalized size	1	1.	1.86	1.7	0.	0.	0.	5.69
time (sec)	N/A	0.237	2.371	0.11	0.	0.	0.	2.237

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	3.198	0.221	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	9.76	0.286	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	358	358	1002	792	0	0	0	3471
normalized size	1	1.	2.8	2.21	0.	0.	0.	9.7
time (sec)	N/A	1.316	16.464	0.171	0.	0.	0.	4.669

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	201	366	0	0	0	1638
normalized size	1	1.	0.93	1.69	0.	0.	0.	7.58
time (sec)	N/A	0.703	1.932	0.132	0.	0.	0.	2.401

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	179	383	393	0	0	0	1621
normalized size	1	1.	2.14	2.2	0.	0.	0.	9.06
time (sec)	N/A	0.428	4.106	0.123	0.	0.	0.	2.906

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	3.336	0.217	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	9.763	0.283	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	223	223	225	167	0	0	0	446
normalized size	1	1.	1.01	0.75	0.	0.	0.	2.
time (sec)	N/A	0.371	0.421	0.085	0.	0.	0.	2.086

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	91	80	0	0	0	188
normalized size	1	1.	0.92	0.81	0.	0.	0.	1.9
time (sec)	N/A	0.171	0.16	0.06	0.	0.	0.	1.821

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	118	85	0	0	0	220
normalized size	1	1.	1.16	0.83	0.	0.	0.	2.16
time (sec)	N/A	0.09	0.082	0.056	0.	0.	0.	1.691

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	3.261	0.199	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	11.961	0.253	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	252	252	273	295	0	0	0	0
normalized size	1	1.	1.08	1.17	0.	0.	0.	0.
time (sec)	N/A	0.388	0.499	0.118	0.	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	124	142	0	0	0	0
normalized size	1	1.	0.95	1.09	0.	0.	0.	0.
time (sec)	N/A	0.154	0.33	0.102	0.	0.	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	137	137	150	150	0	0	0	0
normalized size	1	1.	1.09	1.09	0.	0.	0.	0.
time (sec)	N/A	0.266	0.167	0.092	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	4.118	0.159	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	12.275	0.277	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	292	292	322	659	0	0	0	0
normalized size	1	1.	1.1	2.26	0.	0.	0.	0.
time (sec)	N/A	0.945	2.576	0.148	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	176	311	0	0	0	0
normalized size	1	1.	0.98	1.73	0.	0.	0.	0.
time (sec)	N/A	0.487	0.668	0.114	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	194	324	0	0	0	0
normalized size	1	1.	1.19	1.99	0.	0.	0.	0.
time (sec)	N/A	0.259	1.621	0.111	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	4.212	0.16	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	12.702	0.272	0.	0.	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	158	144	0	0	0	0
normalized size	1	1.	1.32	1.2	0.	0.	0.	0.
time (sec)	N/A	0.067	0.254	0.024	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	66	138	0	0	82	0
normalized size	1	1.	0.53	1.11	0.	0.	0.66	0.
time (sec)	N/A	0.091	0.067	0.007	0.	0.	117.363	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	113	119	0	0	76	0
normalized size	1	1.	1.28	1.35	0.	0.	0.86	0.
time (sec)	N/A	0.046	0.188	0.006	0.	0.	3.349	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	45	98	0	0	0	0
normalized size	1	1.	0.51	1.1	0.	0.	0.	0.
time (sec)	N/A	0.074	0.034	0.008	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	93	85	0	0	0	0
normalized size	1	1.	1.69	1.55	0.	0.	0.	0.
time (sec)	N/A	0.034	0.151	0.006	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	68	129	0	0	0	0
normalized size	1	1.	0.54	1.03	0.	0.	0.	0.
time (sec)	N/A	0.092	0.067	0.007	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	234	0	0	0	0	0
normalized size	1	1.	2.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.139	1.261	0.358	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	176	0	0	0	0	0
normalized size	1	1.	1.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.14	7.42	0.335	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	202	0	0	0	0	0
normalized size	1	1.	1.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.134	0.532	0.352	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	142	0	0	0	0	0
normalized size	1	1.	1.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.125	1.272	0.518	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	129	0	0	0	0	0
normalized size	1	1.	1.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.13	0.433	0.339	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	198	0	0	0	0	0
normalized size	1	1.	1.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.142	0.786	0.342	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	68	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.164	30.752	0.335	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	66	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.167	135.147	0.349	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	64	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.147	10.427	0.356	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	64	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.158	7.313	0.334	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	66	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.167	11.473	0.341	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	3.473	0.238	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	2.271	0.245	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.854	0.24	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	2.946	0.229	0.	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	6.461	0.228	0.	0.	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	4.953	0.246	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	9.581	0.244	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	13.925	0.233	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [30] had the largest ratio of [1.]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.	8	0.375
2	A	4	3	1.	8	0.375
3	A	4	3	1.	8	0.375
4	A	3	3	1.	6	0.5
5	A	2	2	1.	4	0.5
6	A	5	5	1.	8	0.625
7	A	4	4	1.	8	0.5
8	A	2	2	1.	8	0.25
9	A	5	5	1.	8	0.625
10	A	3	3	1.	8	0.375
11	A	6	5	1.	8	0.625
12	A	7	5	1.	10	0.5
13	A	6	4	1.	10	0.4
14	A	5	5	1.	10	0.5
15	A	4	4	1.	8	0.5
16	A	3	3	1.	6	0.5
17	A	6	6	1.	10	0.6
18	A	7	5	1.	10	0.5
19	A	3	3	1.	10	0.3
20	A	9	7	1.	10	0.7
21	A	5	5	1.	10	0.5
22	A	14	7	1.	10	0.7
23	A	11	5	1.	10	0.5
24	A	9	7	1.	10	0.7
25	A	6	5	1.	8	0.625
26	A	4	3	1.	6	0.5
27	A	7	7	1.	10	0.7
28	A	9	6	1.	10	0.6
29	A	7	7	1.	10	0.7
30	A	14	10	1.	10	1.
31	A	10	9	1.	10	0.9

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
32	A	23	4	1.	10	0.4
33	A	19	6	1.	10	0.6
34	A	14	4	1.	10	0.4
35	A	11	6	1.	10	0.6
36	A	7	4	1.	8	0.5
37	A	5	3	1.	6	0.5
38	A	8	7	1.	10	0.7
39	A	11	7	1.	10	0.7
40	A	8	8	1.	10	0.8
41	A	19	10	1.	10	1.
42	A	7	3	1.	10	0.3
43	A	6	3	1.	10	0.3
44	A	6	3	1.	10	0.3
45	A	5	3	1.	10	0.3
46	A	5	3	1.	10	0.3
47	A	4	4	1.	8	0.5
48	A	2	2	1.	6	0.333
49	A	0	0	0.	0	0.
50	A	0	0	0.	0	0.
51	A	6	2	1.	10	0.2
52	A	5	2	1.	10	0.2
53	A	5	2	1.	10	0.2
54	A	4	2	1.	10	0.2
55	A	4	2	1.	10	0.2
56	A	2	2	1.	8	0.25
57	A	3	3	1.	6	0.5
58	A	0	0	0.	0	0.
59	A	0	0	0.	0	0.
60	A	14	5	1.	10	0.5
61	A	12	6	1.	10	0.6
62	A	10	6	1.	10	0.6
63	A	7	7	1.	8	0.875

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	4	4	1.	6	0.667
65	A	0	0	0.	0	0.
66	A	0	0	0.	0	0.
67	A	12	4	1.	10	0.4
68	A	9	4	1.	10	0.4
69	A	10	6	1.	10	0.6
70	A	5	5	1.	8	0.625
71	A	5	4	1.	6	0.667
72	A	0	0	0.	0	0.
73	A	0	0	0.	0	0.
74	A	10	5	1.	12	0.417
75	A	8	5	1.	12	0.417
76	A	8	5	1.	12	0.417
77	A	6	5	1.	10	0.5
78	A	4	4	1.	8	0.5
79	A	0	0	0.	0	0.
80	A	23	8	1.	12	0.667
81	A	16	8	1.	12	0.667
82	A	13	8	1.	12	0.667
83	A	8	8	1.	10	0.8
84	A	5	5	1.	8	0.625
85	A	0	0	0.	0	0.
86	A	26	8	1.	12	0.667
87	A	18	7	1.	12	0.583
88	A	15	8	1.	12	0.667
89	A	9	7	1.	10	0.7
90	A	6	5	1.	8	0.625
91	A	0	0	0.	0	0.
92	A	9	4	1.	12	0.333
93	A	7	4	1.	12	0.333
94	A	7	4	1.	12	0.333
95	A	5	5	1.	10	0.5

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	3	1.	8	0.375
97	A	0	0	0.	0	0.
98	A	0	0	0.	0	0.
99	A	10	3	1.	12	0.25
100	A	8	3	1.	12	0.25
101	A	8	3	1.	12	0.25
102	A	6	3	1.	12	0.25
103	A	6	3	1.	12	0.25
104	A	3	3	1.	10	0.3
105	A	4	4	1.	8	0.5
106	A	0	0	0.	0	0.
107	A	19	6	1.	12	0.5
108	A	15	7	1.	12	0.583
109	A	13	7	1.	12	0.583
110	A	8	8	1.	10	0.8
111	A	5	5	1.	8	0.625
112	A	0	0	0.	0	0.
113	A	17	5	1.	12	0.417
114	A	12	5	1.	12	0.417
115	A	13	7	1.	12	0.583
116	A	6	6	1.	10	0.6
117	A	6	5	1.	8	0.625
118	A	0	0	0.	0	0.
119	A	0	0	0.	0	0.
120	A	0	0	0.	0	0.
121	A	2	2	1.	12	0.167
122	A	2	2	1.	10	0.2
123	A	0	0	0.	0	0.
124	A	0	0	0.	0	0.
125	A	0	0	0.	0	0.
126	A	0	0	0.	0	0.
127	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
128	A	0	0	0.	0	0.
129	A	0	0	0.	0	0.
130	A	9	4	1.	10	0.4
131	A	9	4	1.	10	0.4
132	A	6	5	1.	8	0.625
133	A	4	3	1.	6	0.5
134	A	0	0	0.	0	0.
135	A	0	0	0.	0	0.
136	A	0	0	0.	0	0.
137	A	0	0	0.	0	0.
138	A	0	0	0.	0	0.
139	A	0	0	0.	0	0.
140	A	4	3	1.	12	0.25
141	A	4	3	1.	12	0.25
142	A	3	3	1.	10	0.3
143	A	3	2	1.	8	0.25
144	A	5	5	1.	12	0.417
145	A	4	4	1.	12	0.333
146	A	2	2	1.	12	0.167
147	A	5	5	1.	12	0.417
148	A	5	5	1.	14	0.357
149	A	4	4	1.	12	0.333
150	A	3	3	1.	10	0.3
151	A	6	6	1.	14	0.429
152	A	7	5	1.	14	0.357
153	A	10	7	1.	14	0.5
154	A	6	5	1.	12	0.417
155	A	5	3	1.	10	0.3
156	A	7	7	1.	14	0.5
157	A	9	6	1.	14	0.429
158	A	9	5	0.97	14	0.357
159	A	6	6	1.	12	0.5

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
160	A	4	4	1.	10	0.4
161	A	0	0	0.	0	0.
162	A	0	0	0.	0	0.
163	A	8	4	0.97	14	0.286
164	A	4	4	1.	12	0.333
165	A	5	5	0.95	10	0.5
166	A	0	0	0.	0	0.
167	A	0	0	0.	0	0.
168	A	16	8	1.25	14	0.571
169	A	9	9	1.	12	0.75
170	A	6	6	1.	10	0.6
171	A	0	0	0.	0	0.
172	A	0	0	0.	0	0.
173	A	14	8	1.	16	0.5
174	A	9	8	1.	14	0.571
175	A	7	7	1.	12	0.583
176	A	0	0	0.	0	0.
177	A	0	0	0.	0	0.
178	A	22	11	1.	16	0.688
179	A	11	11	1.	14	0.786
180	A	8	8	1.	12	0.667
181	A	0	0	0.	0	0.
182	A	0	0	0.	0	0.
183	A	24	11	1.	16	0.688
184	A	12	10	1.	14	0.714
185	A	9	8	1.	12	0.667
186	A	0	0	0.	0	0.
187	A	0	0	0.	0	0.
188	A	13	7	1.	16	0.438
189	A	8	8	1.	14	0.571
190	A	6	6	1.	12	0.5
191	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
192	A	0	0	0.	0	0.
193	A	12	6	1.	16	0.375
194	A	6	6	1.	14	0.429
195	A	7	7	1.	12	0.583
196	A	0	0	0.	0	0.
197	A	0	0	0.	0	0.
198	A	22	10	1.	16	0.625
199	A	11	11	1.	14	0.786
200	A	8	8	1.	12	0.667
201	A	0	0	0.	0	0.
202	A	0	0	0.	0	0.
203	A	5	4	1.	16	0.25
204	A	7	7	1.	16	0.438
205	A	4	4	1.	16	0.25
206	A	6	6	1.	16	0.375
207	A	3	3	1.	16	0.188
208	A	7	7	1.	16	0.438
209	A	2	2	1.	18	0.111
210	A	2	2	1.	18	0.111
211	A	2	2	1.	18	0.111
212	A	2	2	1.	18	0.111
213	A	2	2	1.	18	0.111
214	A	2	2	1.	18	0.111
215	A	0	0	0.	0	0.
216	A	0	0	0.	0	0.
217	A	0	0	0.	0	0.
218	A	0	0	0.	0	0.
219	A	0	0	0.	0	0.
220	A	0	0	0.	0	0.
221	A	0	0	0.	0	0.
222	A	0	0	0.	0	0.
223	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
224	A	0	0	0.	0	0.
225	A	0	0	0.	0	0.
226	A	0	0	0.	0	0.
227	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int x^4 \cos^{-1}(ax)\,dx$

Optimal. Leaf size=75

$$-\frac{\left(1-a^2x^2\right)^{5/2}}{25a^5}+\frac{2\left(1-a^2x^2\right)^{3/2}}{15a^5}-\frac{\sqrt{1-a^2x^2}}{5a^5}+\frac{1}{5}x^5\cos^{-1}(ax)$$

```
[Out] -Sqrt[1 - a^2*x^2]/(5*a^5) + (2*(1 - a^2*x^2)^(3/2))/(15*a^5) - (1 - a^2*x^
2)^(5/2)/(25*a^5) + (x^5*ArcCos[a*x])/5
```

Rubi [A] time = 0.0477089, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4628, 266, 43}

$$-\frac{\left(1-a^2x^2\right)^{5/2}}{25a^5}+\frac{2\left(1-a^2x^2\right)^{3/2}}{15a^5}-\frac{\sqrt{1-a^2x^2}}{5a^5}+\frac{1}{5}x^5\cos^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCos[a*x],x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(5*a^5) + (2*(1 - a^2*x^2)^(3/2))/(15*a^5) - (1 - a^2*x^
2)^(5/2)/(25*a^5) + (x^5*ArcCos[a*x])/5
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^4 \cos^{-1}(ax)\,dx &= \frac{1}{5}x^5\cos^{-1}(ax) + \frac{1}{5}a\int \frac{x^5}{\sqrt{1-a^2x^2}}\,dx \\
&= \frac{1}{5}x^5\cos^{-1}(ax) + \frac{1}{10}a\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= \frac{1}{5}x^5\cos^{-1}(ax) + \frac{1}{10}a\,\text{Subst}\left(\int \left(\frac{1}{a^4\sqrt{1-a^2x}} - \frac{2\sqrt{1-a^2x}}{a^4} + \frac{\left(1-a^2x\right)^{3/2}}{a^4}\right)dx, x, x^2\right) \\
&= -\frac{\sqrt{1-a^2x^2}}{5a^5} + \frac{2\left(1-a^2x^2\right)^{3/2}}{15a^5} - \frac{\left(1-a^2x^2\right)^{5/2}}{25a^5} + \frac{1}{5}x^5\cos^{-1}(ax)
\end{aligned}
$$

Mathematica [A] time = 0.0309678, size = 51, normalized size = 0.68

$$
\frac{1}{5}x^5\cos^{-1}(ax) - \frac{\sqrt{1-a^2x^2}\left(3a^4x^4+4a^2x^2+8\right)}{75a^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcCos[a*x],x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]*(8 + 4*a^2*x^2 + 3*a^4*x^4))/(75*a^5) + (x^5*ArcCos[a*x
])/5
```

Maple [A] time = 0.003, size = 72, normalized size = 1.

$$\frac{1}{a^5}\left(\frac{a^5x^5\arccos(ax)}{5}-\frac{a^4x^4}{25}\sqrt{-a^2x^2+1}-\frac{4\,a^2x^2}{75}\sqrt{-a^2x^2+1}-\frac{8}{75}\sqrt{-a^2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccos(a*x),x)
```

```
[Out] 1/a^5*(1/5*a^5*x^5*arccos(a*x)-1/25*a^4*x^4*(-a^2*x^2+1)^(1/2)-4/75*a^2*x^2
*(-a^2*x^2+1)^(1/2)-8/75*(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.48625, size = 96, normalized size = 1.28

$$\frac{1}{5}\,x^5\arccos(ax)-\frac{1}{75}\left(\frac{3\sqrt{-a^2x^2+1}x^4}{a^2}+\frac{4\sqrt{-a^2x^2+1}x^2}{a^4}+\frac{8\sqrt{-a^2x^2+1}}{a^6}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccos(a*x),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arccos(a*x) - 1/75*(3*sqrt(-a^2*x^2 + 1)*x^4/a^2 + 4*sqrt(-a^2*x^2
+ 1)*x^2/a^4 + 8*sqrt(-a^2*x^2 + 1)/a^6)*a
```

Fricas [A] time = 2.01248, size = 113, normalized size = 1.51

$$\frac{15\,a^5x^5\arccos(ax)-\left(3\,a^4x^4+4\,a^2x^2+8\right)\sqrt{-a^2x^2+1}}{75\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccos(a*x),x, algorithm="fricas")
```

```
[Out] 1/75*(15*a^5*x^5*arccos(a*x) - (3*a^4*x^4 + 4*a^2*x^2 + 8)*sqrt(-a^2*x^2 +
1))/a^5
```

Sympy [A] time = 2.18529, size = 75, normalized size = 1.

$$\begin{cases} \frac{x^5 \operatorname{acos}(ax)}{5} - \frac{x^4\sqrt{-a^2x^2+1}}{25a} - \frac{4x^2\sqrt{-a^2x^2+1}}{75a^3} - \frac{8\sqrt{-a^2x^2+1}}{75a^5} & \text{for } a \neq 0 \\ \frac{\pi x^5}{10} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acos(a*x),x)
```

```
[Out] Piecewise((x**5*acos(a*x)/5 - x**4*sqrt(-a**2*x**2 + 1)/(25*a) - 4*x**2*sqr
t(-a**2*x**2 + 1)/(75*a**3) - 8*sqrt(-a**2*x**2 + 1)/(75*a**5), Ne(a, 0)),
(pi*x**5/10, True))
```

Giac [A] time = 1.14711, size = 90, normalized size = 1.2

$$\frac{1}{5}x^5 \arccos(ax) - \frac{\sqrt{-a^2x^2+1}x^4}{25a} - \frac{4\sqrt{-a^2x^2+1}x^2}{75a^3} - \frac{8\sqrt{-a^2x^2+1}}{75a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccos(a*x),x, algorithm="giac")
```

```
[Out] 1/5*x^5*arccos(a*x) - 1/25*sqrt(-a^2*x^2 + 1)*x^4/a - 4/75*sqrt(-a^2*x^2 +
1)*x^2/a^3 - 8/75*sqrt(-a^2*x^2 + 1)/a^5
```

3.2 $\int x^3 \cos^{-1}(ax)\, dx$

Optimal. Leaf size=69

$$-\frac{x^3\sqrt{1-a^2x^2}}{16a}-\frac{3x\sqrt{1-a^2x^2}}{32a^3}+\frac{3\sin^{-1}(ax)}{32a^4}+\frac{1}{4}x^4\cos^{-1}(ax)$$

```
[Out] (-3*x*Sqrt[1 - a^2*x^2])/(32*a^3) - (x^3*Sqrt[1 - a^2*x^2])/(16*a) + (x^4*A
rcCos[a*x])/4 + (3*ArcSin[a*x])/(32*a^4)
```

Rubi [A] time = 0.0308727, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4628, 321, 216}

$$-\frac{x^3\sqrt{1-a^2x^2}}{16a}-\frac{3x\sqrt{1-a^2x^2}}{32a^3}+\frac{3\sin^{-1}(ax)}{32a^4}+\frac{1}{4}x^4\cos^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCos[a*x],x]
```


```
[Out] (-3*x*Sqrt[1 - a^2*x^2])/(32*a^3) - (x^3*Sqrt[1 - a^2*x^2])/(16*a) + (x^4*A
rcCos[a*x])/4 + (3*ArcSin[a*x])/(32*a^4)
```


Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int x^3\cos^{-1}(ax)\,dx &= \frac{1}{4}x^4\cos^{-1}(ax)+\frac{1}{4}a\int\frac{x^4}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{x^3\sqrt{1-a^2x^2}}{16a}+\frac{1}{4}x^4\cos^{-1}(ax)+\frac{3\int\frac{x^2}{\sqrt{1-a^2x^2}}\,dx}{16a}\\
&= -\frac{3x\sqrt{1-a^2x^2}}{32a^3}-\frac{x^3\sqrt{1-a^2x^2}}{16a}+\frac{1}{4}x^4\cos^{-1}(ax)+\frac{3\int\frac{1}{\sqrt{1-a^2x^2}}\,dx}{32a^3}\\
&= -\frac{3x\sqrt{1-a^2x^2}}{32a^3}-\frac{x^3\sqrt{1-a^2x^2}}{16a}+\frac{1}{4}x^4\cos^{-1}(ax)+\frac{3\sin^{-1}(ax)}{32a^4}
\end{aligned}$$

Mathematica [A] time = 0.0320315, size = 54, normalized size = 0.78

$$\frac{-ax\sqrt{1-a^2x^2}\left(2a^2x^2+3\right)+8a^4x^4\cos^{-1}(ax)+3\sin^{-1}(ax)}{32a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCos[a*x],x]
```

```
[Out] (-(a*x*Sqrt[1 - a^2*x^2]*(3 + 2*a^2*x^2)) + 8*a^4*x^4*ArcCos[a*x] + 3*ArcSi
n[a*x])/(32*a^4)
```

Maple [A] time = 0.004, size = 60, normalized size = 0.9

$$\frac{1}{a^4}\left(\frac{a^4x^4\arccos(ax)}{4}-\frac{a^3x^3}{16}\sqrt{-a^2x^2+1}-\frac{3\,ax}{32}\sqrt{-a^2x^2+1}+\frac{3\arcsin(ax)}{32}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccos(a*x),x)
```

```
[Out] 1/a^4*(1/4*a^4*x^4*arccos(a*x)-1/16*a^3*x^3*(-a^2*x^2+1)^(1/2)-3/32*a*x*(-a
^2*x^2+1)^(1/2)+3/32*arcsin(a*x))
```

Maxima [A] time = 1.4617, size = 99, normalized size = 1.43

$$\frac{1}{4}\,x^{4}\arccos\left(ax\right)-\frac{1}{32}\left(\frac{2\,\sqrt{-a^{2}x^{2}+1}x^{3}}{a^{2}}+\frac{3\,\sqrt{-a^{2}x^{2}+1}x}{a^{4}}-\frac{3\,\arcsin\left(\frac{a^{2}x}{\sqrt{a^{2}}}\right)}{\sqrt{a^{2}}a^{4}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccos(a*x) - 1/32*(2*sqrt(-a^2*x^2 + 1)*x^3/a^2 + 3*sqrt(-a^2*x^2
+ 1)*x/a^4 - 3*arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^4))*a
```

Fricas [A] time = 1.95799, size = 109, normalized size = 1.58

$$\frac{\left(8\,a^{4}x^{4}-3\right)\arccos\left(ax\right)-\left(2\,a^{3}x^{3}+3\,ax\right)\sqrt{-a^{2}x^{2}+1}}{32\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x),x, algorithm="fricas")
```

```
[Out] 1/32*((8*a^4*x^4 - 3)*arccos(a*x) - (2*a^3*x^3 + 3*a*x)*sqrt(-a^2*x^2 + 1))
/a^4
```

Sympy [A] time = 1.08707, size = 66, normalized size = 0.96

$$\begin{cases}\frac{x^{4}\operatorname{acos}\left(ax\right)}{4}-\frac{x^{3}\sqrt{-a^{2}x^{2}+1}}{16a}-\frac{3x\sqrt{-a^{2}x^{2}+1}}{32a^{3}}-\frac{3\operatorname{acos}\left(ax\right)}{32a^{4}} & \text{for}\ a\neq 0\\ \frac{\pi x^{4}}{8} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acos(a*x),x)
```

```
[Out] Piecewise((x**4*acos(a*x)/4 - x**3*sqrt(-a**2*x**2 + 1)/(16*a) - 3*x*sqrt(-
a**2*x**2 + 1)/(32*a**3) - 3*acos(a*x)/(32*a**4), Ne(a, 0)), (pi*x**4/8, Tr
```

```
ue))
```

Giac [A] time = 1.17655, size = 77, normalized size = 1.12

$$\frac{1}{4}\,x^{4}\arccos\left(ax\right)-\frac{\sqrt{-a^{2}x^{2}+1}x^{3}}{16\,a}-\frac{3\,\sqrt{-a^{2}x^{2}+1}x}{32\,a^{3}}-\frac{3\,\arccos\left(ax\right)}{32\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x),x, algorithm="giac")
```

```
[Out] 1/4*x^4*arccos(a*x) - 1/16*sqrt(-a^2*x^2 + 1)*x^3/a - 3/32*sqrt(-a^2*x^2 +
1)*x/a^3 - 3/32*arccos(a*x)/a^4
```

3.3 $\int x^2 \cos^{-1}(ax)\,dx$

Optimal. Leaf size=54

$$\frac{\left(1-a^2x^2\right)^{3/2}}{9a^3}-\frac{\sqrt{1-a^2x^2}}{3a^3}+\frac{1}{3}x^3\cos^{-1}(ax)$$

```
[Out] -Sqrt[1 - a^2*x^2]/(3*a^3) + (1 - a^2*x^2)^(3/2)/(9*a^3) + (x^3*ArcCos[a*x]
)/3
```

Rubi [A] time = 0.0346335, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4628, 266, 43}

$$\frac{\left(1-a^2x^2\right)^{3/2}}{9a^3}-\frac{\sqrt{1-a^2x^2}}{3a^3}+\frac{1}{3}x^3\cos^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCos[a*x],x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(3*a^3) + (1 - a^2*x^2)^(3/2)/(9*a^3) + (x^3*ArcCos[a*x]
)/3
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\cos^{-1}(ax)\,dx &= \frac{1}{3}x^3\cos^{-1}(ax)+\frac{1}{3}a\int\frac{x^3}{\sqrt{1-a^2x^2}}\,dx\\ &= \frac{1}{3}x^3\cos^{-1}(ax)+\frac{1}{6}a\,\mathrm{Subst}\left(\int\frac{x}{\sqrt{1-a^2x}}\,dx,x,x^2\right)\\ &= \frac{1}{3}x^3\cos^{-1}(ax)+\frac{1}{6}a\,\mathrm{Subst}\left(\int\left(\frac{1}{a^2\sqrt{1-a^2x}}-\frac{\sqrt{1-a^2x}}{a^2}\right)dx,x,x^2\right)\\ &= -\frac{\sqrt{1-a^2x^2}}{3a^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{9a^3}+\frac{1}{3}x^3\cos^{-1}(ax)\end{aligned}$$

Mathematica [A] time = 0.0246064, size = 42, normalized size = 0.78

$$\frac{1}{3}x^3\cos^{-1}(ax)-\frac{\sqrt{1-a^2x^2}\left(a^2x^2+2\right)}{9a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCos[a*x],x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]*(2 + a^2*x^2))/(9*a^3) + (x^3*ArcCos[a*x])/3
```

Maple [A] time = 0.003, size = 52, normalized size = 1.

$$\frac{1}{a^3}\left(\frac{a^3x^3\arccos(ax)}{3}-\frac{a^2x^2}{9}\sqrt{-a^2x^2+1}-\frac{2}{9}\sqrt{-a^2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccos(a*x),x)
```

```
[Out] 1/a^3*(1/3*a^3*x^3*arccos(a*x)-1/9*a^2*x^2*(-a^2*x^2+1)^(1/2)-2/9*(-a^2*x^2
+1)^(1/2))
```

Maxima [A] time = 1.45727, size = 68, normalized size = 1.26

$$\frac{1}{3}x^3\arccos(ax) - \frac{1}{9}a\left(\frac{\sqrt{-a^2x^2+1}x^2}{a^2} + \frac{2\sqrt{-a^2x^2+1}}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccos(a*x) - 1/9*a*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^2 +
 1)/a^4)
```

Fricas [A] time = 1.89559, size = 92, normalized size = 1.7

$$\frac{3a^3x^3\arccos(ax) - (a^2x^2+2)\sqrt{-a^2x^2+1}}{9a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x),x, algorithm="fricas")
```

```
[Out] 1/9*(3*a^3*x^3*arccos(a*x) - (a^2*x^2 + 2)*sqrt(-a^2*x^2 + 1))/a^3
```

Sympy [A] time = 0.587228, size = 53, normalized size = 0.98

$$\begin{cases} \frac{x^3\operatorname{acos}(ax)}{3} - \frac{x^2\sqrt{-a^2x^2+1}}{9a} - \frac{2\sqrt{-a^2x^2+1}}{9a^3} & \text{for } a \neq 0 \\ \frac{\pi x^3}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acos(a*x),x)
```

```
[Out] Piecewise((x**3*acos(a*x)/3 - x**2*sqrt(-a**2*x**2 + 1)/(9*a) - 2*sqrt(-a**
2*x**2 + 1)/(9*a**3), Ne(a, 0)), (pi*x**3/6, True))
```

<u>Giac</u> [A] time = 1.16271, size = 63, normalized size = 1.17

$$\frac{1}{3}\,x^{3}\arccos\left(ax\right)-\frac{\sqrt{-a^{2}x^{2}+1}x^{2}}{9\,a}-\frac{2\,\sqrt{-a^{2}x^{2}+1}}{9\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arccos(a*x) - 1/9*sqrt(-a^2*x^2 + 1)*x^2/a - 2/9*sqrt(-a^2*x^2 + 1)
/a^3
```

3.4 $\int x \cos^{-1}(ax)\,dx$

Optimal. Leaf size=45

$$-\frac{x\sqrt{1-a^2x^2}}{4a}+\frac{\sin^{-1}(ax)}{4a^2}+\frac{1}{2}x^2\cos^{-1}(ax)$$

```
[Out] -(x*Sqrt[1 - a^2*x^2])/(4*a) + (x^2*ArcCos[a*x])/2 + ArcSin[a*x]/(4*a^2)
```

Rubi [A] time = 0.0160866, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4628, 321, 216}

$$-\frac{x\sqrt{1-a^2x^2}}{4a}+\frac{\sin^{-1}(ax)}{4a^2}+\frac{1}{2}x^2\cos^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCos[a*x],x]
```

```
[Out] -(x*Sqrt[1 - a^2*x^2])/(4*a) + (x^2*ArcCos[a*x])/2 + ArcSin[a*x]/(4*a^2)
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int x\cos^{-1}(ax)\,dx &= \frac{1}{2}x^2\cos^{-1}(ax) + \frac{1}{2}a\int \frac{x^2}{\sqrt{1-a^2x^2}}\,dx \\ &= -\frac{x\sqrt{1-a^2x^2}}{4a} + \frac{1}{2}x^2\cos^{-1}(ax) + \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{4a} \\ &= -\frac{x\sqrt{1-a^2x^2}}{4a} + \frac{1}{2}x^2\cos^{-1}(ax) + \frac{\sin^{-1}(ax)}{4a^2}\end{aligned}$$

Mathematica [A] time = 0.0161218, size = 42, normalized size = 0.93

$$\frac{-ax\sqrt{1-a^2x^2} + 2a^2x^2\cos^{-1}(ax) + \sin^{-1}(ax)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCos[a*x],x]
```

```
[Out] (-(a*x*Sqrt[1 - a^2*x^2]) + 2*a^2*x^2*ArcCos[a*x] + ArcSin[a*x])/(4*a^2)
```

Maple [A] time = 0.002, size = 40, normalized size = 0.9

$$\frac{1}{a^2}\left(\frac{a^2x^2\arccos(ax)}{2} - \frac{ax}{4}\sqrt{-a^2x^2+1} + \frac{\arcsin(ax)}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccos(a*x),x)
```

```
[Out] 1/a^2*(1/2*a^2*x^2*arccos(a*x)-1/4*a*x*(-a^2*x^2+1)^(1/2)+1/4*arcsin(a*x))
```

Maxima [A] time = 1.45635, size = 70, normalized size = 1.56

$$\frac{1}{2}x^2\arccos(ax) - \frac{1}{4}a\left(\frac{\sqrt{-a^2x^2+1}x}{a^2} - \frac{\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccos(a*x) - 1/4*a*(sqrt(-a^2*x^2 + 1)*x/a^2 - arcsin(a^2*x/sqrt(a
^2))/(sqrt(a^2)*a^2))
```

Fricas [A] time = 1.96631, size = 88, normalized size = 1.96

$$-\frac{\sqrt{-a^2x^2+1}\,ax-\left(2\,a^2x^2-1\right)\arccos\left(ax\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(-a^2*x^2 + 1)*a*x - (2*a^2*x^2 - 1)*arccos(a*x))/a^2
```

Sympy [A] time = 0.241072, size = 42, normalized size = 0.93

$$\begin{cases} \frac{x^2\operatorname{acos}\left(ax\right)}{2}-\frac{x\sqrt{-a^2x^2+1}}{4a}-\frac{\operatorname{acos}\left(ax\right)}{4a^2} & \text{for}\ a\neq 0\\ \frac{\pi x^2}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acos(a*x),x)
```

```
[Out] Piecewise((x**2*acos(a*x)/2 - x*sqrt(-a**2*x**2 + 1)/(4*a) - acos(a*x)/(4*a
**2), Ne(a, 0)), (pi*x**2/4, True))
```

Giac [A] time = 1.1384, size = 50, normalized size = 1.11

$$\frac{1}{2}\,x^2\arccos\left(ax\right)-\frac{\sqrt{-a^2x^2+1}\,x}{4\,a}-\frac{\arccos\left(ax\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x),x, algorithm="giac")

[Out] 1/2*x^2*arccos(a*x) - 1/4*sqrt(-a^2*x^2 + 1)*x/a - 1/4*arccos(a*x)/a^2
```

3.5 $\int \cos^{-1}(ax)\,dx$

Optimal. Leaf size=26

$$x\cos^{-1}(ax)-\frac{\sqrt{1-a^2x^2}}{a}$$

```
[Out] -(Sqrt[1 - a^2*x^2]/a) + x*ArcCos[a*x]
```

Rubi [A] time = 0.0075295, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4620, 261}

$$x\cos^{-1}(ax)-\frac{\sqrt{1-a^2x^2}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x],x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/a) + x*ArcCos[a*x]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^{-1}(ax)\,dx &= x\cos^{-1}(ax)+a\int \frac{x}{\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{\sqrt{1-a^2x^2}}{a}+x\cos^{-1}(ax)\end{aligned}$$

Mathematica [A] time = 0.0072581, size = 26, normalized size = 1.

$$x\cos^{-1}(ax) - \frac{\sqrt{1-a^2x^2}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x],x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/a) + x*ArcCos[a*x]
```

Maple [A] time = 0.002, size = 27, normalized size = 1.

$$\frac{1}{a}\left(ax\arccos(ax) - \sqrt{-a^2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x),x)
```

```
[Out] 1/a*(a*x*arccos(a*x)-(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.44063, size = 35, normalized size = 1.35

$$\frac{ax\arccos(ax) - \sqrt{-a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x),x, algorithm="maxima")
```

```
[Out] (a*x*arccos(a*x) - sqrt(-a^2*x^2 + 1))/a
```

Fricas [A] time = 1.91552, size = 57, normalized size = 2.19

$$\frac{ax\arccos(ax) - \sqrt{-a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x),x, algorithm="fricas")
```

```
[Out] (a*x*arccos(a*x) - sqrt(-a^2*x^2 + 1))/a
```

Sympy [A] time = 0.150393, size = 24, normalized size = 0.92

$$\begin{cases} x \operatorname{acos}(ax) - \frac{\sqrt{-a^2x^2+1}}{a} & \text{for } a \neq 0 \\ \frac{\pi x}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x),x)
```

```
[Out] Piecewise((x*acos(a*x) - sqrt(-a**2*x**2 + 1)/a, Ne(a, 0)), (pi*x/2, True))
```

Giac [A] time = 1.15976, size = 35, normalized size = 1.35

$$\frac{ax \arccos(ax) - \sqrt{-a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x),x, algorithm="giac")
```

```
[Out] (a*x*arccos(a*x) - sqrt(-a^2*x^2 + 1))/a
```

3.6 $\int \frac{\cos^{-1}(ax)}{x} \, dx$

Optimal. Leaf size=51

$$-\frac{1}{2}i\text{PolyLog}\left(2,-e^{2i\cos^{-1}(ax)}\right)-\frac{1}{2}i\cos^{-1}(ax)^2+\cos^{-1}(ax)\log\left(1+e^{2i\cos^{-1}(ax)}\right)$$

```
[Out] (-I/2)*ArcCos[a*x]^2 + ArcCos[a*x]*Log[1 + E^((2*I)*ArcCos[a*x])] - (I/2)*P
olyLog[2, -E^((2*I)*ArcCos[a*x])]
```

Rubi [A] time = 0.0567818, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4626, 3719, 2190, 2279, 2391}

$$-\frac{1}{2}i\text{PolyLog}\left(2,-e^{2i\cos^{-1}(ax)}\right)-\frac{1}{2}i\cos^{-1}(ax)^2+\cos^{-1}(ax)\log\left(1+e^{2i\cos^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]/x,x]
```

```
[Out] (-I/2)*ArcCos[a*x]^2 + ArcCos[a*x]*Log[1 + E^((2*I)*ArcCos[a*x])] - (I/2)*P
olyLog[2, -E^((2*I)*ArcCos[a*x])]
```

Rule 4626

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n/Cot[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^{-1}(ax)}{x}\,dx &= -\operatorname{Subst}\left(\int x\tan(x)\,dx, x, \cos^{-1}(ax)\right)\\
&= -\frac{1}{2}i\cos^{-1}(ax)^2 + 2i\operatorname{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \cos^{-1}(ax)\right)\\
&= -\frac{1}{2}i\cos^{-1}(ax)^2 + \cos^{-1}(ax)\log\left(1+e^{2i\cos^{-1}(ax)}\right) - \operatorname{Subst}\left(\int \log\left(1+e^{2ix}\right)\,dx, x, \cos^{-1}(ax)\right)\\
&= -\frac{1}{2}i\cos^{-1}(ax)^2 + \cos^{-1}(ax)\log\left(1+e^{2i\cos^{-1}(ax)}\right) + \frac{1}{2}i\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i\cos^{-1}(ax)}\right)\\
&= -\frac{1}{2}i\cos^{-1}(ax)^2 + \cos^{-1}(ax)\log\left(1+e^{2i\cos^{-1}(ax)}\right) - \frac{1}{2}i\operatorname{Li}_2\left(-e^{2i\cos^{-1}(ax)}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0169056, size = 51, normalized size = 1.

$$-\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\cos^{-1}(ax)}\right) - \frac{1}{2}i\cos^{-1}(ax)^2 + \cos^{-1}(ax)\log\left(1+e^{2i\cos^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x]/x,x]
```

```
[Out] (-I/2)*ArcCos[a*x]^2 + ArcCos[a*x]*Log[1 + E^((2*I)*ArcCos[a*x])] - (I/2)*P
olyLog[2, -E^((2*I)*ArcCos[a*x])]
```

Maple [A] time = 0.063, size = 68, normalized size = 1.3

$$-\frac{i}{2}\left(\arccos\left(ax\right)\right)^{2}+\arccos\left(ax\right)\ln\left(1+\left(i\sqrt{-a^{2}x^{2}+1}+ax\right)^{2}\right)-\frac{i}{2}\mathrm{polylog}\left(2,-\left(i\sqrt{-a^{2}x^{2}+1}+ax\right)^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)/x,x)
```

```
[Out] -1/2*I*arccos(a*x)^2+arccos(a*x)*ln(1+(I*(-a^2*x^2+1)^(1/2)+a*x)^2)-1/2*I*p
olylog(2,-(I*(-a^2*x^2+1)^(1/2)+a*x)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\arccos\left(ax\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(arccos(a*x)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arccos\left(ax\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)/x,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{acos}\left(ax\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)/x,x)
```

```
[Out] Integral(acos(a*x)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)/x,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)/x, x)
```

3.7 $\int \frac{\cos^{-1}(ax)}{x^2} dx$

Optimal. Leaf size=27

$$a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{\cos^{-1}(ax)}{x}$$

```
[Out] -(ArcCos[a*x]/x) + a*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rubi [A] time = 0.0224197, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4628, 266, 63, 208}

$$a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{\cos^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]/x^2,x]
```

```
[Out] -(ArcCos[a*x]/x) + a*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}(ax)}{x^2}\,dx &= -\frac{\cos^{-1}(ax)}{x} - a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\cos^{-1}(ax)}{x} - \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= -\frac{\cos^{-1}(ax)}{x} + \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a} \\
&= -\frac{\cos^{-1}(ax)}{x} + a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0111537, size = 34, normalized size = 1.26

$$a\log\left(\sqrt{1-a^2x^2}+1\right) - a\log(x) - \frac{\cos^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x]/x^2,x]
```

```
[Out] -(ArcCos[a*x]/x) - a*Log[x] + a*Log[1 + Sqrt[1 - a^2*x^2]]
```

Maple [A] time = 0.003, size = 29, normalized size = 1.1

$$a\left(-\frac{\arccos(ax)}{ax} + \text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)/x^2,x)
```

```
[Out] a*(-arccos(a*x)/a/x+arctanh(1/(-a^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.45474, size = 51, normalized size = 1.89

$$a \log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right) - \frac{\arccos(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)/x^2,x, algorithm="maxima")
```

```
[Out] a*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - arccos(a*x)/x
```

Fricas [B] time = 2.47838, size = 203, normalized size = 7.52

$$\frac{ax\log\left(\sqrt{-a^2x^2+1}+1\right) - ax\log\left(\sqrt{-a^2x^2+1}-1\right) + 2\,(x-1)\arccos(ax) - 2\,x\arctan\left(\frac{\sqrt{-a^2x^2+1}ax}{a^2x^2-1}\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*x*log(sqrt(-a^2*x^2 + 1) + 1) - a*x*log(sqrt(-a^2*x^2 + 1) - 1) + 2*
(x - 1)*arccos(a*x) - 2*x*arctan(sqrt(-a^2*x^2 + 1)*a*x/(a^2*x^2 - 1)))/x
```

Sympy [C] time = 1.61538, size = 34, normalized size = 1.26

$$-a\left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise} \end{cases}\right) - \frac{\operatorname{acos}(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)/x**2,x)
```

```
[Out] -a*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), Tru
e)) - acos(a*x)/x
```

Giac [A] time = 1.13227, size = 65, normalized size = 2.41

$$\frac{1}{2}a\left(\log\left(\sqrt{-a^2x^2+1}+1\right)-\log\left(-\sqrt{-a^2x^2+1}+1\right)\right)-\frac{\arccos\left(ax\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*a*(log(sqrt(-a^2*x^2 + 1) + 1) - log(-sqrt(-a^2*x^2 + 1) + 1)) - arccos
(a*x)/x
```

3.8 $\int \frac{\cos^{-1}(ax)}{x^3} \, dx$

Optimal. Leaf size=34

$$\frac{a\sqrt{1-a^2x^2}}{2x} - \frac{\cos^{-1}(ax)}{2x^2}$$

```
[Out] (a*Sqrt[1 - a^2*x^2])/(2*x) - ArcCos[a*x]/(2*x^2)
```

Rubi [A] time = 0.015502, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4628, 264}

$$\frac{a\sqrt{1-a^2x^2}}{2x} - \frac{\cos^{-1}(ax)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]/x^3,x]
```

```
[Out] (a*Sqrt[1 - a^2*x^2])/(2*x) - ArcCos[a*x]/(2*x^2)
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^{-1}(ax)}{x^3}\,dx = -\frac{\cos^{-1}(ax)}{2x^2} - \frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{a\sqrt{1-a^2x^2}}{2x} - \frac{\cos^{-1}(ax)}{2x^2}$$

Mathematica [A] time = 0.0136793, size = 31, normalized size = 0.91

$$\frac{ax\sqrt{1-a^2x^2} - \cos^{-1}(ax)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x]/x^3,x]
```

```
[Out] (a*x*Sqrt[1 - a^2*x^2] - ArcCos[a*x])/(2*x^2)
```

Maple [A] time = 0.003, size = 38, normalized size = 1.1

$$a^2\left(-\frac{\arccos(ax)}{2\,a^2x^2} + \frac{1}{2\,ax}\sqrt{-a^2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)/x^3,x)
```

```
[Out] a^2*(-1/2*arccos(a*x)/a^2/x^2+1/2/a/x*(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.46339, size = 38, normalized size = 1.12

$$\frac{\sqrt{-a^2x^2+1}a}{2\,x} - \frac{\arccos(ax)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*a/x - 1/2*arccos(a*x)/x^2
```

Fricas [A] time = 2.33323, size = 65, normalized size = 1.91

$$\frac{\sqrt{-a^2x^2+1}ax - \arccos(ax)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(-a^2*x^2 + 1)*a*x - arccos(a*x))/x^2
```

Sympy [C] time = 1.3881, size = 53, normalized size = 1.56

$$-\frac{a\left(\begin{cases} -\frac{i\sqrt{a^2x^2-1}}{x} & \text{for } \left|a^2x^2\right| > 1 \\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise} \end{cases}\right)}{2} - \frac{\operatorname{acos}(ax)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)/x**3,x)
```

```
[Out] -a*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x
**2 + 1)/x, True))/2 - acos(a*x)/(2*x**2)
```

Giac [B] time = 1.11412, size = 92, normalized size = 2.71

$$-\frac{1}{4}\left(\frac{a^4x}{\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|} - \frac{\sqrt{-a^2x^2+1}|a|+a}{x|a|}\right)a - \frac{\arccos(ax)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - (sqrt(-a^2*x^2 + 1)*
abs(a) + a)/(x*abs(a)))*a - 1/2*arccos(a*x)/x^2
```

3.9 $\int \frac{\cos^{-1}(ax)}{x^4}\,dx$

Optimal. Leaf size=56

$$\frac{a\sqrt{1-a^2x^2}}{6x^2}+\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{\cos^{-1}(ax)}{3x^3}$$

```
[Out] (a*Sqrt[1 - a^2*x^2])/(6*x^2) - ArcCos[a*x]/(3*x^3) + (a^3*ArcTanh[Sqrt[1 -
 a^2*x^2]])/6
```

Rubi [A] time = 0.0354313, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4628, 266, 51, 63, 208}

$$\frac{a\sqrt{1-a^2x^2}}{6x^2}+\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{\cos^{-1}(ax)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]/x^4,x]
```

```
[Out] (a*Sqrt[1 - a^2*x^2])/(6*x^2) - ArcCos[a*x]/(3*x^3) + (a^3*ArcTanh[Sqrt[1 -
 a^2*x^2]])/6
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
```

```
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}(ax)}{x^4}\,dx &= -\frac{\cos^{-1}(ax)}{3x^3} - \frac{1}{3}a\int \frac{1}{x^3\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\cos^{-1}(ax)}{3x^3} - \frac{1}{6}a\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= \frac{a\sqrt{1-a^2x^2}}{6x^2} - \frac{\cos^{-1}(ax)}{3x^3} - \frac{1}{12}a^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= \frac{a\sqrt{1-a^2x^2}}{6x^2} - \frac{\cos^{-1}(ax)}{3x^3} + \frac{1}{6}a\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right) \\
&= \frac{a\sqrt{1-a^2x^2}}{6x^2} - \frac{\cos^{-1}(ax)}{3x^3} + \frac{1}{6}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0237518, size = 67, normalized size = 1.2

$$\frac{a\sqrt{1-a^2x^2}}{6x^2} + \frac{1}{6}a^3\log\left(\sqrt{1-a^2x^2}+1\right) - \frac{1}{6}a^3\log(x) - \frac{\cos^{-1}(ax)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x]/x^4,x]
```

```
[Out] (a*Sqrt[1 - a^2*x^2])/(6*x^2) - ArcCos[a*x]/(3*x^3) - (a^3*Log[x])/6 + (a^3
*Log[1 + Sqrt[1 - a^2*x^2]])/6
```

Maple [A] time = 0.004, size = 53, normalized size = 1.

$$a^{3}\left(-\frac{\arccos(ax)}{3\,a^{3}x^{3}}+\frac{1}{6\,a^{2}x^{2}}\sqrt{-a^{2}x^{2}+1}+\frac{1}{6}\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)/x^4,x)
```

```
[Out] a^3*(-1/3*arccos(a*x)/a^3/x^3+1/6/a^2/x^2*(-a^2*x^2+1)^(1/2)+1/6*arctanh(1/
(-a^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.45277, size = 81, normalized size = 1.45

$$\frac{1}{6}\left(a^{2}\log\left(\frac{2\sqrt{-a^{2}x^{2}+1}}{|x|}+\frac{2}{|x|}\right)+\frac{\sqrt{-a^{2}x^{2}+1}}{x^{2}}\right)a-\frac{\arccos(ax)}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-a^2*x^2 + 1)/x
^2)*a - 1/3*arccos(a*x)/x^3
```

Fricas [B] time = 2.49061, size = 259, normalized size = 4.62

$$\frac{a^{3}x^{3}\log\left(\sqrt{-a^{2}x^{2}+1}+1\right)-a^{3}x^{3}\log\left(\sqrt{-a^{2}x^{2}+1}-1\right)-4\,x^{3}\arctan\left(\frac{\sqrt{-a^{2}x^{2}+1}ax}{a^{2}x^{2}-1}\right)+2\sqrt{-a^{2}x^{2}+1}ax+4\left(x^{3}-1\right)\mathrm{arccos}}{12\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(a^3*x^3*log(sqrt(-a^2*x^2 + 1) + 1) - a^3*x^3*log(sqrt(-a^2*x^2 + 1)
- 1) - 4*x^3*arctan(sqrt(-a^2*x^2 + 1)*a*x/(a^2*x^2 - 1)) + 2*sqrt(-a^2*x^2
 + 1)*a*x + 4*(x^3 - 1)*arccos(a*x))/x^3
```

Sympy [C] time = 3.00703, size = 110, normalized size = 1.96

$$-\frac{a\left(\begin{cases} -\frac{a^{2}\operatorname{acosh}\left(\frac{1}{ax}\right)}{2}-\frac{a\sqrt{-1+\frac{1}{a^{2}x^{2}}}}{2x} & \text{for}\ \frac{1}{\left|a^{2}x^{2}\right|}>1\\ \frac{ia^{2}\operatorname{asin}\left(\frac{1}{ax}\right)}{2}-\frac{ia}{2x\sqrt{1-\frac{1}{a^{2}x^{2}}}}+\frac{i}{2ax^{3}\sqrt{1-\frac{1}{a^{2}x^{2}}}} & \text{otherwise}\end{cases}\right)}{3}-\frac{\operatorname{acos}\left(ax\right)}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)/x**4,x)
```

```
[Out] -a*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(2*x), 1/
Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**2*x*
*2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))/3 - acos(a*x)/(3*x**3)
```

Giac [A] time = 1.13791, size = 95, normalized size = 1.7

$$\frac{1}{12}\,a^{3}\left(\frac{2\,\sqrt{-a^{2}x^{2}+1}}{a^{2}x^{2}}+\log\left(\sqrt{-a^{2}x^{2}+1}+1\right)-\log\left(-\sqrt{-a^{2}x^{2}+1}+1\right)\right)-\frac{\arccos\left(ax\right)}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*a^3*(2*sqrt(-a^2*x^2 + 1)/(a^2*x^2) + log(sqrt(-a^2*x^2 + 1) + 1) - lo
g(-sqrt(-a^2*x^2 + 1) + 1)) - 1/3*arccos(a*x)/x^3
```

3.10 $\int \frac{\cos^{-1}(ax)}{x^5} dx$

Optimal. Leaf size=58

$$\frac{a^3\sqrt{1-a^2x^2}}{6x}+\frac{a\sqrt{1-a^2x^2}}{12x^3}-\frac{\cos^{-1}(ax)}{4x^4}$$

```
[Out] (a*Sqrt[1 - a^2*x^2])/(12*x^3) + (a^3*Sqrt[1 - a^2*x^2])/(6*x) - ArcCos[a*x
]/(4*x^4)
```

Rubi [A] time = 0.0221729, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4628, 271, 264}

$$\frac{a^3\sqrt{1-a^2x^2}}{6x}+\frac{a\sqrt{1-a^2x^2}}{12x^3}-\frac{\cos^{-1}(ax)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]/x^5,x]
```

```
[Out] (a*Sqrt[1 - a^2*x^2])/(12*x^3) + (a^3*Sqrt[1 - a^2*x^2])/(6*x) - ArcCos[a*x
]/(4*x^4)
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
```

```
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{-1}(ax)}{x^5}\,dx &= -\frac{\cos^{-1}(ax)}{4x^4}-\frac{1}{4}a\int \frac{1}{x^4\sqrt{1-a^2x^2}}\,dx\\ &= \frac{a\sqrt{1-a^2x^2}}{12x^3}-\frac{\cos^{-1}(ax)}{4x^4}-\frac{1}{6}a^3\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx\\ &= \frac{a\sqrt{1-a^2x^2}}{12x^3}+\frac{a^3\sqrt{1-a^2x^2}}{6x}-\frac{\cos^{-1}(ax)}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.0221487, size = 41, normalized size = 0.71

$$\frac{ax\sqrt{1-a^2x^2}\left(2a^2x^2+1\right)-3\cos^{-1}(ax)}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x]/x^5,x]
```

```
[Out] (a*x*Sqrt[1 - a^2*x^2]*(1 + 2*a^2*x^2) - 3*ArcCos[a*x])/(12*x^4)
```

Maple [A] time = 0.003, size = 58, normalized size = 1.

$$a^4\left(-\frac{\arccos(ax)}{4\,a^4x^4}+\frac{1}{12\,a^3x^3}\sqrt{-a^2x^2+1}+\frac{1}{6\,ax}\sqrt{-a^2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)/x^5,x)
```

```
[Out] a^4*(-1/4*arccos(a*x)/a^4/x^4+1/12/a^3/x^3*(-a^2*x^2+1)^(1/2)+1/6/a/x*(-a^2
*x^2+1)^(1/2))
```

Maxima [A] time = 1.43934, size = 68, normalized size = 1.17

$$\frac{1}{12}\left(\frac{2\sqrt{-a^2x^2+1}a^2}{x}+\frac{\sqrt{-a^2x^2+1}}{x^3}\right)a-\frac{\arccos(ax)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*(2*sqrt(-a^2*x^2 + 1)*a^2/x + sqrt(-a^2*x^2 + 1)/x^3)*a - 1/4*arccos(a
*x)/x^4
```

Fricas [A] time = 2.02855, size = 88, normalized size = 1.52

$$\frac{(2a^3x^3+ax)\sqrt{-a^2x^2+1}-3\arccos(ax)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*((2*a^3*x^3 + a*x)*sqrt(-a^2*x^2 + 1) - 3*arccos(a*x))/x^4
```

Sympy [C] time = 2.67278, size = 102, normalized size = 1.76

$$-\frac{a\left(\begin{cases}-\frac{2ia^2\sqrt{a^2x^2-1}}{3x}-\frac{i\sqrt{a^2x^2-1}}{3x^3} & \text{for } \left|a^2x^2\right|>1\\ -\frac{2a^2\sqrt{-a^2x^2+1}}{3x}-\frac{\sqrt{-a^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{4}-\frac{\operatorname{acos}(ax)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)/x**5,x)
```

```
[Out] -a*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(
3*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a
**2*x**2 + 1)/(3*x**3), True))/4 - acos(a*x)/(4*x**4)
```

Giac [B] time = 1.13634, size = 176, normalized size = 3.03

$$-\frac{1}{96}\left(\frac{\left(a^4+\frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{x^2}\right)a^6x^3}{\left(\sqrt{-a^2x^2+1}|a|+a\right)^3|a|}-\frac{\frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)a^4}{x}+\frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{x^3}}{a^2|a|}\right)a-\frac{\arccos(ax)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)/x^5,x, algorithm="giac")
```

```
[Out] -1/96*((a^4 + 9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/x^2)*a^6*x^3/((sqrt(-a^2*
x^2 + 1)*abs(a) + a)^3*abs(a)) - (9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4/x +
 (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/x^3)/(a^2*abs(a)))*a - 1/4*arccos(a*x)/x
^4
```

3.11 $\int \frac{\cos^{-1}(ax)}{x^6} \, dx$

Optimal. Leaf size=80

$$\frac{3a^3\sqrt{1-a^2x^2}}{40x^2}+\frac{a\sqrt{1-a^2x^2}}{20x^4}+\frac{3}{40}a^5\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{\cos^{-1}(ax)}{5x^5}$$

```
[Out] (a*Sqrt[1 - a^2*x^2])/(20*x^4) + (3*a^3*Sqrt[1 - a^2*x^2])/(40*x^2) - ArcCo
s[a*x]/(5*x^5) + (3*a^5*ArcTanh[Sqrt[1 - a^2*x^2]])/40
```

Rubi [A] time = 0.0466105, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4628, 266, 51, 63, 208}

$$\frac{3a^3\sqrt{1-a^2x^2}}{40x^2}+\frac{a\sqrt{1-a^2x^2}}{20x^4}+\frac{3}{40}a^5\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{\cos^{-1}(ax)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]/x^6,x]
```

```
[Out] (a*Sqrt[1 - a^2*x^2])/(20*x^4) + (3*a^3*Sqrt[1 - a^2*x^2])/(40*x^2) - ArcCo
s[a*x]/(5*x^5) + (3*a^5*ArcTanh[Sqrt[1 - a^2*x^2]])/40
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
```

```
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^{-1}(ax)}{x^6}\,dx &= -\frac{\cos^{-1}(ax)}{5x^5} - \frac{1}{5}a\int \frac{1}{x^5\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\cos^{-1}(ax)}{5x^5} - \frac{1}{10}a\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= \frac{a\sqrt{1-a^2x^2}}{20x^4} - \frac{\cos^{-1}(ax)}{5x^5} - \frac{1}{40}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= \frac{a\sqrt{1-a^2x^2}}{20x^4} + \frac{3a^3\sqrt{1-a^2x^2}}{40x^2} - \frac{\cos^{-1}(ax)}{5x^5} - \frac{1}{80}\left(3a^5\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= \frac{a\sqrt{1-a^2x^2}}{20x^4} + \frac{3a^3\sqrt{1-a^2x^2}}{40x^2} - \frac{\cos^{-1}(ax)}{5x^5} + \frac{1}{40}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right) \\
&= \frac{a\sqrt{1-a^2x^2}}{20x^4} + \frac{3a^3\sqrt{1-a^2x^2}}{40x^2} - \frac{\cos^{-1}(ax)}{5x^5} + \frac{3}{40}a^5\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.061937, size = 72, normalized size = 0.9

$$
\frac{1}{40}\left(\frac{a\sqrt{1-a^2x^2}\left(3a^2x^2+2\right)}{x^4} + 3a^5\log\left(\sqrt{1-a^2x^2}+1\right) - 3a^5\log(x) - \frac{8\cos^{-1}(ax)}{x^5}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x]/x^6,x]
```

```
[Out] ((a*Sqrt[1 - a^2*x^2]*(2 + 3*a^2*x^2))/x^4 - (8*ArcCos[a*x])/x^5 - 3*a^5*Lo
g[x] + 3*a^5*Log[1 + Sqrt[1 - a^2*x^2]])/40
```

Maple [A] time = 0.003, size = 73, normalized size = 0.9

$$a^5\left(-\frac{\arccos(ax)}{5a^5x^5}+\frac{1}{20a^4x^4}\sqrt{-a^2x^2+1}+\frac{3}{40a^2x^2}\sqrt{-a^2x^2+1}+\frac{3}{40}\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)/x^6,x)
```

```
[Out] a^5*(-1/5*arccos(a*x)/a^5/x^5+1/20/a^4/x^4*(-a^2*x^2+1)^(1/2)+3/40/a^2/x^2*
(-a^2*x^2+1)^(1/2)+3/40*arctanh(1/(-a^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.45141, size = 111, normalized size = 1.39

$$\frac{1}{40}\left(3a^4\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{3\sqrt{-a^2x^2+1}a^2}{x^2}+\frac{2\sqrt{-a^2x^2+1}}{x^4}\right)a-\frac{\arccos(ax)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)/x^6,x, algorithm="maxima")
```

```
[Out] 1/40*(3*a^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + 3*sqrt(-a^2*x^2 +
 1)*a^2/x^2 + 2*sqrt(-a^2*x^2 + 1)/x^4)*a - 1/5*arccos(a*x)/x^5
```

Fricas [A] time = 2.19651, size = 289, normalized size = 3.61

$$\frac{3a^5x^5\log\left(\sqrt{-a^2x^2+1}+1\right)-3a^5x^5\log\left(\sqrt{-a^2x^2+1}-1\right)-16x^5\arctan\left(\frac{\sqrt{-a^2x^2+1}ax}{a^2x^2-1}\right)+16\left(x^5-1\right)\arccos(ax)+2\left(3a\right.}{80x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)/x^6,x, algorithm="fricas")
```

```
[Out] 1/80*(3*a^5*x^5*log(sqrt(-a^2*x^2 + 1) + 1) - 3*a^5*x^5*log(sqrt(-a^2*x^2 +
 1) - 1) - 16*x^5*arctan(sqrt(-a^2*x^2 + 1)*a*x/(a^2*x^2 - 1)) + 16*(x^5 -
1)*arccos(a*x) + 2*(3*a^3*x^3 + 2*a*x)*sqrt(-a^2*x^2 + 1))/x^5
```

Sympy [C] time = 6.43766, size = 184, normalized size = 2.3

$$-\frac{a\left(\begin{cases}-\frac{3a^4\operatorname{acosh}\left(\frac{1}{ax}\right)}{8}+\frac{3a^3}{8x\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{a}{8x^3\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{1}{4ax^5\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for } \frac{1}{|a^2x^2|}>1\\ \frac{3ia^4\operatorname{asin}\left(\frac{1}{ax}\right)}{8}-\frac{3ia^3}{8x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{ia}{8x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{i}{4ax^5\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise}\end{cases}\right)}{5}-\frac{\operatorname{acos}(ax)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)/x**6,x)
```

```
[Out] -a*Piecewise((-3*a**4*acosh(1/(a*x))/8 + 3*a**3/(8*x*sqrt(-1 + 1/(a**2*x**2
))) - a/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x
**2))), 1/Abs(a**2*x**2) > 1), (3*I*a**4*asin(1/(a*x))/8 - 3*I*a**3/(8*x*sq
rt(1 - 1/(a**2*x**2))) + I*a/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*x**5
*sqrt(1 - 1/(a**2*x**2))), True))/5 - acos(a*x)/(5*x**5)
```

Giac [A] time = 1.15694, size = 120, normalized size = 1.5

$$-\frac{1}{80}a^5\left(\frac{2\left(3\left(-a^2x^2+1\right)^{\frac{3}{2}}-5\sqrt{-a^2x^2+1}\right)}{a^4x^4}-3\log\left(\sqrt{-a^2x^2+1}+1\right)+3\log\left(-\sqrt{-a^2x^2+1}+1\right)\right)-\frac{\arccos(ax)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)/x^6,x, algorithm="giac")
```

```
[Out] -1/80*a^5*(2*(3*(-a^2*x^2 + 1)^(3/2) - 5*sqrt(-a^2*x^2 + 1))/(a^4*x^4) - 3*
log(sqrt(-a^2*x^2 + 1) + 1) + 3*log(-sqrt(-a^2*x^2 + 1) + 1)) - 1/5*arccos(
a*x)/x^5
```

3.12 $\int x^4 \cos^{-1}(ax)^2 \, dx$

Optimal. Leaf size=120

$$-\frac{8x^3}{225a^2}-\frac{2x^4\sqrt{1-a^2x^2}\cos^{-1}(ax)}{25a}-\frac{8x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)}{75a^3}-\frac{16\sqrt{1-a^2x^2}\cos^{-1}(ax)}{75a^5}-\frac{16x}{75a^4}+\frac{1}{5}x^5\cos^{-1}(ax)^2-\frac{2x^5}{125}$$

```
[Out] (-16*x)/(75*a^4) - (8*x^3)/(225*a^2) - (2*x^5)/125 - (16*Sqrt[1 - a^2*x^2]*
ArcCos[a*x])/(75*a^5) - (8*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(75*a^3) - (2
*x^4*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(25*a) + (x^5*ArcCos[a*x]^2)/5
```

Rubi [A] time = 0.195771, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4628, 4708, 4678, 8, 30}

$$-\frac{8x^3}{225a^2}-\frac{2x^4\sqrt{1-a^2x^2}\cos^{-1}(ax)}{25a}-\frac{8x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)}{75a^3}-\frac{16\sqrt{1-a^2x^2}\cos^{-1}(ax)}{75a^5}-\frac{16x}{75a^4}+\frac{1}{5}x^5\cos^{-1}(ax)^2-\frac{2x^5}{125}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCos[a*x]^2,x]
```

```
[Out] (-16*x)/(75*a^4) - (8*x^3)/(225*a^2) - (2*x^5)/125 - (16*Sqrt[1 - a^2*x^2]*
ArcCos[a*x])/(75*a^5) - (8*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(75*a^3) - (2
*x^4*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(25*a) + (x^5*ArcCos[a*x]^2)/5
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

```
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^4 \cos^{-1}(ax)^2\,dx = \frac{1}{5}x^5\cos^{-1}(ax)^2 + \frac{1}{5}(2a)\int \frac{x^5\cos^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{2x^4\sqrt{1-a^2x^2}\cos^{-1}(ax)}{25a} + \frac{1}{5}x^5\cos^{-1}(ax)^2 - \frac{2\int x^4\,dx}{25} + \frac{8\int \frac{x^3\cos^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{25a}$$

$$= -\frac{2x^5}{125} - \frac{8x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)}{75a^3} - \frac{2x^4\sqrt{1-a^2x^2}\cos^{-1}(ax)}{25a} + \frac{1}{5}x^5\cos^{-1}(ax)^2 + \frac{16\int \frac{x\cos^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{75a^3}$$

$$= -\frac{8x^3}{225a^2} - \frac{2x^5}{125} - \frac{16\sqrt{1-a^2x^2}\cos^{-1}(ax)}{75a^5} - \frac{8x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)}{75a^3} - \frac{2x^4\sqrt{1-a^2x^2}\cos^{-1}(ax)}{25a} +$$

$$= -\frac{16x}{75a^4} - \frac{8x^3}{225a^2} - \frac{2x^5}{125} - \frac{16\sqrt{1-a^2x^2}\cos^{-1}(ax)}{75a^5} - \frac{8x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)}{75a^3} - \frac{2x^4\sqrt{1-a^2x^2}\cos^{-}}{25a}$$

Mathematica [A] time = 0.0561761, size = 82, normalized size = 0.68

$$-\frac{8x^3}{225a^2} - \frac{2\sqrt{1-a^2x^2}\left(3a^4x^4+4a^2x^2+8\right)\cos^{-1}(ax)}{75a^5} - \frac{16x}{75a^4} + \frac{1}{5}x^5\cos^{-1}(ax)^2 - \frac{2x^5}{125}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcCos[a*x]^2,x]
```

```
[Out] (-16*x)/(75*a^4) - (8*x^3)/(225*a^2) - (2*x^5)/125 - (2*Sqrt[1 - a^2*x^2]*(
8 + 4*a^2*x^2 + 3*a^4*x^4)*ArcCos[a*x])/(75*a^5) + (x^5*ArcCos[a*x]^2)/5
```

Maple [A] time = 0.05, size = 76, normalized size = 0.6

$$\frac{1}{a^5}\left(\frac{(\arccos(ax))^2 a^5x^5}{5}-\frac{2\arccos(ax)\left(3a^4x^4+4a^2x^2+8\right)}{75}\sqrt{-a^2x^2+1}-\frac{2a^5x^5}{125}-\frac{8a^3x^3}{225}-\frac{16ax}{75}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccos(a*x)^2,x)
```

```
[Out] 1/a^5*(1/5*arccos(a*x)^2*a^5*x^5-2/75*arccos(a*x)*(3*a^4*x^4+4*a^2*x^2+8)*(
-a^2*x^2+1)^(1/2)-2/125*a^5*x^5-8/225*a^3*x^3-16/75*a*x)
```

Maxima [A] time = 1.49582, size = 138, normalized size = 1.15

$$\frac{1}{5}x^5\arccos(ax)^2-\frac{2}{75}\left(\frac{3\sqrt{-a^2x^2+1}x^4}{a^2}+\frac{4\sqrt{-a^2x^2+1}x^2}{a^4}+\frac{8\sqrt{-a^2x^2+1}}{a^6}\right)a\arccos(ax)-\frac{2\left(9a^4x^5+20a^2x^3+120x\right)}{1125a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccos(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arccos(a*x)^2 - 2/75*(3*sqrt(-a^2*x^2 + 1)*x^4/a^2 + 4*sqrt(-a^2*x^
2 + 1)*x^2/a^4 + 8*sqrt(-a^2*x^2 + 1)/a^6)*a*arccos(a*x) - 2/1125*(9*a^4*x^
5 + 20*a^2*x^3 + 120*x)/a^4
```

Fricas [A] time = 2.00245, size = 189, normalized size = 1.58

$$\frac{225a^5x^5\arccos(ax)^2-18a^5x^5-40a^3x^3-30\left(3a^4x^4+4a^2x^2+8\right)\sqrt{-a^2x^2+1}\arccos(ax)-240ax}{1125a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccos(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/1125*(225*a^5*x^5*arccos(a*x)^2 - 18*a^5*x^5 - 40*a^3*x^3 - 30*(3*a^4*x^4
 + 4*a^2*x^2 + 8)*sqrt(-a^2*x^2 + 1)*arccos(a*x) - 240*a*x)/a^5
```

Sympy [A] time = 4.14903, size = 121, normalized size = 1.01

$$\begin{cases} \frac{x^5 \operatorname{acos}^2(ax)}{5} - \frac{2x^5}{125} - \frac{2x^4\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{25a} - \frac{8x^3}{225a^2} - \frac{8x^2\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{75a^3} - \frac{16x}{75a^4} - \frac{16\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{75a^5} & \text{for } a \neq 0 \\ \frac{\pi^2 x^5}{20} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acos(a*x)**2,x)
```

```
[Out] Piecewise((x**5*acos(a*x)**2/5 - 2*x**5/125 - 2*x**4*sqrt(-a**2*x**2 + 1)*a
cos(a*x)/(25*a) - 8*x**3/(225*a**2) - 8*x**2*sqrt(-a**2*x**2 + 1)*acos(a*x)
/(75*a**3) - 16*x/(75*a**4) - 16*sqrt(-a**2*x**2 + 1)*acos(a*x)/(75*a**5),
Ne(a, 0)), (pi**2*x**5/20, True))
```

Giac [A] time = 1.17294, size = 135, normalized size = 1.12

$$\frac{1}{5}x^5 \arccos(ax)^2 - \frac{2}{125}x^5 - \frac{2\sqrt{-a^2x^2+1}x^4 \arccos(ax)}{25a} - \frac{8x^3}{225a^2} - \frac{8\sqrt{-a^2x^2+1}x^2 \arccos(ax)}{75a^3} - \frac{16x}{75a^4} - \frac{16\sqrt{-a^2x^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] 1/5*x^5*arccos(a*x)^2 - 2/125*x^5 - 2/25*sqrt(-a^2*x^2 + 1)*x^4*arccos(a*x)
/a - 8/225*x^3/a^2 - 8/75*sqrt(-a^2*x^2 + 1)*x^2*arccos(a*x)/a^3 - 16/75*x/
a^4 - 16/75*sqrt(-a^2*x^2 + 1)*arccos(a*x)/a^5
```

3.13 $\int x^3 \cos^{-1}(ax)^2 \, dx$

Optimal. Leaf size=98

$$-\frac{3x^2}{32a^2}-\frac{x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)}{8a}-\frac{3x\sqrt{1-a^2x^2}\cos^{-1}(ax)}{16a^3}-\frac{3\cos^{-1}(ax)^2}{32a^4}+\frac{1}{4}x^4\cos^{-1}(ax)^2-\frac{x^4}{32}$$

```
[Out] (-3*x^2)/(32*a^2) - x^4/32 - (3*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(16*a^3) -
 (x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(8*a) - (3*ArcCos[a*x]^2)/(32*a^4) + (
x^4*ArcCos[a*x]^2)/4
```

Rubi [A] time = 0.169663, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4628, 4708, 4642, 30}

$$-\frac{3x^2}{32a^2}-\frac{x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)}{8a}-\frac{3x\sqrt{1-a^2x^2}\cos^{-1}(ax)}{16a^3}-\frac{3\cos^{-1}(ax)^2}{32a^4}+\frac{1}{4}x^4\cos^{-1}(ax)^2-\frac{x^4}{32}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCos[a*x]^2,x]
```

```
[Out] (-3*x^2)/(32*a^2) - x^4/32 - (3*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(16*a^3) -
 (x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(8*a) - (3*ArcCos[a*x]^2)/(32*a^4) + (
x^4*ArcCos[a*x]^2)/4
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

```
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^3 \cos^{-1}(ax)^2\,dx = \frac{1}{4}x^4\cos^{-1}(ax)^2 + \frac{1}{2}a\int \frac{x^4\cos^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)}{8a} + \frac{1}{4}x^4\cos^{-1}(ax)^2 - \frac{\int x^3\,dx}{8} + \frac{3\int \frac{x^2\cos^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{8a}$$

$$= -\frac{x^4}{32} - \frac{3x\sqrt{1-a^2x^2}\cos^{-1}(ax)}{16a^3} - \frac{x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)}{8a} + \frac{1}{4}x^4\cos^{-1}(ax)^2 + \frac{3\int \frac{\cos^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{16a^3} - \frac{3}{1}$$

$$= -\frac{3x^2}{32a^2} - \frac{x^4}{32} - \frac{3x\sqrt{1-a^2x^2}\cos^{-1}(ax)}{16a^3} - \frac{x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)}{8a} - \frac{3\cos^{-1}(ax)^2}{32a^4} + \frac{1}{4}x^4\cos^{-1}(ax)^2$$

Mathematica [A] time = 0.0376655, size = 74, normalized size = 0.76

$$\frac{-a^2x^2\left(a^2x^2+3\right) - 2ax\sqrt{1-a^2x^2}\left(2a^2x^2+3\right)\cos^{-1}(ax) + \left(8a^4x^4-3\right)\cos^{-1}(ax)^2}{32a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCos[a*x]^2,x]
```

```
[Out] (-(a^2*x^2*(3 + a^2*x^2)) - 2*a*x*Sqrt[1 - a^2*x^2]*(3 + 2*a^2*x^2)*ArcCos[
a*x] + (-3 + 8*a^4*x^4)*ArcCos[a*x]^2)/(32*a^4)
```

Maple [A] time = 0.054, size = 93, normalized size = 1.

$$\frac{1}{a^4}\left(\frac{a^4x^4\left(\arccos\left(ax\right)\right)^2}{4}-\frac{\arccos\left(ax\right)}{16}\left(2\,a^3x^3\sqrt{-a^2x^2+1}+3\,ax\sqrt{-a^2x^2+1}+3\,\arccos\left(ax\right)\right)+\frac{3\,\left(\arccos\left(ax\right)\right)^2}{32}-\frac{a^4x^4}{32}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccos(a*x)^2,x)
```

```
[Out] 1/a^4*(1/4*a^4*x^4*arccos(a*x)^2-1/16*arccos(a*x)*(2*a^3*x^3*(-a^2*x^2+1)^(
1/2)+3*a*x*(-a^2*x^2+1)^(1/2)+3*arccos(a*x))+3/32*arccos(a*x)^2-1/32*a^4*x^
4-3/32*a^2*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\,x^4\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^2-a\int\frac{\sqrt{ax+1}\sqrt{-ax+1}x^4\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)}{2\left(a^2x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2 - a*integrate(1/2*sqrt
(a*x + 1)*sqrt(-a*x + 1)*x^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)/(a^
2*x^2 - 1), x)
```

Fricas [A] time = 1.95436, size = 162, normalized size = 1.65

$$-\frac{a^4x^4+3\,a^2x^2-\left(8\,a^4x^4-3\right)\arccos\left(ax\right)^2+2\left(2\,a^3x^3+3\,ax\right)\sqrt{-a^2x^2+1}\arccos\left(ax\right)}{32\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/32*(a^4*x^4 + 3*a^2*x^2 - (8*a^4*x^4 - 3)*arccos(a*x)^2 + 2*(2*a^3*x^3 +
 3*a*x)*sqrt(-a^2*x^2 + 1)*arccos(a*x))/a^4
```

Sympy [A] time = 2.38094, size = 97, normalized size = 0.99

$$\begin{cases} \frac{x^4 \operatorname{acos}^2(ax)}{4} - \frac{x^4}{32} - \frac{x^3\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{8a} - \frac{3x^2}{32a^2} - \frac{3x\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{16a^3} - \frac{3\operatorname{acos}^2(ax)}{32a^4} & \text{for } a \neq 0 \\ \frac{\pi^2 x^4}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acos(a*x)**2,x)
```

```
[Out] Piecewise((x**4*acos(a*x)**2/4 - x**4/32 - x**3*sqrt(-a**2*x**2 + 1)*acos(a
*x)/(8*a) - 3*x**2/(32*a**2) - 3*x*sqrt(-a**2*x**2 + 1)*acos(a*x)/(16*a**3)
 - 3*acos(a*x)**2/(32*a**4), Ne(a, 0)), (pi**2*x**4/16, True))
```

Giac [A] time = 1.1796, size = 117, normalized size = 1.19

$$\frac{1}{4}x^4 \arccos(ax)^2 - \frac{1}{32}x^4 - \frac{\sqrt{-a^2x^2+1}x^3 \arccos(ax)}{8a} - \frac{3x^2}{32a^2} - \frac{3\sqrt{-a^2x^2+1}x \arccos(ax)}{16a^3} - \frac{3\arccos(ax)^2}{32a^4} + \frac{15}{256a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^4*arccos(a*x)^2 - 1/32*x^4 - 1/8*sqrt(-a^2*x^2 + 1)*x^3*arccos(a*x)/a
 - 3/32*x^2/a^2 - 3/16*sqrt(-a^2*x^2 + 1)*x*arccos(a*x)/a^3 - 3/32*arccos(a
*x)^2/a^4 + 15/256/a^4
```

3.14 $\int x^2 \cos^{-1}(ax)^2\,dx$

Optimal. Leaf size=82

$$-\frac{2x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)}{9a}-\frac{4\sqrt{1-a^2x^2}\cos^{-1}(ax)}{9a^3}-\frac{4x}{9a^2}+\frac{1}{3}x^3\cos^{-1}(ax)^2-\frac{2x^3}{27}$$

```
[Out] (-4*x)/(9*a^2) - (2*x^3)/27 - (4*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(9*a^3) - (
2*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(9*a) + (x^3*ArcCos[a*x]^2)/3
```

Rubi [A] time = 0.125599, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4628, 4708, 4678, 8, 30}

$$-\frac{2x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)}{9a}-\frac{4\sqrt{1-a^2x^2}\cos^{-1}(ax)}{9a^3}-\frac{4x}{9a^2}+\frac{1}{3}x^3\cos^{-1}(ax)^2-\frac{2x^3}{27}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCos[a*x]^2,x]
```

```
[Out] (-4*x)/(9*a^2) - (2*x^3)/27 - (4*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(9*a^3) - (
2*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(9*a) + (x^3*ArcCos[a*x]^2)/3
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2 \cos^{-1}(ax)^2\,dx &= \frac{1}{3}x^3\cos^{-1}(ax)^2 + \frac{1}{3}(2a)\int \frac{x^3\cos^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{2x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)}{9a} + \frac{1}{3}x^3\cos^{-1}(ax)^2 - \frac{2\int x^2\,dx}{9} + \frac{4\int \frac{x\cos^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{9a} \\
&= -\frac{2x^3}{27} - \frac{4\sqrt{1-a^2x^2}\cos^{-1}(ax)}{9a^3} - \frac{2x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)}{9a} + \frac{1}{3}x^3\cos^{-1}(ax)^2 - \frac{4\int 1\,dx}{9a^2} \\
&= -\frac{4x}{9a^2} - \frac{2x^3}{27} - \frac{4\sqrt{1-a^2x^2}\cos^{-1}(ax)}{9a^3} - \frac{2x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)}{9a} + \frac{1}{3}x^3\cos^{-1}(ax)^2
\end{aligned}$$

Mathematica [A] time = 0.0463105, size = 63, normalized size = 0.77

$$-\frac{2\sqrt{1-a^2x^2}\left(a^2x^2+2\right)\cos^{-1}(ax)}{9a^3} - \frac{4x}{9a^2} + \frac{1}{3}x^3\cos^{-1}(ax)^2 - \frac{2x^3}{27}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCos[a*x]^2,x]
```

```
[Out] (-4*x)/(9*a^2) - (2*x^3)/27 - (2*Sqrt[1 - a^2*x^2]*(2 + a^2*x^2)*ArcCos[a*x
])/(9*a^3) + (x^3*ArcCos[a*x]^2)/3
```

Maple [A] time = 0.048, size = 59, normalized size = 0.7

$$\frac{1}{a^{3}}\left(\frac{a^{3}x^{3}\left(\arccos\left(ax\right)\right)^{2}}{3}-\frac{2\arccos\left(ax\right)\left(a^{2}x^{2}+2\right)}{9}\sqrt{-a^{2}x^{2}+1}-\frac{2a^{3}x^{3}}{27}-\frac{4ax}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccos(a*x)^2,x)
```

```
[Out] 1/a^3*(1/3*a^3*x^3*arccos(a*x)^2-2/9*arccos(a*x)*(a^2*x^2+2)*(-a^2*x^2+1)^(
1/2)-2/27*a^3*x^3-4/9*a*x)
```

Maxima [A] time = 1.46577, size = 97, normalized size = 1.18

$$\frac{1}{3}x^{3}\arccos\left(ax\right)^{2}-\frac{2}{9}a\left(\frac{\sqrt{-a^{2}x^{2}+1}x^{2}}{a^{2}}+\frac{2\sqrt{-a^{2}x^{2}+1}}{a^{4}}\right)\arccos\left(ax\right)-\frac{2\left(a^{2}x^{3}+6x\right)}{27a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccos(a*x)^2 - 2/9*a*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^2
 + 1)/a^4)*arccos(a*x) - 2/27*(a^2*x^3 + 6*x)/a^2
```

Fricas [A] time = 1.97583, size = 143, normalized size = 1.74

$$\frac{9a^{3}x^{3}\arccos\left(ax\right)^{2}-2a^{3}x^{3}-6\left(a^{2}x^{2}+2\right)\sqrt{-a^{2}x^{2}+1}\arccos\left(ax\right)-12ax}{27a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/27*(9*a^3*x^3*arccos(a*x)^2 - 2*a^3*x^3 - 6*(a^2*x^2 + 2)*sqrt(-a^2*x^2 +
 1)*arccos(a*x) - 12*a*x)/a^3
```

Sympy [A] time = 1.15989, size = 83, normalized size = 1.01

$$\begin{cases} \frac{x^3 \operatorname{acos}^2(ax)}{3} - \frac{2x^3}{27} - \frac{2x^2\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{9a} - \frac{4x}{9a^2} - \frac{4\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{9a^3} & \text{for } a \neq 0 \\ \frac{\pi^2x^3}{12} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acos(a*x)**2,x)
```

```
[Out] Piecewise((x**3*acos(a*x)**2/3 - 2*x**3/27 - 2*x**2*sqrt(-a**2*x**2 + 1)*ac
os(a*x)/(9*a) - 4*x/(9*a**2) - 4*sqrt(-a**2*x**2 + 1)*acos(a*x)/(9*a**3), N
e(a, 0)), (pi**2*x**3/12, True))
```

Giac [A] time = 1.16626, size = 92, normalized size = 1.12

$$\frac{1}{3}\,x^3 \arccos(ax)^2 - \frac{2}{27}\,x^3 - \frac{2\,\sqrt{-a^2x^2+1}x^2 \arccos(ax)}{9\,a} - \frac{4\,x}{9\,a^2} - \frac{4\,\sqrt{-a^2x^2+1}\arccos(ax)}{9\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3*arccos(a*x)^2 - 2/27*x^3 - 2/9*sqrt(-a^2*x^2 + 1)*x^2*arccos(a*x)/a
 - 4/9*x/a^2 - 4/9*sqrt(-a^2*x^2 + 1)*arccos(a*x)/a^3
```

3.15 $\int x \cos^{-1}(ax)^2 \, dx$

Optimal. Leaf size=60

$$-\frac{x\sqrt{1-a^2x^2}\cos^{-1}(ax)}{2a}-\frac{\cos^{-1}(ax)^2}{4a^2}+\frac{1}{2}x^2\cos^{-1}(ax)^2-\frac{x^2}{4}$$

```
[Out] -x^2/4 - (x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(2*a) - ArcCos[a*x]^2/(4*a^2) +
(x^2*ArcCos[a*x]^2)/2
```

Rubi [A] time = 0.0966761, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4628, 4708, 4642, 30}

$$-\frac{x\sqrt{1-a^2x^2}\cos^{-1}(ax)}{2a}-\frac{\cos^{-1}(ax)^2}{4a^2}+\frac{1}{2}x^2\cos^{-1}(ax)^2-\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCos[a*x]^2,x]
```

```
[Out] -x^2/4 - (x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(2*a) - ArcCos[a*x]^2/(4*a^2) +
(x^2*ArcCos[a*x]^2)/2
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\cos^{-1}(ax)^2\,dx &= \frac{1}{2}x^2\cos^{-1}(ax)^2 + a\int\frac{x^2\cos^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{x\sqrt{1-a^2x^2}\cos^{-1}(ax)}{2a} + \frac{1}{2}x^2\cos^{-1}(ax)^2 - \frac{\int x\,dx}{2} + \frac{\int\frac{\cos^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{2a}\\ &= -\frac{x^2}{4} - \frac{x\sqrt{1-a^2x^2}\cos^{-1}(ax)}{2a} - \frac{\cos^{-1}(ax)^2}{4a^2} + \frac{1}{2}x^2\cos^{-1}(ax)^2\end{aligned}$$

Mathematica [A] time = 0.0249349, size = 57, normalized size = 0.95

$$-\frac{x\sqrt{1-a^2x^2}\cos^{-1}(ax)}{2a} + \frac{\left(2a^2x^2-1\right)\cos^{-1}(ax)^2}{4a^2} - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCos[a*x]^2,x]
```

```
[Out] -x^2/4 - (x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(2*a) + ((-1 + 2*a^2*x^2)*ArcCos
[a*x]^2)/(4*a^2)
```

Maple [A] time = 0.048, size = 63, normalized size = 1.1

$$\frac{1}{a^2}\left(\frac{a^2x^2\left(\arccos(ax)\right)^2}{2} - \frac{\arccos(ax)}{2}\left(ax\sqrt{-a^2x^2+1} + \arccos(ax)\right) + \frac{\left(\arccos(ax)\right)^2}{4} - \frac{a^2x^2}{4} + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccos(a*x)^2,x)
```

```
[Out] 1/a^2*(1/2*a^2*x^2*arccos(a*x)^2-1/2*arccos(a*x)*(a*x*(-a^2*x^2+1)^(1/2)+ar
ccos(a*x))+1/4*arccos(a*x)^2-1/4*a^2*x^2+1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x^2\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^2 - a\int\frac{\sqrt{ax+1}\sqrt{-ax+1}x^2\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2 - a*integrate(sqrt(a*x
 + 1)*sqrt(-a*x + 1)*x^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)/(a^2*x^
2 - 1), x)
```

Fricas [A] time = 1.92655, size = 123, normalized size = 2.05

$$-\frac{a^2x^2 + 2\sqrt{-a^2x^2+1}ax\arccos(ax) - \left(2a^2x^2-1\right)\arccos(ax)^2}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(a^2*x^2 + 2*sqrt(-a^2*x^2 + 1)*a*x*arccos(a*x) - (2*a^2*x^2 - 1)*arcc
os(a*x)^2)/a^2
```

Sympy [A] time = 0.589721, size = 58, normalized size = 0.97

$$\begin{cases} \frac{x^2\operatorname{acos}^2(ax)}{2} - \frac{x^2}{4} - \frac{x\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{2a} - \frac{\operatorname{acos}^2(ax)}{4a^2} & \text{for } a \neq 0 \\ \frac{\pi^2x^2}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acos(a*x)**2,x)
```

```
[Out] Piecewise((x**2*acos(a*x)**2/2 - x**2/4 - x*sqrt(-a**2*x**2 + 1)*acos(a*x)/
(2*a) - acos(a*x)**2/(4*a**2), Ne(a, 0)), (pi**2*x**2/8, True))
```

Giac [A] time = 1.13698, size = 74, normalized size = 1.23

$$\frac{1}{2}x^2\arccos(ax)^2 - \frac{1}{4}x^2 - \frac{\sqrt{-a^2x^2+1}x\arccos(ax)}{2a} - \frac{\arccos(ax)^2}{4a^2} + \frac{1}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2*arccos(a*x)^2 - 1/4*x^2 - 1/2*sqrt(-a^2*x^2 + 1)*x*arccos(a*x)/a -
1/4*arccos(a*x)^2/a^2 + 1/8/a^2
```

3.16 $\int \cos^{-1}(ax)^2\,dx$

Optimal. Leaf size=35

$$-\frac{2\sqrt{1-a^2x^2}\cos^{-1}(ax)}{a}+x\cos^{-1}(ax)^2-2x$$

```
[Out] -2*x - (2*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a + x*ArcCos[a*x]^2
```

Rubi [A] time = 0.0468431, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4620, 4678, 8}

$$-\frac{2\sqrt{1-a^2x^2}\cos^{-1}(ax)}{a}+x\cos^{-1}(ax)^2-2x$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^2,x]
```

```
[Out] -2*x - (2*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a + x*ArcCos[a*x]^2
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^{-1}(ax)^2\,dx = x\cos^{-1}(ax)^2 + (2a)\int \frac{x\cos^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{2\sqrt{1-a^2x^2}\cos^{-1}(ax)}{a} + x\cos^{-1}(ax)^2 - 2\int 1\,dx$$
$$= -2x - \frac{2\sqrt{1-a^2x^2}\cos^{-1}(ax)}{a} + x\cos^{-1}(ax)^2$$

Mathematica [A] time = 0.0161168, size = 35, normalized size = 1.

$$-\frac{2\sqrt{1-a^2x^2}\cos^{-1}(ax)}{a} + x\cos^{-1}(ax)^2 - 2x$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x]^2,x]
```

```
[Out] -2*x - (2*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a + x*ArcCos[a*x]^2
```

Maple [A] time = 0.046, size = 37, normalized size = 1.1

$$\frac{1}{a}\left(ax\left(\arccos\left(ax\right)\right)^{2} - 2\,ax - 2\,\arccos\left(ax\right)\sqrt{-a^2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^2,x)
```

```
[Out] 1/a*(a*x*arccos(a*x)^2-2*a*x-2*arccos(a*x)*(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.43443, size = 45, normalized size = 1.29

$$x\arccos\left(ax\right)^{2} - 2\,x - \frac{2\,\sqrt{-a^2x^2+1}\arccos\left(ax\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^2,x, algorithm="maxima")
```

```
[Out] x*arccos(a*x)^2 - 2*x - 2*sqrt(-a^2*x^2 + 1)*arccos(a*x)/a
```

Fricas [A] time = 1.87394, size = 89, normalized size = 2.54

$$\frac{ax \arccos(ax)^2 - 2\,ax - 2\sqrt{-a^2x^2+1}\arccos(ax)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^2,x, algorithm="fricas")
```

```
[Out] (a*x*arccos(a*x)^2 - 2*a*x - 2*sqrt(-a^2*x^2 + 1)*arccos(a*x))/a
```

Sympy [A] time = 0.238852, size = 37, normalized size = 1.06

$$\begin{cases} x\operatorname{acos}^2(ax) - 2x - \frac{2\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{a} & \text{for } a \neq 0 \\ \frac{\pi^2 x}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**2,x)
```

```
[Out] Piecewise((x*acos(a*x)**2 - 2*x - 2*sqrt(-a**2*x**2 + 1)*acos(a*x)/a, Ne(a,
 0)), (pi**2*x/4, True))
```

Giac [A] time = 1.13932, size = 45, normalized size = 1.29

$$x \arccos(ax)^2 - 2\,x - \frac{2\sqrt{-a^2x^2+1}\arccos(ax)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] x*arccos(a*x)^2 - 2*x - 2*sqrt(-a^2*x^2 + 1)*arccos(a*x)/a
```

3.17 $\int \frac{\cos^{-1}(ax)^2}{x}\,dx$

Optimal. Leaf size=73

$$-i\cos^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{2i\cos^{-1}(ax)}\right)+\frac{1}{2}\mathrm{PolyLog}\left(3,-e^{2i\cos^{-1}(ax)}\right)-\frac{1}{3}i\cos^{-1}(ax)^3+\cos^{-1}(ax)^2\log\left(1+e^{2i\cos^{-1}(ax)}\right)$$

```
[Out] (-I/3)*ArcCos[a*x]^3 + ArcCos[a*x]^2*Log[1 + E^((2*I)*ArcCos[a*x])] - I*Arc
Cos[a*x]*PolyLog[2, -E^((2*I)*ArcCos[a*x])] + PolyLog[3, -E^((2*I)*ArcCos[a
*x])]/2
```

Rubi [A] time = 0.0933503, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4626, 3719, 2190, 2531, 2282, 6589}

$$-i\cos^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{2i\cos^{-1}(ax)}\right)+\frac{1}{2}\mathrm{PolyLog}\left(3,-e^{2i\cos^{-1}(ax)}\right)-\frac{1}{3}i\cos^{-1}(ax)^3+\cos^{-1}(ax)^2\log\left(1+e^{2i\cos^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^2/x,x]
```

```
[Out] (-I/3)*ArcCos[a*x]^3 + ArcCos[a*x]^2*Log[1 + E^((2*I)*ArcCos[a*x])] - I*Arc
Cos[a*x]*PolyLog[2, -E^((2*I)*ArcCos[a*x])] + PolyLog[3, -E^((2*I)*ArcCos[a
*x])]/2
```

Rule 4626

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n/Cot[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cos^{-1}(ax)^2}{x}\,dx = -\operatorname{Subst}\left(\int x^2\tan(x)\,dx, x, \cos^{-1}(ax)\right)$$

$$= -\frac{1}{3}i\cos^{-1}(ax)^3 + 2i\operatorname{Subst}\left(\int \frac{e^{2ix}x^2}{1+e^{2ix}}\,dx, x, \cos^{-1}(ax)\right)$$

$$= -\frac{1}{3}i\cos^{-1}(ax)^3 + \cos^{-1}(ax)^2\log\left(1+e^{2i\cos^{-1}(ax)}\right) - 2\operatorname{Subst}\left(\int x\log\left(1+e^{2ix}\right)dx, x, \cos^{-1}(ax)\right)$$

$$= -\frac{1}{3}i\cos^{-1}(ax)^3 + \cos^{-1}(ax)^2\log\left(1+e^{2i\cos^{-1}(ax)}\right) - i\cos^{-1}(ax)\mathrm{Li}_2\left(-e^{2i\cos^{-1}(ax)}\right) + i\operatorname{Subst}\left(\int \mathrm{Li}_2\left(-\right.\right.$$

$$= -\frac{1}{3}i\cos^{-1}(ax)^3 + \cos^{-1}(ax)^2\log\left(1+e^{2i\cos^{-1}(ax)}\right) - i\cos^{-1}(ax)\mathrm{Li}_2\left(-e^{2i\cos^{-1}(ax)}\right) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{\mathrm{Li}_2(-}{x}\right.$$

$$= -\frac{1}{3}i\cos^{-1}(ax)^3 + \cos^{-1}(ax)^2\log\left(1+e^{2i\cos^{-1}(ax)}\right) - i\cos^{-1}(ax)\mathrm{Li}_2\left(-e^{2i\cos^{-1}(ax)}\right) + \frac{1}{2}\mathrm{Li}_3\left(-e^{2i\cos^{-1}(ax)}\right.$$

Mathematica [A] time = 0.0196891, size = 73, normalized size = 1.

$$-i\cos^{-1}(ax)\text{PolyLog}\left(2,-e^{2i\cos^{-1}(ax)}\right)+\frac{1}{2}\text{PolyLog}\left(3,-e^{2i\cos^{-1}(ax)}\right)-\frac{1}{3}i\cos^{-1}(ax)^3+\cos^{-1}(ax)^2\log\left(1+e^{2i\cos^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x]^2/x,x]
```

```
[Out] (-I/3)*ArcCos[a*x]^3 + ArcCos[a*x]^2*Log[1 + E^((2*I)*ArcCos[a*x])] - I*Arc
Cos[a*x]*PolyLog[2, -E^((2*I)*ArcCos[a*x])] + PolyLog[3, -E^((2*I)*ArcCos[a
*x])]/2
```

Maple [A] time = 0.054, size = 101, normalized size = 1.4

$$-\frac{i}{3}(\arccos(ax))^3+(\arccos(ax))^2\ln\left(1+\left(i\sqrt{-a^2x^2+1}+ax\right)^2\right)-i\arccos(ax)\operatorname{polylog}\left(2,-\left(i\sqrt{-a^2x^2+1}+ax\right)^2\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^2/x,x)
```

```
[Out] -1/3*I*arccos(a*x)^3+arccos(a*x)^2*ln(1+(I*(-a^2*x^2+1)^(1/2)+a*x)^2)-I*arc
cos(a*x)*polylog(2,-(I*(-a^2*x^2+1)^(1/2)+a*x)^2)+1/2*polylog(3,-(I*(-a^2*x
^2+1)^(1/2)+a*x)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\arccos(ax)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(arccos(a*x)^2/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arccos(ax)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)^2/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acos}^2(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**2/x,x)
```

```
[Out] Integral(acos(a*x)**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(ax)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^2/x, x)
```

3.18 $\int \frac{\cos^{-1}(ax)^2}{x^2}\,dx$

Optimal. Leaf size=74

$$2ia\text{PolyLog}\left(2,-ie^{i\cos^{-1}(ax)}\right)-2ia\text{PolyLog}\left(2,ie^{i\cos^{-1}(ax)}\right)-\frac{\cos^{-1}(ax)^2}{x}-4ia\cos^{-1}(ax)\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right)$$

```
[Out] -(ArcCos[a*x]^2/x) - (4*I)*a*ArcCos[a*x]*ArcTan[E^(I*ArcCos[a*x])] + (2*I)*
a*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] - (2*I)*a*PolyLog[2, I*E^(I*ArcCos[a*x
])]
```

Rubi [A] time = 0.106937, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4628, 4710, 4181, 2279, 2391}

$$2ia\text{PolyLog}\left(2,-ie^{i\cos^{-1}(ax)}\right)-2ia\text{PolyLog}\left(2,ie^{i\cos^{-1}(ax)}\right)-\frac{\cos^{-1}(ax)^2}{x}-4ia\cos^{-1}(ax)\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^2/x^2,x]
```

```
[Out] -(ArcCos[a*x]^2/x) - (4*I)*a*ArcCos[a*x]*ArcTan[E^(I*ArcCos[a*x])] + (2*I)*
a*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] - (2*I)*a*PolyLog[2, I*E^(I*ArcCos[a*x
])]
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4710

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> -Dist[(c^(m + 1)*Sqrt[d])^(-1), Subst[Int[(a + b*x)^n
*Cos[x]^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{-1}(ax)^2}{x^2}\,dx &= -\frac{\cos^{-1}(ax)^2}{x} - (2a)\int \frac{\cos^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{\cos^{-1}(ax)^2}{x} + (2a)\,\text{Subst}\left(\int x\sec(x)\,dx, x, \cos^{-1}(ax)\right)\\ &= -\frac{\cos^{-1}(ax)^2}{x} - 4ia\cos^{-1}(ax)\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right) - (2a)\,\text{Subst}\left(\int \log\left(1-ie^{ix}\right)dx, x, \cos^{-1}(ax)\right) + (2a\\ &= -\frac{\cos^{-1}(ax)^2}{x} - 4ia\cos^{-1}(ax)\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right) + (2ia)\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{i\cos^{-1}(ax)}\right) - (2ia)\\ &= -\frac{\cos^{-1}(ax)^2}{x} - 4ia\cos^{-1}(ax)\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right) + 2ia\text{Li}_2\left(-ie^{i\cos^{-1}(ax)}\right) - 2ia\text{Li}_2\left(ie^{i\cos^{-1}(ax)}\right)\end{aligned}$$

Mathematica [A] time = 0.131581, size = 98, normalized size = 1.32

$$2ia\text{PolyLog}\left(2,-ie^{i\cos^{-1}(ax)}\right) - 2ia\text{PolyLog}\left(2,ie^{i\cos^{-1}(ax)}\right) - \frac{\cos^{-1}(ax)\left(\cos^{-1}(ax) + 2ax\left(\log\left(1+ie^{i\cos^{-1}(ax)}\right) - \log\left(1-\right.\right.\right.}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCos[a*x]^2/x^2,x]
```

```
[Out] -((ArcCos[a*x]*(ArcCos[a*x] + 2*a*x*(-Log[1 - I*E^(I*ArcCos[a*x])] + Log[1
+ I*E^(I*ArcCos[a*x])])))/x) + (2*I)*a*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] -
```

```
(2*I)*a*PolyLog[2, I*E^(I*ArcCos[a*x])]
```

Maple [A] time = 0.105, size = 135, normalized size = 1.8

$$-\frac{(\arccos(ax))^2}{x} - 2\,a\arccos(ax)\ln\left(1+i\left(i\sqrt{-a^2x^2+1}+ax\right)\right) + 2\,a\arccos(ax)\ln\left(1-i\left(i\sqrt{-a^2x^2+1}+ax\right)\right) + 2\,ia\mathrm{di}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^2/x^2,x)
```

```
[Out] -arccos(a*x)^2/x-2*a*arccos(a*x)*ln(1+I*(I*(-a^2*x^2+1)^(1/2)+a*x))+2*a*arc
cos(a*x)*ln(1-I*(I*(-a^2*x^2+1)^(1/2)+a*x))+2*I*a*dilog(1+I*(I*(-a^2*x^2+1)
^(1/2)+a*x))-2*I*a*dilog(1-I*(I*(-a^2*x^2+1)^(1/2)+a*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,ax\int\frac{\sqrt{-ax+1}\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)}{\sqrt{ax+1}(ax-1)x}\,dx - \arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] (2*a*x*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x + 1)*sqrt(-a
*x + 1), a*x)/(a^2*x^3 - x), x) - arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x
)^2)/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arccos(ax)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)^2/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acos}^2(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**2/x**2,x)
```

```
[Out] Integral(acos(a*x)**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(ax)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^2/x^2, x)
```

3.19 $\int \frac{\cos^{-1}(ax)^2}{x^3}\,dx$

Optimal. Leaf size=43

$$\frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)}{x}+a^2\log(x)-\frac{\cos^{-1}(ax)^2}{2x^2}$$

```
[Out] (a*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/x - ArcCos[a*x]^2/(2*x^2) + a^2*Log[x]
```

Rubi [A] time = 0.0783459, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4628, 4682, 29}

$$\frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)}{x}+a^2\log(x)-\frac{\cos^{-1}(ax)^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^2/x^3,x]
```

```
[Out] (a*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/x - ArcCos[a*x]^2/(2*x^2) + a^2*Log[x]
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4682

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b
*ArcCos[c*x])^n)/(d*f*(m + 1)), x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^F
racPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 - c
^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &
& NeQ[m, -1]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{-1}(ax)^2}{x^3}\,dx &= -\frac{\cos^{-1}(ax)^2}{2x^2} - a\int \frac{\cos^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx\\ &= \frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)}{x} - \frac{\cos^{-1}(ax)^2}{2x^2} + a^2\int \frac{1}{x}\,dx\\ &= \frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)}{x} - \frac{\cos^{-1}(ax)^2}{2x^2} + a^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.027813, size = 43, normalized size = 1.

$$\frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)}{x} + a^2\log(x) - \frac{\cos^{-1}(ax)^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x]^2/x^3,x]
```

```
[Out] (a*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/x - ArcCos[a*x]^2/(2*x^2) + a^2*Log[x]
```

Maple [A] time = 0.051, size = 42, normalized size = 1.

$$-\frac{(\arccos(ax))^2}{2x^2} + \frac{a\arccos(ax)}{x}\sqrt{-a^2x^2+1} + a^2\ln(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^2/x^3,x)
```

```
[Out] -1/2*arccos(a*x)^2/x^2+a*arccos(a*x)*(-a^2*x^2+1)^(1/2)/x+a^2*ln(a*x)
```

Maxima [A] time = 1.52371, size = 53, normalized size = 1.23

$$a^2\log(x) + \frac{\sqrt{-a^2x^2+1}\,a\arccos(ax)}{x} - \frac{\arccos(ax)^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + sqrt(-a^2*x^2 + 1)*a*arccos(a*x)/x - 1/2*arccos(a*x)^2/x^2
```

Fricas [A] time = 2.39595, size = 112, normalized size = 2.6

$$\frac{2\,a^2x^2\log(x) + 2\sqrt{-a^2x^2+1}ax\arccos(ax) - \arccos(ax)^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*x^2*log(x) + 2*sqrt(-a^2*x^2 + 1)*a*x*arccos(a*x) - arccos(a*x)^
2)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acos}^2(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**2/x**3,x)
```

```
[Out] Integral(acos(a*x)**2/x**3, x)
```

Giac [B] time = 1.19005, size = 117, normalized size = 2.72

$$-\frac{1}{2}\left(\left(\frac{a^4x}{\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|} - \frac{\sqrt{-a^2x^2+1}|a|+a}{x|a|}\right)\arccos(ax) - a\log\left(a^2x^2\right)\right)a - \frac{\arccos(ax)^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] -1/2*((a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - (sqrt(-a^2*x^2 + 1)
*abs(a) + a)/(x*abs(a)))*arccos(a*x) - a*log(a^2*x^2))*a - 1/2*arccos(a*x)^
2/x^2
```

3.20 $\int \frac{\cos^{-1}(ax)^2}{x^4}\,dx$

Optimal. Leaf size=124

$$\frac{1}{3}ia^3\text{PolyLog}\left(2,-ie^{i\cos^{-1}(ax)}\right)-\frac{1}{3}ia^3\text{PolyLog}\left(2,ie^{i\cos^{-1}(ax)}\right)+\frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)}{3x^2}-\frac{a^2}{3x}-\frac{2}{3}ia^3\cos^{-1}(ax)\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right)$$

```
[Out] -a^2/(3*x) + (a*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(3*x^2) - ArcCos[a*x]^2/(3*x
^3) - ((2*I)/3)*a^3*ArcCos[a*x]*ArcTan[E^(I*ArcCos[a*x])] + (I/3)*a^3*PolyL
og[2, (-I)*E^(I*ArcCos[a*x])] - (I/3)*a^3*PolyLog[2, I*E^(I*ArcCos[a*x])]
```

Rubi [A] time = 0.176919, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {4628, 4702, 4710, 4181, 2279, 2391, 30}

$$\frac{1}{3}ia^3\text{PolyLog}\left(2,-ie^{i\cos^{-1}(ax)}\right)-\frac{1}{3}ia^3\text{PolyLog}\left(2,ie^{i\cos^{-1}(ax)}\right)+\frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)}{3x^2}-\frac{a^2}{3x}-\frac{2}{3}ia^3\cos^{-1}(ax)\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^2/x^4,x]
```

```
[Out] -a^2/(3*x) + (a*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(3*x^2) - ArcCos[a*x]^2/(3*x
^3) - ((2*I)/3)*a^3*ArcCos[a*x]*ArcTan[E^(I*ArcCos[a*x])] + (I/3)*a^3*PolyL
og[2, (-I)*E^(I*ArcCos[a*x])] - (I/3)*a^3*PolyLog[2, I*E^(I*ArcCos[a*x])]
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4702

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b
*ArcCos[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)
), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart
[p]), Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1)
```

```
, x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 4710

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> -Dist[(c^(m + 1)*Sqrt[d])^(-1), Subst[Int[(a + b*x)^n
*Cos[x]^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^{-1}(ax)^2}{x^4}\,dx = -\frac{\cos^{-1}(ax)^2}{3x^3} - \frac{1}{3}(2a)\int \frac{\cos^{-1}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx$$

$$= \frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)}{3x^2} - \frac{\cos^{-1}(ax)^2}{3x^3} + \frac{1}{3}a^2\int \frac{1}{x^2}\,dx - \frac{1}{3}a^3\int \frac{\cos^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{a^2}{3x} + \frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)}{3x^2} - \frac{\cos^{-1}(ax)^2}{3x^3} + \frac{1}{3}a^3\,\text{Subst}\left(\int x\sec(x)\,dx, x, \cos^{-1}(ax)\right)$$

$$= -\frac{a^2}{3x} + \frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)}{3x^2} - \frac{\cos^{-1}(ax)^2}{3x^3} - \frac{2}{3}ia^3\cos^{-1}(ax)\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right) - \frac{1}{3}a^3\,\text{Subst}\left(\int \log$$

$$= -\frac{a^2}{3x} + \frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)}{3x^2} - \frac{\cos^{-1}(ax)^2}{3x^3} - \frac{2}{3}ia^3\cos^{-1}(ax)\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right) + \frac{1}{3}\left(ia^3\right)\text{Subst}\left(\int$$

$$= -\frac{a^2}{3x} + \frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)}{3x^2} - \frac{\cos^{-1}(ax)^2}{3x^3} - \frac{2}{3}ia^3\cos^{-1}(ax)\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right) + \frac{1}{3}ia^3\text{Li}_2\left(-ie^{i\cos^{-1}}$$

Mathematica [A] time = 0.571418, size = 152, normalized size = 1.23

$$-\frac{-ia^3x^3\text{PolyLog}\left(2,-ie^{i\cos^{-1}(ax)}\right) + ia^3x^3\text{PolyLog}\left(2,ie^{i\cos^{-1}(ax)}\right) + a^2x^2 - ax\sqrt{1-a^2x^2}\cos^{-1}(ax) - a^3x^3\cos^{-1}(ax)\log}{3x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCos[a*x]^2/x^4,x]
```

```
[Out] -(a^2*x^2 - a*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x] + ArcCos[a*x]^2 - a^3*x^3*Arc
Cos[a*x]*Log[1 - I*E^(I*ArcCos[a*x])] + a^3*x^3*ArcCos[a*x]*Log[1 + I*E^(I*
ArcCos[a*x])] - I*a^3*x^3*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] + I*a^3*x^3*Po
lyLog[2, I*E^(I*ArcCos[a*x])])/(3*x^3)
```

Maple [A] time = 0.19, size = 173, normalized size = 1.4

$$\frac{a\arccos(ax)}{3x^2}\sqrt{-a^2x^2+1} - \frac{a^2}{3x} - \frac{(\arccos(ax))^2}{3x^3} - \frac{a^3\arccos(ax)}{3}\ln\left(1+i\left(i\sqrt{-a^2x^2+1}+ax\right)\right) + \frac{a^3\arccos(ax)}{3}\ln\left(1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^2/x^4,x)
```

```
[Out] 1/3*a*arccos(a*x)*(-a^2*x^2+1)^(1/2)/x^2-1/3*a^2/x-1/3*arccos(a*x)^2/x^3-1/
3*a^3*arccos(a*x)*ln(1+I*(I*(-a^2*x^2+1)^(1/2)+a*x))+1/3*a^3*arccos(a*x)*ln
(1-I*(I*(-a^2*x^2+1)^(1/2)+a*x))+1/3*I*a^3*dilog(1+I*(I*(-a^2*x^2+1)^(1/2)+
a*x))-1/3*I*a^3*dilog(1-I*(I*(-a^2*x^2+1)^(1/2)+a*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,ax^3 \int \frac{\sqrt{ax+1}\sqrt{-ax+1}\,\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)}{a^2x^5-x^3}\,dx - \arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(6*a*x^3*integrate(1/3*sqrt(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x +
1)*sqrt(-a*x + 1), a*x)/(a^2*x^5 - x^3), x) - arctan2(sqrt(a*x + 1)*sqrt(-a
*x + 1), a*x)^2)/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arccos\left(ax\right)^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)^2/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acos}^2\left(ax\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**2/x**4,x)
```

```
[Out] Integral(acos(a*x)**2/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos{(ax)}^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^2/x^4, x)
```

3.21 $\int \frac{\cos^{-1}(ax)^2}{x^5} \, dx$

Optimal. Leaf size=87

$$-\frac{a^2}{12x^2}+\frac{a^3\sqrt{1-a^2x^2}\cos^{-1}(ax)}{3x}+\frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)}{6x^3}+\frac{1}{3}a^4\log(x)-\frac{\cos^{-1}(ax)^2}{4x^4}$$

```
[Out] -a^2/(12*x^2) + (a*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(6*x^3) + (a^3*Sqrt[1 - a
^2*x^2]*ArcCos[a*x])/(3*x) - ArcCos[a*x]^2/(4*x^4) + (a^4*Log[x])/3
```

Rubi [A] time = 0.151741, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4628, 4702, 4682, 29, 30}

$$-\frac{a^2}{12x^2}+\frac{a^3\sqrt{1-a^2x^2}\cos^{-1}(ax)}{3x}+\frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)}{6x^3}+\frac{1}{3}a^4\log(x)-\frac{\cos^{-1}(ax)^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^2/x^5,x]
```

```
[Out] -a^2/(12*x^2) + (a*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(6*x^3) + (a^3*Sqrt[1 - a
^2*x^2]*ArcCos[a*x])/(3*x) - ArcCos[a*x]^2/(4*x^4) + (a^4*Log[x])/3
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4702

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b
*ArcCos[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)
), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart
[p]), Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 4682

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b
*ArcCos[c*x])^n)/(d*f*(m + 1)), x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^F
racPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 - c
^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &
& NeQ[m, -1]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}(ax)^2}{x^5}\,dx &= -\frac{\cos^{-1}(ax)^2}{4x^4} - \frac{1}{2}a\int \frac{\cos^{-1}(ax)}{x^4\sqrt{1-a^2x^2}}\,dx \\
&= \frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)}{6x^3} - \frac{\cos^{-1}(ax)^2}{4x^4} + \frac{1}{6}a^2\int \frac{1}{x^3}\,dx - \frac{1}{3}a^3\int \frac{\cos^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{a^2}{12x^2} + \frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)}{6x^3} + \frac{a^3\sqrt{1-a^2x^2}\cos^{-1}(ax)}{3x} - \frac{\cos^{-1}(ax)^2}{4x^4} + \frac{1}{3}a^4\int \frac{1}{x}\,dx \\
&= -\frac{a^2}{12x^2} + \frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)}{6x^3} + \frac{a^3\sqrt{1-a^2x^2}\cos^{-1}(ax)}{3x} - \frac{\cos^{-1}(ax)^2}{4x^4} + \frac{1}{3}a^4\log(x)
\end{aligned}$$

Mathematica [A] time = 0.0408128, size = 69, normalized size = 0.79

$$-\frac{a^2}{12x^2} + \frac{a\sqrt{1-a^2x^2}\left(2a^2x^2+1\right)\cos^{-1}(ax)}{6x^3} + \frac{1}{3}a^4\log(x) - \frac{\cos^{-1}(ax)^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x]^2/x^5,x]
```

```
[Out] -a^2/(12*x^2) + (a*Sqrt[1 - a^2*x^2]*(1 + 2*a^2*x^2)*ArcCos[a*x])/(6*x^3) -
 ArcCos[a*x]^2/(4*x^4) + (a^4*Log[x])/3
```

Maple [A] time = 0.053, size = 76, normalized size = 0.9

$$-\frac{(\arccos(ax))^2}{4x^4}+\frac{a\arccos(ax)}{6x^3}\sqrt{-a^2x^2+1}-\frac{a^2}{12x^2}+\frac{a^3\arccos(ax)}{3x}\sqrt{-a^2x^2+1}+\frac{a^4\ln(ax)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^2/x^5,x)
```

```
[Out] -1/4*arccos(a*x)^2/x^4+1/6*a*arccos(a*x)*(-a^2*x^2+1)^(1/2)/x^3-1/12*a^2/x^
2+1/3*a^3*arccos(a*x)*(-a^2*x^2+1)^(1/2)/x+1/3*a^4*ln(a*x)
```

Maxima [A] time = 1.49606, size = 100, normalized size = 1.15

$$\frac{1}{12}\left(4a^2\log(x)-\frac{1}{x^2}\right)a^2+\frac{1}{6}\left(\frac{2\sqrt{-a^2x^2+1}a^2}{x}+\frac{\sqrt{-a^2x^2+1}}{x^3}\right)a\arccos(ax)-\frac{\arccos(ax)^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*(4*a^2*log(x) - 1/x^2)*a^2 + 1/6*(2*sqrt(-a^2*x^2 + 1)*a^2/x + sqrt(-a
^2*x^2 + 1)/x^3)*a*arccos(a*x) - 1/4*arccos(a*x)^2/x^4
```

Fricas [A] time = 2.38229, size = 149, normalized size = 1.71

$$\frac{4a^4x^4\log(x)-a^2x^2+2\left(2a^3x^3+ax\right)\sqrt{-a^2x^2+1}\arccos(ax)-3\arccos(ax)^2}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*a^4*x^4*log(x) - a^2*x^2 + 2*(2*a^3*x^3 + a*x)*sqrt(-a^2*x^2 + 1)*a
rccos(a*x) - 3*arccos(a*x)^2)/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acos}^2(ax)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**2/x**5,x)
```

```
[Out] Integral(acos(a*x)**2/x**5, x)
```

Giac [B] time = 1.31776, size = 223, normalized size = 2.56

$$-\frac{1}{48}\left(\left(\left(\frac{\left(a^4+\frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{x^2}\right)a^6x^3}{\left(\sqrt{-a^2x^2+1}|a|+a\right)^3|a|}-\frac{\frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)a^4}{x}+\frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{x^3}}{a^2|a|}\right)\arccos(ax)-\frac{4\left(2a^4\log\left(a^2x^2\right)-\frac{a^2}{x^2}\right)}{a}\right)a-\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^2/x^5,x, algorithm="giac")
```

```
[Out] -1/48*(((a^4 + 9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/x^2)*a^6*x^3/((sqrt(-a^2
*x^2 + 1)*abs(a) + a)^3*abs(a)) - (9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4/x
+ (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/x^3)/(a^2*abs(a)))*arccos(a*x) - 4*(2*a
^4*log(a^2*x^2) - a^2/x^2)/a)*a - 1/4*arccos(a*x)^2/x^4
```

3.22 $\int x^4 \cos^{-1}(ax)^3 \, dx$

Optimal. Leaf size=201

$$\frac{6\left(1-a^2x^2\right)^{5/2}}{625a^5}-\frac{76\left(1-a^2x^2\right)^{3/2}}{1125a^5}+\frac{298\sqrt{1-a^2x^2}}{375a^5}-\frac{3x^4\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{25a}-\frac{8x^3\cos^{-1}(ax)}{75a^2}-\frac{4x^2\sqrt{1-a^2x^2}\cos^{-1}(ax}{25a^3}$$

```
[Out] (298*Sqrt[1 - a^2*x^2])/(375*a^5) - (76*(1 - a^2*x^2)^(3/2))/(1125*a^5) + (
6*(1 - a^2*x^2)^(5/2))/(625*a^5) - (16*x*ArcCos[a*x])/(25*a^4) - (8*x^3*Arc
Cos[a*x])/(75*a^2) - (6*x^5*ArcCos[a*x])/125 - (8*Sqrt[1 - a^2*x^2]*ArcCos[
a*x]^2)/(25*a^5) - (4*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/(25*a^3) - (3*x^
4*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/(25*a) + (x^5*ArcCos[a*x]^3)/5
```

Rubi [A] time = 0.402615, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {4628, 4708, 4678, 4620, 261, 266, 43}

$$\frac{6\left(1-a^2x^2\right)^{5/2}}{625a^5}-\frac{76\left(1-a^2x^2\right)^{3/2}}{1125a^5}+\frac{298\sqrt{1-a^2x^2}}{375a^5}-\frac{3x^4\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{25a}-\frac{8x^3\cos^{-1}(ax)}{75a^2}-\frac{4x^2\sqrt{1-a^2x^2}\cos^{-1}(ax}{25a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCos[a*x]^3,x]
```


```
[Out] (298*Sqrt[1 - a^2*x^2])/(375*a^5) - (76*(1 - a^2*x^2)^(3/2))/(1125*a^5) + (
6*(1 - a^2*x^2)^(5/2))/(625*a^5) - (16*x*ArcCos[a*x])/(25*a^4) - (8*x^3*Arc
Cos[a*x])/(75*a^2) - (6*x^5*ArcCos[a*x])/125 - (8*Sqrt[1 - a^2*x^2]*ArcCos[
a*x]^2)/(25*a^5) - (4*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/(25*a^3) - (3*x^
4*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/(25*a) + (x^5*ArcCos[a*x]^3)/5
```


Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
```

```
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^4 \cos^{-1}(ax)^3\,dx = \frac{1}{5}x^5\cos^{-1}(ax)^3 + \frac{1}{5}(3a)\int \frac{x^5\cos^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{3x^4\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{25a} + \frac{1}{5}x^5\cos^{-1}(ax)^3 - \frac{6}{25}\int x^4\cos^{-1}(ax)\,dx + \frac{12\int \frac{x^3\cos^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{25a}$$

$$= -\frac{6}{125}x^5\cos^{-1}(ax) - \frac{4x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{25a^3} - \frac{3x^4\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{25a} + \frac{1}{5}x^5\cos^{-1}(ax)^3 + \frac{8\int}{}$$

$$= -\frac{8x^3\cos^{-1}(ax)}{75a^2} - \frac{6}{125}x^5\cos^{-1}(ax) - \frac{8\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{25a^5} - \frac{4x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{25a^3} - \frac{3x^4\sqrt{1-}}{}$$

$$= -\frac{16x\cos^{-1}(ax)}{25a^4} - \frac{8x^3\cos^{-1}(ax)}{75a^2} - \frac{6}{125}x^5\cos^{-1}(ax) - \frac{8\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{25a^5} - \frac{4x^2\sqrt{1-a^2x^2}\cos^{-}}{25a^3}$$

$$= \frac{86\sqrt{1-a^2x^2}}{125a^5} - \frac{4\left(1-a^2x^2\right)^{3/2}}{125a^5} + \frac{6\left(1-a^2x^2\right)^{5/2}}{625a^5} - \frac{16x\cos^{-1}(ax)}{25a^4} - \frac{8x^3\cos^{-1}(ax)}{75a^2} - \frac{6}{125}x^5\cos^{-1}(a$$

$$= \frac{298\sqrt{1-a^2x^2}}{375a^5} - \frac{76\left(1-a^2x^2\right)^{3/2}}{1125a^5} + \frac{6\left(1-a^2x^2\right)^{5/2}}{625a^5} - \frac{16x\cos^{-1}(ax)}{25a^4} - \frac{8x^3\cos^{-1}(ax)}{75a^2} - \frac{6}{125}x^5\cos^{-}$$

Mathematica [A] time = 0.0682192, size = 122, normalized size = 0.61

$$\frac{2\sqrt{1-a^2x^2}\left(27a^4x^4+136a^2x^2+2072\right)+1125a^5x^5\cos^{-1}(ax)^3-30ax\left(9a^4x^4+20a^2x^2+120\right)\cos^{-1}(ax)-225\sqrt{1-a^2x^2}}{5625a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcCos[a*x]^3,x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2]*(2072 + 136*a^2*x^2 + 27*a^4*x^4) - 30*a*x*(120 + 20*a
^2*x^2 + 9*a^4*x^4)*ArcCos[a*x] - 225*Sqrt[1 - a^2*x^2]*(8 + 4*a^2*x^2 + 3*
a^4*x^4)*ArcCos[a*x]^2 + 1125*a^5*x^5*ArcCos[a*x]^3)/(5625*a^5)
```

Maple [A] time = 0.056, size = 159, normalized size = 0.8

$$\frac{1}{a^5}\left(\frac{(\arccos(ax))^3 a^5x^5}{5} - \frac{(\arccos(ax))^2\left(3a^4x^4+4a^2x^2+8\right)}{25}\sqrt{-a^2x^2+1} + \frac{16}{25}\sqrt{-a^2x^2+1} - \frac{16\,ax\arccos(ax)}{25} - \frac{6\,a^5x^5}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccos(a*x)^3,x)
```

```
[Out] 1/a^5*(1/5*arccos(a*x)^3*a^5*x^5-1/25*arccos(a*x)^2*(3*a^4*x^4+4*a^2*x^2+8)
*(-a^2*x^2+1)^(1/2)+16/25*(-a^2*x^2+1)^(1/2)-16/25*a*x*arccos(a*x)-6/125*a^
5*x^5*arccos(a*x)+2/625*(3*a^4*x^4+4*a^2*x^2+8)*(-a^2*x^2+1)^(1/2)-8/75*a^3
*x^3*arccos(a*x)+8/225*(a^2*x^2+2)*(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.50085, size = 231, normalized size = 1.15

$$\frac{1}{5}x^5\arccos(ax)^3 - \frac{1}{25}\left(\frac{3\sqrt{-a^2x^2+1}x^4}{a^2} + \frac{4\sqrt{-a^2x^2+1}x^2}{a^4} + \frac{8\sqrt{-a^2x^2+1}}{a^6}\right)a\arccos(ax)^2 + \frac{2}{5625}a\left(\frac{27\sqrt{-a^2x^2+1}a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccos(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arccos(a*x)^3 - 1/25*(3*sqrt(-a^2*x^2 + 1)*x^4/a^2 + 4*sqrt(-a^2*x^
2 + 1)*x^2/a^4 + 8*sqrt(-a^2*x^2 + 1)/a^6)*a*arccos(a*x)^2 + 2/5625*a*((27*
sqrt(-a^2*x^2 + 1)*a^2*x^4 + 136*sqrt(-a^2*x^2 + 1)*x^2 + 2072*sqrt(-a^2*x^
2 + 1)/a^2)/a^4 - 15*(9*a^4*x^5 + 20*a^2*x^3 + 120*x)*arccos(a*x)/a^5)
```

Fricas [A] time = 2.3728, size = 265, normalized size = 1.32

$$\frac{1125\,a^5x^5\arccos(ax)^3 - 30\left(9\,a^5x^5 + 20\,a^3x^3 + 120\,ax\right)\arccos(ax) + \left(54\,a^4x^4 + 272\,a^2x^2 - 225\left(3\,a^4x^4 + 4\,a^2x^2 + 8\right)a}{5625\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccos(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/5625*(1125*a^5*x^5*arccos(a*x)^3 - 30*(9*a^5*x^5 + 20*a^3*x^3 + 120*a*x)*
arccos(a*x) + (54*a^4*x^4 + 272*a^2*x^2 - 225*(3*a^4*x^4 + 4*a^2*x^2 + 8)*a
rccos(a*x)^2 + 4144)*sqrt(-a^2*x^2 + 1))/a^5
```

Sympy [A] time = 7.92169, size = 202, normalized size = 1.

$$\begin{cases} \frac{x^5\operatorname{acos}^3(ax)}{5} - \frac{6x^5\operatorname{acos}(ax)}{125} - \frac{3x^4\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{25a} + \frac{6x^4\sqrt{-a^2x^2+1}}{625a} - \frac{8x^3\operatorname{acos}(ax)}{75a^2} - \frac{4x^2\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{25a^3} + \frac{272x^2\sqrt{-a^2x^2+1}}{5625a^3} - \frac{16x\operatorname{acos}}{25a^4} \\ \frac{\pi^3x^5}{40} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acos(a*x)**3,x)
```

```
[Out] Piecewise((x**5*acos(a*x)**3/5 - 6*x**5*acos(a*x)/125 - 3*x**4*sqrt(-a**2*x
**2 + 1)*acos(a*x)**2/(25*a) + 6*x**4*sqrt(-a**2*x**2 + 1)/(625*a) - 8*x**3
*acos(a*x)/(75*a**2) - 4*x**2*sqrt(-a**2*x**2 + 1)*acos(a*x)**2/(25*a**3) +
 272*x**2*sqrt(-a**2*x**2 + 1)/(5625*a**3) - 16*x*acos(a*x)/(25*a**4) - 8*s
qrt(-a**2*x**2 + 1)*acos(a*x)**2/(25*a**5) + 4144*sqrt(-a**2*x**2 + 1)/(562
5*a**5), Ne(a, 0)), (pi**3*x**5/40, True))
```

Giac [A] time = 1.14352, size = 236, normalized size = 1.17

$$\frac{1}{5}x^5\arccos(ax)^3 - \frac{6}{125}x^5\arccos(ax) - \frac{3\sqrt{-a^2x^2+1}x^4\arccos(ax)^2}{25a} + \frac{6\sqrt{-a^2x^2+1}x^4}{625a} - \frac{8x^3\arccos(ax)}{75a^2} - \frac{4\sqrt{-a^2x^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccos(a*x)^3,x, algorithm="giac")
```

```
[Out] 1/5*x^5*arccos(a*x)^3 - 6/125*x^5*arccos(a*x) - 3/25*sqrt(-a^2*x^2 + 1)*x^4
*arccos(a*x)^2/a + 6/625*sqrt(-a^2*x^2 + 1)*x^4/a - 8/75*x^3*arccos(a*x)/a^
2 - 4/25*sqrt(-a^2*x^2 + 1)*x^2*arccos(a*x)^2/a^3 + 272/5625*sqrt(-a^2*x^2
+ 1)*x^2/a^3 - 16/25*x*arccos(a*x)/a^4 - 8/25*sqrt(-a^2*x^2 + 1)*arccos(a*x
)^2/a^5 + 4144/5625*sqrt(-a^2*x^2 + 1)/a^5
```

3.23 $\int x^3 \cos^{-1}(ax)^3\,dx$

Optimal. Leaf size=167

$$\frac{3x^3\sqrt{1-a^2x^2}}{128a}+\frac{45x\sqrt{1-a^2x^2}}{256a^3}-\frac{3x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{16a}-\frac{9x^2\cos^{-1}(ax)}{32a^2}-\frac{9x\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{32a^3}-\frac{45\sin^{-1}(ax)}{256a^4}$$

```
[Out] (45*x*Sqrt[1 - a^2*x^2])/(256*a^3) + (3*x^3*Sqrt[1 - a^2*x^2])/(128*a) - (9
*x^2*ArcCos[a*x])/(32*a^2) - (3*x^4*ArcCos[a*x])/32 - (9*x*Sqrt[1 - a^2*x^2
]*ArcCos[a*x]^2)/(32*a^3) - (3*x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/(16*a)
- (3*ArcCos[a*x]^3)/(32*a^4) + (x^4*ArcCos[a*x]^3)/4 - (45*ArcSin[a*x])/(25
6*a^4)
```

Rubi [A] time = 0.31498, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4628, 4708, 4642, 321, 216}

$$\frac{3x^3\sqrt{1-a^2x^2}}{128a}+\frac{45x\sqrt{1-a^2x^2}}{256a^3}-\frac{3x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{16a}-\frac{9x^2\cos^{-1}(ax)}{32a^2}-\frac{9x\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{32a^3}-\frac{45\sin^{-1}(ax)}{256a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCos[a*x]^3,x]
```


```
[Out] (45*x*Sqrt[1 - a^2*x^2])/(256*a^3) + (3*x^3*Sqrt[1 - a^2*x^2])/(128*a) - (9
*x^2*ArcCos[a*x])/(32*a^2) - (3*x^4*ArcCos[a*x])/32 - (9*x*Sqrt[1 - a^2*x^2
]*ArcCos[a*x]^2)/(32*a^3) - (3*x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/(16*a)
- (3*ArcCos[a*x]^3)/(32*a^4) + (x^4*ArcCos[a*x]^3)/4 - (45*ArcSin[a*x])/(25
6*a^4)
```


Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
```

```
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int x^3 \cos^{-1}(ax)^3\,dx = \frac{1}{4}x^4\cos^{-1}(ax)^3 + \frac{1}{4}(3a)\int \frac{x^4\cos^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{3x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{16a} + \frac{1}{4}x^4\cos^{-1}(ax)^3 - \frac{3}{8}\int x^3\cos^{-1}(ax)\,dx + \frac{9\int \frac{x^2\cos^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{16a}$$

$$= -\frac{3}{32}x^4\cos^{-1}(ax) - \frac{9x\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{32a^3} - \frac{3x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{16a} + \frac{1}{4}x^4\cos^{-1}(ax)^3 + \cdots$$

$$= \frac{3x^3\sqrt{1-a^2x^2}}{128a} - \frac{9x^2\cos^{-1}(ax)}{32a^2} - \frac{3}{32}x^4\cos^{-1}(ax) - \frac{9x\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{32a^3} - \frac{3x^3\sqrt{1-a^2x^2}\cos^{-1}\cdots}{16a}$$

$$= \frac{45x\sqrt{1-a^2x^2}}{256a^3} + \frac{3x^3\sqrt{1-a^2x^2}}{128a} - \frac{9x^2\cos^{-1}(ax)}{32a^2} - \frac{3}{32}x^4\cos^{-1}(ax) - \frac{9x\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{32a^3} - \cdots$$

$$= \frac{45x\sqrt{1-a^2x^2}}{256a^3} + \frac{3x^3\sqrt{1-a^2x^2}}{128a} - \frac{9x^2\cos^{-1}(ax)}{32a^2} - \frac{3}{32}x^4\cos^{-1}(ax) - \frac{9x\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{32a^3} - \cdots$$

Mathematica [A] time = 0.0756178, size = 115, normalized size = 0.69

$$\frac{3ax\sqrt{1-a^2x^2}\left(2a^2x^2+15\right)+8\left(8a^4x^4-3\right)\cos^{-1}(ax)^3-24ax\sqrt{1-a^2x^2}\left(2a^2x^2+3\right)\cos^{-1}(ax)^2-24a^2x^2\left(a^2x^2+3\right)\cos}{256a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCos[a*x]^3,x]
```

```
[Out] (3*a*x*Sqrt[1 - a^2*x^2]*(15 + 2*a^2*x^2) - 24*a^2*x^2*(3 + a^2*x^2)*ArcCos
[a*x] - 24*a*x*Sqrt[1 - a^2*x^2]*(3 + 2*a^2*x^2)*ArcCos[a*x]^2 + 8*(-3 + 8*
a^4*x^4)*ArcCos[a*x]^3 - 45*ArcSin[a*x])/(256*a^4)
```

Maple [A] time = 0.056, size = 151, normalized size = 0.9

$$\frac{1}{a^4}\left(\frac{a^4x^4(\arccos(ax))^3}{4}-\frac{3\,(\arccos(ax))^2}{32}\left(2\,a^3x^3\sqrt{-a^2x^2+1}+3\,ax\sqrt{-a^2x^2+1}+3\,\arccos(ax)\right)-\frac{3\,a^4x^4\arccos(ax)}{32}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccos(a*x)^3,x)
```

```
[Out] 1/a^4*(1/4*a^4*x^4*arccos(a*x)^3-3/32*arccos(a*x)^2*(2*a^3*x^3*(-a^2*x^2+1)
^(1/2)+3*a*x*(-a^2*x^2+1)^(1/2)+3*arccos(a*x))-3/32*a^4*x^4*arccos(a*x)+3/2
56*a*x*(2*a^2*x^2+3)*(-a^2*x^2+1)^(1/2)+45/256*arccos(a*x)-9/32*a^2*x^2*arc
cos(a*x)+9/64*a*x*(-a^2*x^2+1)^(1/2)+3/16*arccos(a*x)^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\,x^4\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^3-3\,a\int\frac{\sqrt{ax+1}\sqrt{-ax+1}x^4\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^2}{4\left(a^2x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3 - 3*a*integrate(1/4*sq
rt(a*x + 1)*sqrt(-a*x + 1)*x^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2
```

```
/(a^2*x^2 - 1), x)
```

Fricas [A] time = 2.43175, size = 234, normalized size = 1.4

$$\frac{8\left(8\,a^4x^4-3\right)\arccos\left(ax\right)^3-3\left(8\,a^4x^4+24\,a^2x^2-15\right)\arccos\left(ax\right)+3\left(2\,a^3x^3-8\left(2\,a^3x^3+3\,ax\right)\arccos\left(ax\right)^2+15\,ax\right)\sqrt{}}{256\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/256*(8*(8*a^4*x^4 - 3)*arccos(a*x)^3 - 3*(8*a^4*x^4 + 24*a^2*x^2 - 15)*ar
ccos(a*x) + 3*(2*a^3*x^3 - 8*(2*a^3*x^3 + 3*a*x)*arccos(a*x)^2 + 15*a*x)*sq
rt(-a^2*x^2 + 1))/a^4
```

Sympy [A] time = 4.58681, size = 167, normalized size = 1.

$$\begin{cases}\frac{x^4\operatorname{acos}^3(ax)}{4}-\frac{3x^4\operatorname{acos}(ax)}{32}-\frac{3x^3\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{16a}+\frac{3x^3\sqrt{-a^2x^2+1}}{128a}-\frac{9x^2\operatorname{acos}(ax)}{32a^2}-\frac{9x\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{32a^3}+\frac{45x\sqrt{-a^2x^2+1}}{256a^3}-\frac{3\operatorname{acos}^3(ax)}{32a^4} \\ \frac{\pi^3x^4}{32}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acos(a*x)**3,x)
```

```
[Out] Piecewise((x**4*acos(a*x)**3/4 - 3*x**4*acos(a*x)/32 - 3*x**3*sqrt(-a**2*x*
*2 + 1)*acos(a*x)**2/(16*a) + 3*x**3*sqrt(-a**2*x**2 + 1)/(128*a) - 9*x**2*
acos(a*x)/(32*a**2) - 9*x*sqrt(-a**2*x**2 + 1)*acos(a*x)**2/(32*a**3) + 45*
x*sqrt(-a**2*x**2 + 1)/(256*a**3) - 3*acos(a*x)**3/(32*a**4) + 45*acos(a*x)
/(256*a**4), Ne(a, 0)), (pi**3*x**4/32, True))
```

Giac [A] time = 1.21932, size = 190, normalized size = 1.14

$$\frac{1}{4}\,x^4\arccos\left(ax\right)^3-\frac{3}{32}\,x^4\arccos\left(ax\right)-\frac{3\sqrt{-a^2x^2+1}x^3\arccos\left(ax\right)^2}{16\,a}+\frac{3\sqrt{-a^2x^2+1}x^3}{128\,a}-\frac{9\,x^2\arccos\left(ax\right)}{32\,a^2}-\frac{9\sqrt{-a^2x^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x)^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4*arccos(a*x)^3 - 3/32*x^4*arccos(a*x) - 3/16*sqrt(-a^2*x^2 + 1)*x^3*
arccos(a*x)^2/a + 3/128*sqrt(-a^2*x^2 + 1)*x^3/a - 9/32*x^2*arccos(a*x)/a^2
 - 9/32*sqrt(-a^2*x^2 + 1)*x*arccos(a*x)^2/a^3 - 3/32*arccos(a*x)^3/a^4 + 4
5/256*sqrt(-a^2*x^2 + 1)*x/a^3 + 45/256*arccos(a*x)/a^4
```

3.24 $\int x^2 \cos^{-1}(ax)^3 \, dx$

Optimal. Leaf size=136

$$-\frac{2\left(1-a^2x^2\right)^{3/2}}{27a^3}+\frac{14\sqrt{1-a^2x^2}}{9a^3}-\frac{x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{3a}-\frac{2\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{3a^3}-\frac{4x\cos^{-1}(ax)}{3a^2}+\frac{1}{3}x^3\cos^{-1}(ax)^3-$$

```
[Out] (14*Sqrt[1 - a^2*x^2])/(9*a^3) - (2*(1 - a^2*x^2)^(3/2))/(27*a^3) - (4*x*Ar
cCos[a*x])/(3*a^2) - (2*x^3*ArcCos[a*x])/9 - (2*Sqrt[1 - a^2*x^2]*ArcCos[a*
x]^2)/(3*a^3) - (x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/(3*a) + (x^3*ArcCos[a
*x]^3)/3
```

Rubi [A] time = 0.240902, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {4628, 4708, 4678, 4620, 261, 266, 43}

$$-\frac{2\left(1-a^2x^2\right)^{3/2}}{27a^3}+\frac{14\sqrt{1-a^2x^2}}{9a^3}-\frac{x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{3a}-\frac{2\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{3a^3}-\frac{4x\cos^{-1}(ax)}{3a^2}+\frac{1}{3}x^3\cos^{-1}(ax)^3-$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCos[a*x]^3,x]
```

```
[Out] (14*Sqrt[1 - a^2*x^2])/(9*a^3) - (2*(1 - a^2*x^2)^(3/2))/(27*a^3) - (4*x*Ar
cCos[a*x])/(3*a^2) - (2*x^3*ArcCos[a*x])/9 - (2*Sqrt[1 - a^2*x^2]*ArcCos[a*
x]^2)/(3*a^3) - (x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/(3*a) + (x^3*ArcCos[a
*x]^3)/3
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
```

```
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2 \cos^{-1}(ax)^3 \, dx = \frac{1}{3}x^3 \cos^{-1}(ax)^3 + a \int \frac{x^3 \cos^{-1}(ax)^2}{\sqrt{1-a^2x^2}} \, dx$$

$$= -\frac{x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{3a} + \frac{1}{3}x^3\cos^{-1}(ax)^3 - \frac{2}{3}\int x^2\cos^{-1}(ax)\,dx + \frac{2\int \frac{x\cos^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{3a}$$

$$= -\frac{2}{9}x^3\cos^{-1}(ax) - \frac{2\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{3a^3} - \frac{x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{3a} + \frac{1}{3}x^3\cos^{-1}(ax)^3 - \frac{4\int \cos^{-1}(}{3a^2}$$

$$= -\frac{4x\cos^{-1}(ax)}{3a^2} - \frac{2}{9}x^3\cos^{-1}(ax) - \frac{2\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{3a^3} - \frac{x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{3a} + \frac{1}{3}x^3\cos^{-1}(a$$

$$= \frac{4\sqrt{1-a^2x^2}}{3a^3} - \frac{4x\cos^{-1}(ax)}{3a^2} - \frac{2}{9}x^3\cos^{-1}(ax) - \frac{2\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{3a^3} - \frac{x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{3a} +$$

$$= \frac{14\sqrt{1-a^2x^2}}{9a^3} - \frac{2\left(1-a^2x^2\right)^{3/2}}{27a^3} - \frac{4x\cos^{-1}(ax)}{3a^2} - \frac{2}{9}x^3\cos^{-1}(ax) - \frac{2\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{3a^3} - \frac{x^2\sqrt{1-}}{}$$

Mathematica [A] time = 0.0496641, size = 95, normalized size = 0.7

$$\frac{2\sqrt{1-a^2x^2}\left(a^2x^2+20\right) + 9a^3x^3\cos^{-1}(ax)^3 - 9\sqrt{1-a^2x^2}\left(a^2x^2+2\right)\cos^{-1}(ax)^2 - 6ax\left(a^2x^2+6\right)\cos^{-1}(ax)}{27a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCos[a*x]^3,x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2]*(20 + a^2*x^2) - 6*a*x*(6 + a^2*x^2)*ArcCos[a*x] - 9*S
qrt[1 - a^2*x^2]*(2 + a^2*x^2)*ArcCos[a*x]^2 + 9*a^3*x^3*ArcCos[a*x]^3)/(27
*a^3)
```

Maple [A] time = 0.051, size = 106, normalized size = 0.8

$$\frac{1}{a^3}\left(\frac{a^3x^3\left(\arccos(ax)\right)^3}{3} - \frac{\left(\arccos(ax)\right)^2\left(a^2x^2+2\right)}{3}\sqrt{-a^2x^2+1} + \frac{4}{3}\sqrt{-a^2x^2+1} - \frac{4\,ax\arccos(ax)}{3} - \frac{2\,a^3x^3\arccos(ax)}{9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccos(a*x)^3,x)
```

```
[Out] 1/a^3*(1/3*a^3*x^3*arccos(a*x)^3-1/3*arccos(a*x)^2*(a^2*x^2+2)*(-a^2*x^2+1)
^(1/2)+4/3*(-a^2*x^2+1)^(1/2)-4/3*a*x*arccos(a*x)-2/9*a^3*x^3*arccos(a*x)+2
/27*(a^2*x^2+2)*(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.51405, size = 162, normalized size = 1.19

$$\frac{1}{3}x^3\arccos(ax)^3-\frac{1}{3}a\left(\frac{\sqrt{-a^2x^2+1}x^2}{a^2}+\frac{2\sqrt{-a^2x^2+1}}{a^4}\right)\arccos(ax)^2+\frac{2}{27}a\left(\frac{\sqrt{-a^2x^2+1}x^2+\frac{20\sqrt{-a^2x^2+1}}{a^2}}{a^2}-\frac{3\left(a^2x^3+6\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccos(a*x)^3 - 1/3*a*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^2
 + 1)/a^4)*arccos(a*x)^2 + 2/27*a*((sqrt(-a^2*x^2 + 1)*x^2 + 20*sqrt(-a^2*x
^2 + 1)/a^2)/a^2 - 3*(a^2*x^3 + 6*x)*arccos(a*x)/a^3)
```

Fricas [A] time = 2.2562, size = 189, normalized size = 1.39

$$\frac{9\,a^3x^3\arccos(ax)^3-6\left(a^3x^3+6\,ax\right)\arccos(ax)+\left(2\,a^2x^2-9\left(a^2x^2+2\right)\arccos(ax)^2+40\right)\sqrt{-a^2x^2+1}}{27\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/27*(9*a^3*x^3*arccos(a*x)^3 - 6*(a^3*x^3 + 6*a*x)*arccos(a*x) + (2*a^2*x^
2 - 9*(a^2*x^2 + 2)*arccos(a*x)^2 + 40)*sqrt(-a^2*x^2 + 1))/a^3
```

Sympy [A] time = 2.45764, size = 134, normalized size = 0.99

$$\begin{cases}\frac{x^3\operatorname{acos}^3(ax)}{3}-\frac{2x^3\operatorname{acos}(ax)}{9}-\frac{x^2\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{3a}+\frac{2x^2\sqrt{-a^2x^2+1}}{27a}-\frac{4x\operatorname{acos}(ax)}{3a^2}-\frac{2\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{3a^3}+\frac{40\sqrt{-a^2x^2+1}}{27a^3} & \text{for } a\neq 0\\ \frac{\pi^3x^3}{24} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acos(a*x)**3,x)
```

```
[Out] Piecewise((x**3*acos(a*x)**3/3 - 2*x**3*acos(a*x)/9 - x**2*sqrt(-a**2*x**2
+ 1)*acos(a*x)**2/(3*a) + 2*x**2*sqrt(-a**2*x**2 + 1)/(27*a) - 4*x*acos(a*x
)/(3*a**2) - 2*sqrt(-a**2*x**2 + 1)*acos(a*x)**2/(3*a**3) + 40*sqrt(-a**2*x
**2 + 1)/(27*a**3), Ne(a, 0)), (pi**3*x**3/24, True))
```

Giac [A] time = 1.15595, size = 158, normalized size = 1.16

$$\frac{1}{3}\,x^3\arccos\left(ax\right)^3 - \frac{2}{9}\,x^3\arccos\left(ax\right) - \frac{\sqrt{-a^2x^2+1}x^2\arccos\left(ax\right)^2}{3\,a} + \frac{2\,\sqrt{-a^2x^2+1}x^2}{27\,a} - \frac{4\,x\arccos\left(ax\right)}{3\,a^2} - \frac{2\,\sqrt{-a^2x^2+1}\,a}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x)^3,x, algorithm="giac")
```

```
[Out] 1/3*x^3*arccos(a*x)^3 - 2/9*x^3*arccos(a*x) - 1/3*sqrt(-a^2*x^2 + 1)*x^2*ar
ccos(a*x)^2/a + 2/27*sqrt(-a^2*x^2 + 1)*x^2/a - 4/3*x*arccos(a*x)/a^2 - 2/3
*sqrt(-a^2*x^2 + 1)*arccos(a*x)^2/a^3 + 40/27*sqrt(-a^2*x^2 + 1)/a^3
```

3.25 $\int x \cos^{-1}(ax)^3 \, dx$

Optimal. Leaf size=99

$$\frac{3x\sqrt{1-a^2x^2}}{8a} - \frac{3x\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{4a} - \frac{3\sin^{-1}(ax)}{8a^2} - \frac{\cos^{-1}(ax)^3}{4a^2} + \frac{1}{2}x^2\cos^{-1}(ax)^3 - \frac{3}{4}x^2\cos^{-1}(ax)$$

```
[Out] (3*x*Sqrt[1 - a^2*x^2])/(8*a) - (3*x^2*ArcCos[a*x])/4 - (3*x*Sqrt[1 - a^2*x
^2]*ArcCos[a*x]^2)/(4*a) - ArcCos[a*x]^3/(4*a^2) + (x^2*ArcCos[a*x]^3)/2 -
(3*ArcSin[a*x])/(8*a^2)
```

Rubi [A] time = 0.155969, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4628, 4708, 4642, 321, 216}

$$\frac{3x\sqrt{1-a^2x^2}}{8a} - \frac{3x\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{4a} - \frac{3\sin^{-1}(ax)}{8a^2} - \frac{\cos^{-1}(ax)^3}{4a^2} + \frac{1}{2}x^2\cos^{-1}(ax)^3 - \frac{3}{4}x^2\cos^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCos[a*x]^3,x]
```

```
[Out] (3*x*Sqrt[1 - a^2*x^2])/(8*a) - (3*x^2*ArcCos[a*x])/4 - (3*x*Sqrt[1 - a^2*x
^2]*ArcCos[a*x]^2)/(4*a) - ArcCos[a*x]^3/(4*a^2) + (x^2*ArcCos[a*x]^3)/2 -
(3*ArcSin[a*x])/(8*a^2)
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

```
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int x\cos^{-1}(ax)^3\,dx &= \frac{1}{2}x^2\cos^{-1}(ax)^3+\frac{1}{2}(3a)\int\frac{x^2\cos^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{3x\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{4a}+\frac{1}{2}x^2\cos^{-1}(ax)^3-\frac{3}{2}\int x\cos^{-1}(ax)\,dx+\frac{3\int\frac{\cos^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{4a}\\
&= -\frac{3}{4}x^2\cos^{-1}(ax)-\frac{3x\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{4a}-\frac{\cos^{-1}(ax)^3}{4a^2}+\frac{1}{2}x^2\cos^{-1}(ax)^3-\frac{1}{4}(3a)\int\frac{x^2}{\sqrt{1-a^2x^2}}\,dx\\
&= \frac{3x\sqrt{1-a^2x^2}}{8a}-\frac{3}{4}x^2\cos^{-1}(ax)-\frac{3x\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{4a}-\frac{\cos^{-1}(ax)^3}{4a^2}+\frac{1}{2}x^2\cos^{-1}(ax)^3-\frac{3\int\frac{1}{\sqrt{1-a\ldots}}}{8a\ldots}\\
&= \frac{3x\sqrt{1-a^2x^2}}{8a}-\frac{3}{4}x^2\cos^{-1}(ax)-\frac{3x\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{4a}-\frac{\cos^{-1}(ax)^3}{4a^2}+\frac{1}{2}x^2\cos^{-1}(ax)^3-\frac{3\sin^{-1}(\ldots}{8a^2}
\end{aligned}$$

Mathematica [A] time = 0.0413478, size = 85, normalized size = 0.86

$$\frac{3ax\sqrt{1-a^2x^2}+\left(4a^2x^2-2\right)\cos^{-1}(ax)^3-6ax\sqrt{1-a^2x^2}\cos^{-1}(ax)^2-6a^2x^2\cos^{-1}(ax)-3\sin^{-1}(ax)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCos[a*x]^3,x]
```

```
[Out] (3*a*x*Sqrt[1 - a^2*x^2] - 6*a^2*x^2*ArcCos[a*x] - 6*a*x*Sqrt[1 - a^2*x^2]*
ArcCos[a*x]^2 + (-2 + 4*a^2*x^2)*ArcCos[a*x]^3 - 3*ArcSin[a*x])/(8*a^2)
```

Maple [A] time = 0.05, size = 90, normalized size = 0.9

$$\frac{1}{a^2}\left(\frac{a^2x^2(\arccos(ax))^3}{2}-\frac{3(\arccos(ax))^2}{4}\left(ax\sqrt{-a^2x^2+1}+\arccos(ax)\right)-\frac{3a^2x^2\arccos(ax)}{4}+\frac{3ax}{8}\sqrt{-a^2x^2+1}+\frac{3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccos(a*x)^3,x)
```

```
[Out] 1/a^2*(1/2*a^2*x^2*arccos(a*x)^3-3/4*arccos(a*x)^2*(a*x*(-a^2*x^2+1)^(1/2)+
arccos(a*x))-3/4*a^2*x^2*arccos(a*x)+3/8*a*x*(-a^2*x^2+1)^(1/2)+3/8*arccos(
a*x)+1/2*arccos(a*x)^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x^2\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^3-3a\int\frac{\sqrt{ax+1}\sqrt{-ax+1}x^2\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^2}{2\left(a^2x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3 - 3*a*integrate(1/2*sq
rt(a*x + 1)*sqrt(-a*x + 1)*x^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2
/(a^2*x^2 - 1), x)
```

Fricas [A] time = 2.42596, size = 170, normalized size = 1.72

$$\frac{2\left(2a^2x^2-1\right)\arccos(ax)^3-3\left(2a^2x^2-1\right)\arccos(ax)-3\sqrt{-a^2x^2+1}\left(2ax\arccos(ax)^2-ax\right)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^2*x^2 - 1)*arccos(a*x)^3 - 3*(2*a^2*x^2 - 1)*arccos(a*x) - 3*sq
rt(-a^2*x^2 + 1)*(2*a*x*arccos(a*x)^2 - a*x))/a^2
```

Sympy [A] time = 1.19576, size = 99, normalized size = 1.

$$\begin{cases} \frac{x^2 \operatorname{acos}^3(ax)}{2} - \frac{3x^2 \operatorname{acos}(ax)}{4} - \frac{3x\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{4a} + \frac{3x\sqrt{-a^2x^2+1}}{8a} - \frac{\operatorname{acos}^3(ax)}{4a^2} + \frac{3\operatorname{acos}(ax)}{8a^2} & \text{for } a \neq 0 \\ \frac{\pi^3 x^2}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acos(a*x)**3,x)
```

```
[Out] Piecewise((x**2*acos(a*x)**3/2 - 3*x**2*acos(a*x)/4 - 3*x*sqrt(-a**2*x**2 +
 1)*acos(a*x)**2/(4*a) + 3*x*sqrt(-a**2*x**2 + 1)/(8*a) - acos(a*x)**3/(4*a
**2) + 3*acos(a*x)/(8*a**2), Ne(a, 0)), (pi**3*x**2/16, True))
```

Giac [A] time = 1.14073, size = 112, normalized size = 1.13

$$\frac{1}{2}x^2 \arccos(ax)^3 - \frac{3}{4}x^2 \arccos(ax) - \frac{3\sqrt{-a^2x^2+1}x \arccos(ax)^2}{4a} - \frac{\arccos(ax)^3}{4a^2} + \frac{3\sqrt{-a^2x^2+1}x}{8a} + \frac{3\arccos(ax)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x)^3,x, algorithm="giac")
```

```
[Out] 1/2*x^2*arccos(a*x)^3 - 3/4*x^2*arccos(a*x) - 3/4*sqrt(-a^2*x^2 + 1)*x*arcc
os(a*x)^2/a - 1/4*arccos(a*x)^3/a^2 + 3/8*sqrt(-a^2*x^2 + 1)*x/a + 3/8*arcc
os(a*x)/a^2
```

3.26 $\int \cos^{-1}(ax)^3\,dx$

Optimal. Leaf size=60

$$\frac{6\sqrt{1-a^2x^2}}{a}-\frac{3\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{a}+x\cos^{-1}(ax)^3-6x\cos^{-1}(ax)$$

```
[Out] (6*Sqrt[1 - a^2*x^2])/a - 6*x*ArcCos[a*x] - (3*Sqrt[1 - a^2*x^2]*ArcCos[a*x
]^2)/a + x*ArcCos[a*x]^3
```

Rubi [A] time = 0.0810922, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4620, 4678, 261}

$$\frac{6\sqrt{1-a^2x^2}}{a}-\frac{3\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{a}+x\cos^{-1}(ax)^3-6x\cos^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^3,x]
```

```
[Out] (6*Sqrt[1 - a^2*x^2])/a - 6*x*ArcCos[a*x] - (3*Sqrt[1 - a^2*x^2]*ArcCos[a*x
]^2)/a + x*ArcCos[a*x]^3
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \cos^{-1}(ax)^3\,dx &= x\cos^{-1}(ax)^3+(3a)\int \frac{x\cos^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{3\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{a}+x\cos^{-1}(ax)^3-6\int \cos^{-1}(ax)\,dx\\
&= -6x\cos^{-1}(ax)-\frac{3\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{a}+x\cos^{-1}(ax)^3-(6a)\int \frac{x}{\sqrt{1-a^2x^2}}\,dx\\
&= \frac{6\sqrt{1-a^2x^2}}{a}-6x\cos^{-1}(ax)-\frac{3\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{a}+x\cos^{-1}(ax)^3
\end{aligned}
$$

Mathematica [A] time = 0.0186135, size = 60, normalized size = 1.

$$\frac{6\sqrt{1-a^2x^2}}{a}-\frac{3\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{a}+x\cos^{-1}(ax)^3-6x\cos^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x]^3,x]
```

```
[Out] (6*Sqrt[1 - a^2*x^2])/a - 6*x*ArcCos[a*x] - (3*Sqrt[1 - a^2*x^2]*ArcCos[a*x
]^2)/a + x*ArcCos[a*x]^3
```

Maple [A] time = 0.049, size = 57, normalized size = 1.

$$\frac{1}{a}\left(ax\left(\arccos(ax)\right)^3-3\left(\arccos(ax)\right)^2\sqrt{-a^2x^2+1}+6\sqrt{-a^2x^2+1}-6\,ax\arccos(ax)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^3,x)
```

```
[Out] 1/a*(a*x*arccos(a*x)^3-3*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)+6*(-a^2*x^2+1)^(1
/2)-6*a*x*arccos(a*x))
```

Maxima [A] time = 1.4488, size = 80, normalized size = 1.33

$$x \arccos(ax)^3 - \frac{3\sqrt{-a^2x^2+1}\arccos(ax)^2}{a} - \frac{6\left(ax\arccos(ax) - \sqrt{-a^2x^2+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^3,x, algorithm="maxima")
```

```
[Out] x*arccos(a*x)^3 - 3*sqrt(-a^2*x^2 + 1)*arccos(a*x)^2/a - 6*(a*x*arccos(a*x)
 - sqrt(-a^2*x^2 + 1))/a
```

Fricas [A] time = 2.26834, size = 116, normalized size = 1.93

$$\frac{ax\arccos(ax)^3 - 6\,ax\arccos(ax) - 3\sqrt{-a^2x^2+1}\left(\arccos(ax)^2 - 2\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^3,x, algorithm="fricas")
```

```
[Out] (a*x*arccos(a*x)^3 - 6*a*x*arccos(a*x) - 3*sqrt(-a^2*x^2 + 1)*(arccos(a*x)^
2 - 2))/a
```

Sympy [A] time = 0.572868, size = 60, normalized size = 1.

$$\begin{cases} x\operatorname{acos}^{3}(ax) - 6x\operatorname{acos}(ax) - \frac{3\sqrt{-a^2x^2+1}\operatorname{acos}^{2}(ax)}{a} + \frac{6\sqrt{-a^2x^2+1}}{a} & \text{for } a \neq 0 \\ \frac{\pi^3 x}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**3,x)
```

```
[Out] Piecewise((x*acos(a*x)**3 - 6*x*acos(a*x) - 3*sqrt(-a**2*x**2 + 1)*acos(a*x
)**2/a + 6*sqrt(-a**2*x**2 + 1)/a, Ne(a, 0)), (pi**3*x/8, True))
```

Giac [A] time = 1.11909, size = 76, normalized size = 1.27

$$x \arccos(ax)^3 - 6\,x \arccos(ax) - \frac{3\,\sqrt{-a^2x^2+1}\arccos(ax)^2}{a} + \frac{6\,\sqrt{-a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^3,x, algorithm="giac")
```

```
[Out] x*arccos(a*x)^3 - 6*x*arccos(a*x) - 3*sqrt(-a^2*x^2 + 1)*arccos(a*x)^2/a +
6*sqrt(-a^2*x^2 + 1)/a
```

3.27 $\int \frac{\cos^{-1}(ax)^3}{x} dx$

Optimal. Leaf size=101

$$-\frac{3}{2} i \cos^{-1}(ax)^2 \text{PolyLog}\left(2, -e^{2i\cos^{-1}(ax)}\right) + \frac{3}{2}\cos^{-1}(ax)\text{PolyLog}\left(3, -e^{2i\cos^{-1}(ax)}\right) + \frac{3}{4} i \text{PolyLog}\left(4, -e^{2i\cos^{-1}(ax)}\right) - \frac{1}{4} i \text{co}$$

```
[Out] (-I/4)*ArcCos[a*x]^4 + ArcCos[a*x]^3*Log[1 + E^((2*I)*ArcCos[a*x])] - ((3*I
)/2)*ArcCos[a*x]^2*PolyLog[2, -E^((2*I)*ArcCos[a*x])] + (3*ArcCos[a*x]*Poly
Log[3, -E^((2*I)*ArcCos[a*x])])/2 + ((3*I)/4)*PolyLog[4, -E^((2*I)*ArcCos[a
*x])]
```

Rubi [A] time = 0.110519, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {4626, 3719, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3}{2} i \cos^{-1}(ax)^2 \text{PolyLog}\left(2, -e^{2i\cos^{-1}(ax)}\right) + \frac{3}{2}\cos^{-1}(ax)\text{PolyLog}\left(3, -e^{2i\cos^{-1}(ax)}\right) + \frac{3}{4} i \text{PolyLog}\left(4, -e^{2i\cos^{-1}(ax)}\right) - \frac{1}{4} i \text{co}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^3/x,x]
```



```
[Out] (-I/4)*ArcCos[a*x]^4 + ArcCos[a*x]^3*Log[1 + E^((2*I)*ArcCos[a*x])] - ((3*I
)/2)*ArcCos[a*x]^2*PolyLog[2, -E^((2*I)*ArcCos[a*x])] + (3*ArcCos[a*x]*Poly
Log[3, -E^((2*I)*ArcCos[a*x])])/2 + ((3*I)/4)*PolyLog[4, -E^((2*I)*ArcCos[a
*x])]
```


Rule 4626

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n/Cot[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cos^{-1}(ax)^3}{x}\,dx = -\operatorname{Subst}\left(\int x^3\tan(x)\,dx, x, \cos^{-1}(ax)\right)$$
$$= -\frac{1}{4}i\cos^{-1}(ax)^4 + 2i\operatorname{Subst}\left(\int \frac{e^{2ix}x^3}{1+e^{2ix}}\,dx, x, \cos^{-1}(ax)\right)$$
$$= -\frac{1}{4}i\cos^{-1}(ax)^4 + \cos^{-1}(ax)^3\log\left(1+e^{2i\cos^{-1}(ax)}\right) - 3\operatorname{Subst}\left(\int x^2\log\left(1+e^{2ix}\right)dx, x, \cos^{-1}(ax)\right)$$
$$= -\frac{1}{4}i\cos^{-1}(ax)^4 + \cos^{-1}(ax)^3\log\left(1+e^{2i\cos^{-1}(ax)}\right) - \frac{3}{2}i\cos^{-1}(ax)^2\text{Li}_2\left(-e^{2i\cos^{-1}(ax)}\right) + 3i\operatorname{Subst}\left(\int x$$
$$= -\frac{1}{4}i\cos^{-1}(ax)^4 + \cos^{-1}(ax)^3\log\left(1+e^{2i\cos^{-1}(ax)}\right) - \frac{3}{2}i\cos^{-1}(ax)^2\text{Li}_2\left(-e^{2i\cos^{-1}(ax)}\right) + \frac{3}{2}\cos^{-1}(ax)\text{L}$$
$$= -\frac{1}{4}i\cos^{-1}(ax)^4 + \cos^{-1}(ax)^3\log\left(1+e^{2i\cos^{-1}(ax)}\right) - \frac{3}{2}i\cos^{-1}(ax)^2\text{Li}_2\left(-e^{2i\cos^{-1}(ax)}\right) + \frac{3}{2}\cos^{-1}(ax)\text{L}$$
$$= -\frac{1}{4}i\cos^{-1}(ax)^4 + \cos^{-1}(ax)^3\log\left(1+e^{2i\cos^{-1}(ax)}\right) - \frac{3}{2}i\cos^{-1}(ax)^2\text{Li}_2\left(-e^{2i\cos^{-1}(ax)}\right) + \frac{3}{2}\cos^{-1}(ax)\text{L}$$

Mathematica [A] time = 0.0196128, size = 101, normalized size = 1.

$$-\frac{3}{2}i\cos^{-1}(ax)^2\text{PolyLog}\left(2,-e^{2i\cos^{-1}(ax)}\right) + \frac{3}{2}\cos^{-1}(ax)\text{PolyLog}\left(3,-e^{2i\cos^{-1}(ax)}\right) + \frac{3}{4}i\text{PolyLog}\left(4,-e^{2i\cos^{-1}(ax)}\right) - \frac{1}{4}i\,\text{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x]^3/x,x]
```

```
[Out] (-I/4)*ArcCos[a*x]^4 + ArcCos[a*x]^3*Log[1 + E^((2*I)*ArcCos[a*x])] - ((3*I
)/2)*ArcCos[a*x]^2*PolyLog[2, -E^((2*I)*ArcCos[a*x])] + (3*ArcCos[a*x]*Poly
Log[3, -E^((2*I)*ArcCos[a*x])])/2 + ((3*I)/4)*PolyLog[4, -E^((2*I)*ArcCos[a
*x])]
```

Maple [A] time = 0.07, size = 135, normalized size = 1.3

$$-\frac{i}{4}(\arccos(ax))^4 + (\arccos(ax))^3\ln\left(1+\left(i\sqrt{-a^2x^2+1}+ax\right)^2\right) - \frac{3i}{2}(\arccos(ax))^2\operatorname{polylog}\left(2,-\left(i\sqrt{-a^2x^2+1}+ax\right)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^3/x,x)
```

```
[Out] -1/4*I*arccos(a*x)^4+arccos(a*x)^3*ln(1+(I*(-a^2*x^2+1)^(1/2)+a*x)^2)-3/2*I
*arccos(a*x)^2*polylog(2,-(I*(-a^2*x^2+1)^(1/2)+a*x)^2)+3/2*arccos(a*x)*pol
ylog(3,-(I*(-a^2*x^2+1)^(1/2)+a*x)^2)+3/4*I*polylog(4,-(I*(-a^2*x^2+1)^(1/2
)+a*x)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos{(ax)}^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^3/x,x, algorithm="maxima")
```

```
[Out] integrate(arccos(a*x)^3/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arccos{(ax)}^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^3/x,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)^3/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acos}^3{(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**3/x,x)
```

```
[Out] Integral(acos(a*x)**3/x, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(ax)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^3/x,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^3/x, x)
```

3.28 $\int \frac{\cos^{-1}(ax)^3}{x^2} dx$

Optimal. Leaf size=122

$$6ia\cos^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\cos^{-1}(ax)}\right)-6ia\cos^{-1}(ax)\text{PolyLog}\left(2,ie^{i\cos^{-1}(ax)}\right)-6a\text{PolyLog}\left(3,-ie^{i\cos^{-1}(ax)}\right)+6a\text{PolyL}$$

```
[Out] -(ArcCos[a*x]^3/x) - (6*I)*a*ArcCos[a*x]^2*ArcTan[E^(I*ArcCos[a*x])] + (6*I
)*a*ArcCos[a*x]*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] - (6*I)*a*ArcCos[a*x]*Po
lyLog[2, I*E^(I*ArcCos[a*x])] - 6*a*PolyLog[3, (-I)*E^(I*ArcCos[a*x])] + 6*
a*PolyLog[3, I*E^(I*ArcCos[a*x])]
```

Rubi [A] time = 0.174938, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4628, 4710, 4181, 2531, 2282, 6589}

$$6ia\cos^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\cos^{-1}(ax)}\right)-6ia\cos^{-1}(ax)\text{PolyLog}\left(2,ie^{i\cos^{-1}(ax)}\right)-6a\text{PolyLog}\left(3,-ie^{i\cos^{-1}(ax)}\right)+6a\text{PolyL}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^3/x^2,x]
```

```
[Out] -(ArcCos[a*x]^3/x) - (6*I)*a*ArcCos[a*x]^2*ArcTan[E^(I*ArcCos[a*x])] + (6*I
)*a*ArcCos[a*x]*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] - (6*I)*a*ArcCos[a*x]*Po
lyLog[2, I*E^(I*ArcCos[a*x])] - 6*a*PolyLog[3, (-I)*E^(I*ArcCos[a*x])] + 6*
a*PolyLog[3, I*E^(I*ArcCos[a*x])]
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4710

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> -Dist[(c^(m + 1)*Sqrt[d])^(-1), Subst[Int[(a + b*x)^n
*Cos[x]^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{-1}(ax)^3}{x^2}\,dx &= -\frac{\cos^{-1}(ax)^3}{x} - (3a)\int \frac{\cos^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx \\ &= -\frac{\cos^{-1}(ax)^3}{x} + (3a)\,\mathrm{Subst}\left(\int x^2\sec(x)\,dx, x, \cos^{-1}(ax)\right) \\ &= -\frac{\cos^{-1}(ax)^3}{x} - 6ia\cos^{-1}(ax)^2\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right) - (6a)\,\mathrm{Subst}\left(\int x\log\left(1-ie^{ix}\right)\,dx, x, \cos^{-1}(ax)\right) + (\\ &= -\frac{\cos^{-1}(ax)^3}{x} - 6ia\cos^{-1}(ax)^2\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right) + 6ia\cos^{-1}(ax)\mathrm{Li}_2\left(-ie^{i\cos^{-1}(ax)}\right) - 6ia\cos^{-1}(ax)\mathrm{Li}_2 \\ &= -\frac{\cos^{-1}(ax)^3}{x} - 6ia\cos^{-1}(ax)^2\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right) + 6ia\cos^{-1}(ax)\mathrm{Li}_2\left(-ie^{i\cos^{-1}(ax)}\right) - 6ia\cos^{-1}(ax)\mathrm{Li}_2 \\ &= -\frac{\cos^{-1}(ax)^3}{x} - 6ia\cos^{-1}(ax)^2\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right) + 6ia\cos^{-1}(ax)\mathrm{Li}_2\left(-ie^{i\cos^{-1}(ax)}\right) - 6ia\cos^{-1}(ax)\mathrm{Li}_2\end{aligned}$$

Mathematica [A] time = 0.109815, size = 139, normalized size = 1.14

$$-\frac{\cos^{-1}(ax)^3}{x} + 3a\left(2i\cos^{-1}(ax)\left(\mathrm{PolyLog}\left(2,-ie^{i\cos^{-1}(ax)}\right) - \mathrm{PolyLog}\left(2,ie^{i\cos^{-1}(ax)}\right)\right) - 2\mathrm{PolyLog}\left(3,-ie^{i\cos^{-1}(ax)}\right) + 2\mathrm{P}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCos[a*x]^3/x^2,x]
```

```
[Out] -(ArcCos[a*x]^3/x) + 3*a*(ArcCos[a*x]^2*(Log[1 - I*E^(I*ArcCos[a*x])] - Log
[1 + I*E^(I*ArcCos[a*x])]) + (2*I)*ArcCos[a*x]*(PolyLog[2, (-I)*E^(I*ArcCos
[a*x])] - PolyLog[2, I*E^(I*ArcCos[a*x])]) - 2*PolyLog[3, (-I)*E^(I*ArcCos[
a*x])] + 2*PolyLog[3, I*E^(I*ArcCos[a*x])])
```

Maple [F] time = 0.207, size = 0, normalized size = 0.

$$\int \frac{(\arccos(ax))^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^3/x^2,x)
```

```
[Out] int(arccos(a*x)^3/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\arctan\left(\sqrt{ax+1}\sqrt{-ax+1}, ax\right)^3 - 3\,ax\int\frac{\sqrt{-ax+1}\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^2}{\sqrt{ax+1}(ax-1)x}\,dx}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^3/x^2,x, algorithm="maxima")
```

```
[Out] -(arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3 - 3*a*x*integrate(sqrt(a*x +
 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2/(a^2*x^3 -
x), x))/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arccos\left(ax\right)^3}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)^3/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{acos}^3\left(ax\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**3/x**2,x)
```

```
[Out] Integral(acos(a*x)**3/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(ax)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^3/x^2, x)
```

3.29 $\int \frac{\cos^{-1}(ax)^3}{x^3} dx$

Optimal. Leaf size=102

$$-\frac{3}{2}ia^2\text{PolyLog}\left(2,-e^{2i\cos^{-1}(ax)}\right)+\frac{3a\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{2x}-\frac{3}{2}ia^2\cos^{-1}(ax)^2+3a^2\cos^{-1}(ax)\log\left(1+e^{2i\cos^{-1}(ax)}\right)-\frac{\text{co}}{}$$

```
[Out] ((-3*I)/2)*a^2*ArcCos[a*x]^2 + (3*a*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/(2*x)
- ArcCos[a*x]^3/(2*x^2) + 3*a^2*ArcCos[a*x]*Log[1 + E^((2*I)*ArcCos[a*x])]
- ((3*I)/2)*a^2*PolyLog[2, -E^((2*I)*ArcCos[a*x])]
```

Rubi [A] time = 0.175343, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {4628, 4682, 4626, 3719, 2190, 2279, 2391}

$$-\frac{3}{2}ia^2\text{PolyLog}\left(2,-e^{2i\cos^{-1}(ax)}\right)+\frac{3a\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{2x}-\frac{3}{2}ia^2\cos^{-1}(ax)^2+3a^2\cos^{-1}(ax)\log\left(1+e^{2i\cos^{-1}(ax)}\right)-\frac{\text{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^3/x^3,x]
```

```
[Out] ((-3*I)/2)*a^2*ArcCos[a*x]^2 + (3*a*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/(2*x)
- ArcCos[a*x]^3/(2*x^2) + 3*a^2*ArcCos[a*x]*Log[1 + E^((2*I)*ArcCos[a*x])]
- ((3*I)/2)*a^2*PolyLog[2, -E^((2*I)*ArcCos[a*x])]
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4682

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b
*ArcCos[c*x])^n)/(d*f*(m + 1)), x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^F
racPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 - c
^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &
```

```
& NeQ[m, -1]
```

Rule 4626

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n/Cot[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\cos^{-1}(ax)^3}{x^3}\,dx = -\frac{\cos^{-1}(ax)^3}{2x^2} - \frac{1}{2}(3a)\int \frac{\cos^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{3a\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{2x} - \frac{\cos^{-1}(ax)^3}{2x^2} + \left(3a^2\right)\int \frac{\cos^{-1}(ax)}{x}\,dx$$
$$= \frac{3a\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{2x} - \frac{\cos^{-1}(ax)^3}{2x^2} - \left(3a^2\right)\text{Subst}\left(\int x\tan(x)\,dx, x, \cos^{-1}(ax)\right)$$
$$= -\frac{3}{2}ia^2\cos^{-1}(ax)^2 + \frac{3a\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{2x} - \frac{\cos^{-1}(ax)^3}{2x^2} + \left(6ia^2\right)\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \cos^{-1}(\right.$$
$$= -\frac{3}{2}ia^2\cos^{-1}(ax)^2 + \frac{3a\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{2x} - \frac{\cos^{-1}(ax)^3}{2x^2} + 3a^2\cos^{-1}(ax)\log\left(1+e^{2i\cos^{-1}(ax)}\right) - (3$$
$$= -\frac{3}{2}ia^2\cos^{-1}(ax)^2 + \frac{3a\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{2x} - \frac{\cos^{-1}(ax)^3}{2x^2} + 3a^2\cos^{-1}(ax)\log\left(1+e^{2i\cos^{-1}(ax)}\right) + \frac{1}{2}$$
$$= -\frac{3}{2}ia^2\cos^{-1}(ax)^2 + \frac{3a\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{2x} - \frac{\cos^{-1}(ax)^3}{2x^2} + 3a^2\cos^{-1}(ax)\log\left(1+e^{2i\cos^{-1}(ax)}\right) - \frac{3}{2}$$

Mathematica [A] time = 0.192443, size = 92, normalized size = 0.9

$$\frac{1}{2}\left(-3ia^2\text{PolyLog}\left(2, -e^{2i\cos^{-1}(ax)}\right) + \frac{3a\left(\sqrt{1-a^2x^2} - iax\right)\cos^{-1}(ax)^2}{x} + 6a^2\cos^{-1}(ax)\log\left(1+e^{2i\cos^{-1}(ax)}\right) - \frac{\cos^{-1}(ax)}{x^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x]^3/x^3,x]
```

```
[Out] ((3*a*((-I)*a*x + Sqrt[1 - a^2*x^2])*ArcCos[a*x]^2)/x - ArcCos[a*x]^3/x^2 +
 6*a^2*ArcCos[a*x]*Log[1 + E^((2*I)*ArcCos[a*x])] - (3*I)*a^2*PolyLog[2, -E
^((2*I)*ArcCos[a*x])])/2
```

Maple [A] time = 0.139, size = 113, normalized size = 1.1

$$-\frac{3\,i}{2}a^2\left(\arccos\left(ax\right)\right)^2 - \frac{\left(\arccos\left(ax\right)\right)^3}{2\,x^2} + 3\,a^2\arccos\left(ax\right)\ln\left(1+\left(i\sqrt{-a^2x^2+1}+ax\right)^2\right) - \frac{3\,i}{2}a^2\text{polylog}\left(2, -\left(i\sqrt{-a^2x^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^3/x^3,x)
```

```
[Out] -3/2*I*a^2*arccos(a*x)^2-1/2*arccos(a*x)^3/x^2+3*a^2*arccos(a*x)*ln(1+(I*(-
a^2*x^2+1)^(1/2)+a*x)^2)-3/2*I*a^2*polylog(2,-(I*(-a^2*x^2+1)^(1/2)+a*x)^2)
+3/2*a*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\frac{3}{4}\left(\sqrt{ax+1}\sqrt{-ax+1}\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^2+4x\int\frac{3\sqrt{ax+1}\sqrt{-ax+1}\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^2+2\left(a^3x^3-ax\right)\arctan\left(\sqrt{ax+1}\sqrt{\cdot}\right.}{4\left(a^2x^4-x^2\right)}\right.}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(6*a*x^2*integrate(1/2*sqrt(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x +
1)*sqrt(-a*x + 1), a*x)^2/(a^2*x^4 - x^2), x) - arctan2(sqrt(a*x + 1)*sqrt(
-a*x + 1), a*x)^3)/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arccos(ax)^3}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)^3/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{acos}^3(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**3/x**3,x)
```

```
[Out] Integral(acos(a*x)**3/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos{(ax)}^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^3/x^3,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^3/x^3, x)
```

3.30 $\int \frac{\cos^{-1}(ax)^3}{x^4}\,dx$

Optimal. Leaf size=192

$$ia^3\cos^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\cos^{-1}(ax)}\right)-ia^3\cos^{-1}(ax)\text{PolyLog}\left(2,ie^{i\cos^{-1}(ax)}\right)-a^3\text{PolyLog}\left(3,-ie^{i\cos^{-1}(ax)}\right)+a^3\text{PolyLo}$$

```
[Out] -((a^2*ArcCos[a*x])/x) + (a*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/(2*x^2) - ArcC
os[a*x]^3/(3*x^3) - I*a^3*ArcCos[a*x]^2*ArcTan[E^(I*ArcCos[a*x])] + a^3*Arc
Tanh[Sqrt[1 - a^2*x^2]] + I*a^3*ArcCos[a*x]*PolyLog[2, (-I)*E^(I*ArcCos[a*x
])] - I*a^3*ArcCos[a*x]*PolyLog[2, I*E^(I*ArcCos[a*x])] - a^3*PolyLog[3, (-
I)*E^(I*ArcCos[a*x])] + a^3*PolyLog[3, I*E^(I*ArcCos[a*x])]
```

Rubi [A] time = 0.303869, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {4628, 4702, 4710, 4181, 2531, 2282, 6589, 266, 63, 208}

$$ia^3\cos^{-1}(ax)\text{PolyLog}\left(2,-ie^{i\cos^{-1}(ax)}\right)-ia^3\cos^{-1}(ax)\text{PolyLog}\left(2,ie^{i\cos^{-1}(ax)}\right)-a^3\text{PolyLog}\left(3,-ie^{i\cos^{-1}(ax)}\right)+a^3\text{PolyLo}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^3/x^4,x]
```

```
[Out] -((a^2*ArcCos[a*x])/x) + (a*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/(2*x^2) - ArcC
os[a*x]^3/(3*x^3) - I*a^3*ArcCos[a*x]^2*ArcTan[E^(I*ArcCos[a*x])] + a^3*Arc
Tanh[Sqrt[1 - a^2*x^2]] + I*a^3*ArcCos[a*x]*PolyLog[2, (-I)*E^(I*ArcCos[a*x
])] - I*a^3*ArcCos[a*x]*PolyLog[2, I*E^(I*ArcCos[a*x])] - a^3*PolyLog[3, (-
I)*E^(I*ArcCos[a*x])] + a^3*PolyLog[3, I*E^(I*ArcCos[a*x])]
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4702

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b
```

```
*ArcCos[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)
), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart
[p]), Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 4710

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> -Dist[(c^(m + 1)*Sqrt[d])^(-1), Subst[Int[(a + b*x)^n
*Cos[x]^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^{-1}(ax)^3}{x^4}\,dx = -\frac{\cos^{-1}(ax)^3}{3x^3} - a\int \frac{\cos^{-1}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx$$

$$= \frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{2x^2} - \frac{\cos^{-1}(ax)^3}{3x^3} + a^2\int \frac{\cos^{-1}(ax)}{x^2}\,dx - \frac{1}{2}a^3\int \frac{\cos^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{a^2\cos^{-1}(ax)}{x} + \frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{2x^2} - \frac{\cos^{-1}(ax)^3}{3x^3} + \frac{1}{2}a^3\,\mathrm{Subst}\left(\int x^2\sec(x)\,dx, x, \cos^{-1}(ax)\right) - $$

$$= -\frac{a^2\cos^{-1}(ax)}{x} + \frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{2x^2} - \frac{\cos^{-1}(ax)^3}{3x^3} - ia^3\cos^{-1}(ax)^2\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right) - \frac{1}{2}a^3\,\mathrm{Subs}$$

$$= -\frac{a^2\cos^{-1}(ax)}{x} + \frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{2x^2} - \frac{\cos^{-1}(ax)^3}{3x^3} - ia^3\cos^{-1}(ax)^2\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right) + ia^3\cos^{-1}($$

$$= -\frac{a^2\cos^{-1}(ax)}{x} + \frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{2x^2} - \frac{\cos^{-1}(ax)^3}{3x^3} - ia^3\cos^{-1}(ax)^2\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right) + a^3\tanh^{-}$$

$$= -\frac{a^2\cos^{-1}(ax)}{x} + \frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{2x^2} - \frac{\cos^{-1}(ax)^3}{3x^3} - ia^3\cos^{-1}(ax)^2\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right) + a^3\tanh^{-}$$

Mathematica [A] time = 0.73871, size = 165, normalized size = 0.86

$$-\frac{\cos^{-1}(ax)\left(12a^2x^2 + 4\cos^{-1}(ax)^2 - 3\cos^{-1}(ax)\sin\left(2\cos^{-1}(ax)\right)\right)}{12x^3} + a^3\left(i\cos^{-1}(ax)\mathrm{PolyLog}\left(2, -ie^{i\cos^{-1}(ax)}\right) - i\cos^{-1}(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x]^3/x^4,x]
```

```
[Out] a^3*((-I)*ArcCos[a*x]^2*ArcTan[E^(I*ArcCos[a*x])] + ArcTanh[Sqrt[1 - a^2*x^
2]] + I*ArcCos[a*x]*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] - I*ArcCos[a*x]*Poly
Log[2, I*E^(I*ArcCos[a*x])] - PolyLog[3, (-I)*E^(I*ArcCos[a*x])] + PolyLog[
3, I*E^(I*ArcCos[a*x])]) - (ArcCos[a*x]*(12*a^2*x^2 + 4*ArcCos[a*x]^2 - 3*A
rcCos[a*x]*Sin[2*ArcCos[a*x]]))/(12*x^3)
```

Maple [A] time = 0.175, size = 272, normalized size = 1.4

$$\frac{a(\arccos(ax))^2}{2x^2}\sqrt{-a^2x^2+1}-\frac{a^2\arccos(ax)}{x}-\frac{(\arccos(ax))^3}{3x^3}-\frac{a^3(\arccos(ax))^2}{2}\ln\left(1+i\left(i\sqrt{-a^2x^2+1}+ax\right)\right)+ia^3 a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^3/x^4,x)
```

```
[Out] 1/2*a*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)/x^2-a^2*arccos(a*x)/x-1/3*arccos(a*x
)^3/x^3-1/2*a^3*arccos(a*x)^2*ln(1+I*(I*(-a^2*x^2+1)^(1/2)+a*x))+I*a^3*arcc
os(a*x)*polylog(2,-I*(I*(-a^2*x^2+1)^(1/2)+a*x))-a^3*polylog(3,-I*(I*(-a^2*
x^2+1)^(1/2)+a*x))+1/2*a^3*arccos(a*x)^2*ln(1-I*(I*(-a^2*x^2+1)^(1/2)+a*x))
-I*a^3*arccos(a*x)*polylog(2,I*(I*(-a^2*x^2+1)^(1/2)+a*x))+a^3*polylog(3,I*
(I*(-a^2*x^2+1)^(1/2)+a*x))-2*I*a^3*arctan(I*(-a^2*x^2+1)^(1/2)+a*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\,ax^3\int\frac{\sqrt{-ax+1}\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^2}{\sqrt{ax+1}(ax-1)x^3}\,dx-\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*a*x^3*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x + 1)*s
qrt(-a*x + 1), a*x)^2/(a^2*x^5 - x^3), x) - arctan2(sqrt(a*x + 1)*sqrt(-a*x
 + 1), a*x)^3)/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arccos(ax)^3}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^3/x^4,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)^3/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acos}^3(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**3/x**4,x)
```

```
[Out] Integral(acos(a*x)**3/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(ax)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^3/x^4,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^3/x^4, x)
```

3.31 $\int \frac{\cos^{-1}(ax)^3}{x^5}\,dx$

Optimal. Leaf size=169

$$-\frac{1}{2}ia^4\text{PolyLog}\left(2,-e^{2i\cos^{-1}(ax)}\right)+\frac{a^3\sqrt{1-a^2x^2}}{4x}+\frac{a^3\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{2x}-\frac{a^2\cos^{-1}(ax)}{4x^2}+\frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{4x^3}-\frac{1}{2}$$

```
[Out] (a^3*Sqrt[1 - a^2*x^2])/(4*x) - (a^2*ArcCos[a*x])/(4*x^2) - (I/2)*a^4*ArcCo
s[a*x]^2 + (a*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/(4*x^3) + (a^3*Sqrt[1 - a^2*
x^2]*ArcCos[a*x]^2)/(2*x) - ArcCos[a*x]^3/(4*x^4) + a^4*ArcCos[a*x]*Log[1 +
 E^((2*I)*ArcCos[a*x])] - (I/2)*a^4*PolyLog[2, -E^((2*I)*ArcCos[a*x])]
```

Rubi [A] time = 0.289569, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {4628, 4702, 4682, 4626, 3719, 2190, 2279, 2391, 264}

$$-\frac{1}{2}ia^4\text{PolyLog}\left(2,-e^{2i\cos^{-1}(ax)}\right)+\frac{a^3\sqrt{1-a^2x^2}}{4x}+\frac{a^3\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{2x}-\frac{a^2\cos^{-1}(ax)}{4x^2}+\frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{4x^3}-\frac{1}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^3/x^5,x]
```


```
[Out] (a^3*Sqrt[1 - a^2*x^2])/(4*x) - (a^2*ArcCos[a*x])/(4*x^2) - (I/2)*a^4*ArcCo
s[a*x]^2 + (a*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/(4*x^3) + (a^3*Sqrt[1 - a^2*
x^2]*ArcCos[a*x]^2)/(2*x) - ArcCos[a*x]^3/(4*x^4) + a^4*ArcCos[a*x]*Log[1 +
 E^((2*I)*ArcCos[a*x])] - (I/2)*a^4*PolyLog[2, -E^((2*I)*ArcCos[a*x])]
```


Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4702

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b
*ArcCos[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)
), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Dist[(b*
```

```
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart
[p]), Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 4682

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b
*ArcCos[c*x])^n)/(d*f*(m + 1)), x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^F
racPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 - c
^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &
& NeQ[m, -1]
```

Rule 4626

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n/Cot[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^{-1}(ax)^3}{x^5}\,dx = -\frac{\cos^{-1}(ax)^3}{4x^4} - \frac{1}{4}(3a)\int \frac{\cos^{-1}(ax)^2}{x^4\sqrt{1-a^2x^2}}\,dx$$

$$= \frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{4x^3} - \frac{\cos^{-1}(ax)^3}{4x^4} + \frac{1}{2}a^2\int \frac{\cos^{-1}(ax)}{x^3}\,dx - \frac{1}{2}a^3\int \frac{\cos^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{a^2\cos^{-1}(ax)}{4x^2} + \frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{4x^3} + \frac{a^3\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{2x} - \frac{\cos^{-1}(ax)^3}{4x^4} - \frac{1}{4}a^3\int \frac{1}{x^2\sqrt{1-}$$

$$= \frac{a^3\sqrt{1-a^2x^2}}{4x} - \frac{a^2\cos^{-1}(ax)}{4x^2} + \frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{4x^3} + \frac{a^3\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{2x} - \frac{\cos^{-1}(ax)^3}{4x^4} - a^4$$

$$= \frac{a^3\sqrt{1-a^2x^2}}{4x} - \frac{a^2\cos^{-1}(ax)}{4x^2} - \frac{1}{2}ia^4\cos^{-1}(ax)^2 + \frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{4x^3} + \frac{a^3\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{2x}$$

$$= \frac{a^3\sqrt{1-a^2x^2}}{4x} - \frac{a^2\cos^{-1}(ax)}{4x^2} - \frac{1}{2}ia^4\cos^{-1}(ax)^2 + \frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{4x^3} + \frac{a^3\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{2x}$$

$$= \frac{a^3\sqrt{1-a^2x^2}}{4x} - \frac{a^2\cos^{-1}(ax)}{4x^2} - \frac{1}{2}ia^4\cos^{-1}(ax)^2 + \frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{4x^3} + \frac{a^3\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{2x}$$

$$= \frac{a^3\sqrt{1-a^2x^2}}{4x} - \frac{a^2\cos^{-1}(ax)}{4x^2} - \frac{1}{2}ia^4\cos^{-1}(ax)^2 + \frac{a\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{4x^3} + \frac{a^3\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}{2x}$$

Mathematica [A] time = 0.470525, size = 149, normalized size = 0.88

$$-\frac{\cos^{-1}(ax)^3}{4x^4} + \frac{1}{4}a^4\sqrt{1-a^2x^2}\left(-\frac{2i\text{PolyLog}\left(2,-e^{2i\cos^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} + \frac{\frac{\cos^{-1}(ax)^2}{a^2x^2} + 2\cos^{-1}(ax)^2 + 1}{ax} + \frac{\cos^{-1}(ax)\left(-\frac{1}{a^2x^2} - 2i\cos\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x]^3/x^5,x]
```

```
[Out] -ArcCos[a*x]^3/(4*x^4) + (a^4*Sqrt[1 - a^2*x^2]*((1 + 2*ArcCos[a*x]^2 + Arc
Cos[a*x]^2/(a^2*x^2))/(a*x) + (ArcCos[a*x]*(-(1/(a^2*x^2)) - (2*I)*ArcCos[a
*x] + 4*Log[1 + E^((2*I)*ArcCos[a*x])]))/Sqrt[1 - a^2*x^2] - ((2*I)*PolyLog
[2, -E^((2*I)*ArcCos[a*x])])/Sqrt[1 - a^2*x^2]))/4
```

Maple [A] time = 0.188, size = 176, normalized size = 1.

$$-\frac{i}{2}a^4(\arccos(ax))^2+\frac{i}{4}a^4+\frac{a^3(\arccos(ax))^2}{2x}\sqrt{-a^2x^2+1}+\frac{a^3}{4x}\sqrt{-a^2x^2+1}+\frac{a(\arccos(ax))^2}{4x^3}\sqrt{-a^2x^2+1}-\frac{a^2\arccos(}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^3/x^5,x)
```

```
[Out] -1/2*I*a^4*arccos(a*x)^2+1/4*I*a^4+1/2*a^3*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)
/x+1/4*a^3*(-a^2*x^2+1)^(1/2)/x+1/4*a*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)/x^3-
1/4*a^2*arccos(a*x)/x^2-1/4*arccos(a*x)^3/x^4+a^4*arccos(a*x)*ln(1+(I*(-a^2
*x^2+1)^(1/2)+a*x)^2)-1/2*I*a^4*polylog(2,-(I*(-a^2*x^2+1)^(1/2)+a*x)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\frac{1}{4}\left(\left(2a^2x^2+1\right)\sqrt{ax+1}\sqrt{-ax+1}\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^2+12x^3\int\frac{9\sqrt{ax+1}\sqrt{-ax+1}\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^2+2\left(2a^5x^5-a\right.}{12\left(a^2x^6-x^4\right)}\right.}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^3/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*(12*a*x^4*integrate(1/4*sqrt(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x +
 1)*sqrt(-a*x + 1), a*x)^2/(a^2*x^6 - x^4), x) - arctan2(sqrt(a*x + 1)*sqrt
(-a*x + 1), a*x)^3)/x^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arccos(ax)^3}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^3/x^5,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)^3/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acos}^3(ax)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**3/x**5,x)
```

```
[Out] Integral(acos(a*x)**3/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(ax)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^3/x^5,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^3/x^5, x)
```

3.32 $\int x^5 \cos^{-1}(ax)^4 \, dx$

Optimal. Leaf size=282

$$\frac{65x^4}{3456a^2}+\frac{245x^2}{1152a^4}-\frac{x^5\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{9a}+\frac{x^5\sqrt{1-a^2x^2}\cos^{-1}(ax)}{54a}-\frac{5x^4\cos^{-1}(ax)^2}{48a^2}-\frac{5x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{36a^3}+\frac{65}{}$$

```
[Out] (245*x^2)/(1152*a^4) + (65*x^4)/(3456*a^2) + x^6/324 + (245*x*Sqrt[1 - a^2*
x^2]*ArcCos[a*x])/(576*a^5) + (65*x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(864*a
^3) + (x^5*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(54*a) + (245*ArcCos[a*x]^2)/(115
2*a^6) - (5*x^2*ArcCos[a*x]^2)/(16*a^4) - (5*x^4*ArcCos[a*x]^2)/(48*a^2) -
(x^6*ArcCos[a*x]^2)/18 - (5*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/(24*a^5) - (
5*x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/(36*a^3) - (x^5*Sqrt[1 - a^2*x^2]*Ar
cCos[a*x]^3)/(9*a) - (5*ArcCos[a*x]^4)/(96*a^6) + (x^6*ArcCos[a*x]^4)/6
```

Rubi [A] time = 0.874223, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4628, 4708, 4642, 30}

$$\frac{65x^4}{3456a^2}+\frac{245x^2}{1152a^4}-\frac{x^5\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{9a}+\frac{x^5\sqrt{1-a^2x^2}\cos^{-1}(ax)}{54a}-\frac{5x^4\cos^{-1}(ax)^2}{48a^2}-\frac{5x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{36a^3}+\frac{65}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*ArcCos[a*x]^4,x]
```


```
[Out] (245*x^2)/(1152*a^4) + (65*x^4)/(3456*a^2) + x^6/324 + (245*x*Sqrt[1 - a^2*
x^2]*ArcCos[a*x])/(576*a^5) + (65*x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(864*a
^3) + (x^5*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(54*a) + (245*ArcCos[a*x]^2)/(115
2*a^6) - (5*x^2*ArcCos[a*x]^2)/(16*a^4) - (5*x^4*ArcCos[a*x]^2)/(48*a^2) -
(x^6*ArcCos[a*x]^2)/18 - (5*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/(24*a^5) - (
5*x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/(36*a^3) - (x^5*Sqrt[1 - a^2*x^2]*Ar
cCos[a*x]^3)/(9*a) - (5*ArcCos[a*x]^4)/(96*a^6) + (x^6*ArcCos[a*x]^4)/6
```


Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^5 \cos^{-1}(ax)^4\,dx = \frac{1}{6}x^6\cos^{-1}(ax)^4 + \frac{1}{3}(2a)\int \frac{x^6\cos^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{x^5\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{9a} + \frac{1}{6}x^6\cos^{-1}(ax)^4 - \frac{1}{3}\int x^5\cos^{-1}(ax)^2\,dx + \frac{5\int \frac{x^4\cos^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx}{9a}$$

$$= -\frac{1}{18}x^6\cos^{-1}(ax)^2 - \frac{5x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{36a^3} - \frac{x^5\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{9a} + \frac{1}{6}x^6\cos^{-1}(ax)^4 + \frac{5\int}{}$$

$$= \frac{x^5\sqrt{1-a^2x^2}\cos^{-1}(ax)}{54a} - \frac{5x^4\cos^{-1}(ax)^2}{48a^2} - \frac{1}{18}x^6\cos^{-1}(ax)^2 - \frac{5x\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{24a^5} - \frac{5x^3\sqrt{1-}}{}$$

$$= \frac{x^6}{324} + \frac{65x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)}{864a^3} + \frac{x^5\sqrt{1-a^2x^2}\cos^{-1}(ax)}{54a} - \frac{5x^2\cos^{-1}(ax)^2}{16a^4} - \frac{5x^4\cos^{-1}(ax)^2}{48a^2} -$$

$$= \frac{65x^4}{3456a^2} + \frac{x^6}{324} + \frac{245x\sqrt{1-a^2x^2}\cos^{-1}(ax)}{576a^5} + \frac{65x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)}{864a^3} + \frac{x^5\sqrt{1-a^2x^2}\cos^{-1}(ax)}{54a}$$

$$= \frac{245x^2}{1152a^4} + \frac{65x^4}{3456a^2} + \frac{x^6}{324} + \frac{245x\sqrt{1-a^2x^2}\cos^{-1}(ax)}{576a^5} + \frac{65x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)}{864a^3} + \frac{x^5\sqrt{1-a^2x^2}}{54}$$

Mathematica [A] time = 0.087507, size = 167, normalized size = 0.59

$$\frac{a^2x^2\left(32a^4x^4+195a^2x^2+2205\right)+108\left(16a^6x^6-5\right)\cos^{-1}(ax)^4-144ax\sqrt{1-a^2x^2}\left(8a^4x^4+10a^2x^2+15\right)\cos^{-1}(ax)^3-9\left(\right.}{10368a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*ArcCos[a*x]^4,x]
```

```
[Out] (a^2*x^2*(2205 + 195*a^2*x^2 + 32*a^4*x^4) + 6*a*x*Sqrt[1 - a^2*x^2]*(735 +
 130*a^2*x^2 + 32*a^4*x^4)*ArcCos[a*x] - 9*(-245 + 360*a^2*x^2 + 120*a^4*x^
4 + 64*a^6*x^6)*ArcCos[a*x]^2 - 144*a*x*Sqrt[1 - a^2*x^2]*(15 + 10*a^2*x^2
+ 8*a^4*x^4)*ArcCos[a*x]^3 + 108*(-5 + 16*a^6*x^6)*ArcCos[a*x]^4)/(10368*a^
6)
```

Maple [A] time = 0.09, size = 318, normalized size = 1.1

$$\frac{1}{a^6}\left(\frac{a^6x^6(\arccos(ax))^4}{6}-\frac{(\arccos(ax))^3}{72}\left(8a^5x^5\sqrt{-a^2x^2+1}+10a^3x^3\sqrt{-a^2x^2+1}+15ax\sqrt{-a^2x^2+1}+15\arccos(ax)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arccos(a*x)^4,x)
```

```
[Out] 1/a^6*(1/6*a^6*x^6*arccos(a*x)^4-1/72*arccos(a*x)^3*(8*a^5*x^5*(-a^2*x^2+1)
^(1/2)+10*a^3*x^3*(-a^2*x^2+1)^(1/2)+15*a*x*(-a^2*x^2+1)^(1/2)+15*arccos(a*
x))-1/18*arccos(a*x)^2*a^6*x^6+1/432*arccos(a*x)*(8*a^5*x^5*(-a^2*x^2+1)^(1
/2)+10*a^3*x^3*(-a^2*x^2+1)^(1/2)+15*a*x*(-a^2*x^2+1)^(1/2)+15*arccos(a*x))
-245/1152*arccos(a*x)^2+1/324*x^6*a^6+65/3456*a^4*x^4+245/1152*a^2*x^2-5/48
*a^4*x^4*arccos(a*x)^2+5/192*arccos(a*x)*(2*a^3*x^3*(-a^2*x^2+1)^(1/2)+3*a*
x*(-a^2*x^2+1)^(1/2)+3*arccos(a*x))-5/16*a^2*x^2*arccos(a*x)^2+5/16*arccos(
a*x)*(a*x*(-a^2*x^2+1)^(1/2)+arccos(a*x))-5/32+5/32*arccos(a*x)^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}x^6\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^4-2a\int\frac{\sqrt{ax+1}\sqrt{-ax+1}x^6\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^3}{3\left(a^2x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/6*x^6*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^4 - 2*a*integrate(1/3*sq
rt(a*x + 1)*sqrt(-a*x + 1)*x^6*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3
/(a^2*x^2 - 1), x)
```

Fricas [A] time = 2.45637, size = 383, normalized size = 1.36

$$\frac{32\,a^6x^6 + 195\,a^4x^4 + 108\left(16\,a^6x^6 - 5\right)\arccos\left(ax\right)^4 + 2205\,a^2x^2 - 9\left(64\,a^6x^6 + 120\,a^4x^4 + 360\,a^2x^2 - 245\right)\arccos\left(ax\right)^2}{10368\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccos(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/10368*(32*a^6*x^6 + 195*a^4*x^4 + 108*(16*a^6*x^6 - 5)*arccos(a*x)^4 + 22
05*a^2*x^2 - 9*(64*a^6*x^6 + 120*a^4*x^4 + 360*a^2*x^2 - 245)*arccos(a*x)^2
 - 6*sqrt(-a^2*x^2 + 1)*(24*(8*a^5*x^5 + 10*a^3*x^3 + 15*a*x)*arccos(a*x)^3
 - (32*a^5*x^5 + 130*a^3*x^3 + 735*a*x)*arccos(a*x)))/a^6
```

Sympy [A] time = 24.1654, size = 275, normalized size = 0.98

$$\begin{cases} \frac{x^6\operatorname{acos}^4(ax)}{6} - \frac{x^6\operatorname{acos}^2(ax)}{18} + \frac{x^6}{324} - \frac{x^5\sqrt{-a^2x^2+1}\operatorname{acos}^3(ax)}{9a} + \frac{x^5\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{54a} - \frac{5x^4\operatorname{acos}^2(ax)}{48a^2} + \frac{65x^4}{3456a^2} - \frac{5x^3\sqrt{-a^2x^2+1}\operatorname{acos}^3(ax)}{36a^3} + \\ \frac{\pi^4x^6}{96} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*acos(a*x)**4,x)
```

```
[Out] Piecewise((x**6*acos(a*x)**4/6 - x**6*acos(a*x)**2/18 + x**6/324 - x**5*sqr
t(-a**2*x**2 + 1)*acos(a*x)**3/(9*a) + x**5*sqrt(-a**2*x**2 + 1)*acos(a*x)/
(54*a) - 5*x**4*acos(a*x)**2/(48*a**2) + 65*x**4/(3456*a**2) - 5*x**3*sqrt(
-a**2*x**2 + 1)*acos(a*x)**3/(36*a**3) + 65*x**3*sqrt(-a**2*x**2 + 1)*acos(
a*x)/(864*a**3) - 5*x**2*acos(a*x)**2/(16*a**4) + 245*x**2/(1152*a**4) - 5*
x*sqrt(-a**2*x**2 + 1)*acos(a*x)**3/(24*a**5) + 245*x*sqrt(-a**2*x**2 + 1)*
acos(a*x)/(576*a**5) - 5*acos(a*x)**4/(96*a**6) + 245*acos(a*x)**2/(1152*a*
*6), Ne(a, 0)), (pi**4*x**6/96, True))
```

Giac [A] time = 1.18754, size = 331, normalized size = 1.17

$$\frac{1}{6}x^6\arccos(ax)^4 - \frac{1}{18}x^6\arccos(ax)^2 - \frac{\sqrt{-a^2x^2+1}x^5\arccos(ax)^3}{9a} + \frac{1}{324}x^6 + \frac{\sqrt{-a^2x^2+1}x^5\arccos(ax)}{54a} - \frac{5x^4\arccos}{48a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] 1/6*x^6*arccos(a*x)^4 - 1/18*x^6*arccos(a*x)^2 - 1/9*sqrt(-a^2*x^2 + 1)*x^5
*arccos(a*x)^3/a + 1/324*x^6 + 1/54*sqrt(-a^2*x^2 + 1)*x^5*arccos(a*x)/a -
5/48*x^4*arccos(a*x)^2/a^2 - 5/36*sqrt(-a^2*x^2 + 1)*x^3*arccos(a*x)^3/a^3
+ 65/3456*x^4/a^2 + 65/864*sqrt(-a^2*x^2 + 1)*x^3*arccos(a*x)/a^3 - 5/16*x^
2*arccos(a*x)^2/a^4 - 5/24*sqrt(-a^2*x^2 + 1)*x*arccos(a*x)^3/a^5 + 245/115
2*x^2/a^4 - 5/96*arccos(a*x)^4/a^6 + 245/576*sqrt(-a^2*x^2 + 1)*x*arccos(a*
x)/a^5 + 245/1152*arccos(a*x)^2/a^6 - 9485/82944/a^6
```

3.33 $\int x^4 \cos^{-1}(ax)^4\,dx$

Optimal. Leaf size=250

$$\frac{1088x^3}{16875a^2}-\frac{4x^4\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{25a}+\frac{24x^4\sqrt{1-a^2x^2}\cos^{-1}(ax)}{625a}-\frac{16x^3\cos^{-1}(ax)^2}{75a^2}-\frac{16x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{75a^3}+108$$

```
[Out] (16576*x)/(5625*a^4) + (1088*x^3)/(16875*a^2) + (24*x^5)/3125 + (16576*Sqrt
[1 - a^2*x^2]*ArcCos[a*x])/(5625*a^5) + (1088*x^2*Sqrt[1 - a^2*x^2]*ArcCos[
a*x])/(5625*a^3) + (24*x^4*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(625*a) - (32*x*A
rcCos[a*x]^2)/(25*a^4) - (16*x^3*ArcCos[a*x]^2)/(75*a^2) - (12*x^5*ArcCos[a
*x]^2)/125 - (32*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/(75*a^5) - (16*x^2*Sqrt[1
 - a^2*x^2]*ArcCos[a*x]^3)/(75*a^3) - (4*x^4*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^
3)/(25*a) + (x^5*ArcCos[a*x]^4)/5
```

Rubi [A] time = 0.669239, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4628, 4708, 4678, 4620, 8, 30}

$$\frac{1088x^3}{16875a^2}-\frac{4x^4\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{25a}+\frac{24x^4\sqrt{1-a^2x^2}\cos^{-1}(ax)}{625a}-\frac{16x^3\cos^{-1}(ax)^2}{75a^2}-\frac{16x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{75a^3}+108$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCos[a*x]^4,x]
```

```
[Out] (16576*x)/(5625*a^4) + (1088*x^3)/(16875*a^2) + (24*x^5)/3125 + (16576*Sqrt
[1 - a^2*x^2]*ArcCos[a*x])/(5625*a^5) + (1088*x^2*Sqrt[1 - a^2*x^2]*ArcCos[
a*x])/(5625*a^3) + (24*x^4*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(625*a) - (32*x*A
rcCos[a*x]^2)/(25*a^4) - (16*x^3*ArcCos[a*x]^2)/(75*a^2) - (12*x^5*ArcCos[a
*x]^2)/125 - (32*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/(75*a^5) - (16*x^2*Sqrt[1
 - a^2*x^2]*ArcCos[a*x]^3)/(75*a^3) - (4*x^4*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^
3)/(25*a) + (x^5*ArcCos[a*x]^4)/5
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^4 \cos^{-1}(ax)^4\,dx = \frac{1}{5}x^5\cos^{-1}(ax)^4 + \frac{1}{5}(4a)\int \frac{x^5\cos^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{4x^4\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{25a} + \frac{1}{5}x^5\cos^{-1}(ax)^4 - \frac{12}{25}\int x^4\cos^{-1}(ax)^2\,dx + \frac{16\int \frac{x^3\cos^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx}{25a}$$

$$= -\frac{12}{125}x^5\cos^{-1}(ax)^2 - \frac{16x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{75a^3} - \frac{4x^4\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{25a} + \frac{1}{5}x^5\cos^{-1}(ax)^4 +$$

$$= \frac{24x^4\sqrt{1-a^2x^2}\cos^{-1}(ax)}{625a} - \frac{16x^3\cos^{-1}(ax)^2}{75a^2} - \frac{12}{125}x^5\cos^{-1}(ax)^2 - \frac{32\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{75a^5} - 16$$

$$= \frac{24x^5}{3125} + \frac{1088x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)}{5625a^3} + \frac{24x^4\sqrt{1-a^2x^2}\cos^{-1}(ax)}{625a} - \frac{32x\cos^{-1}(ax)^2}{25a^4} - \frac{16x^3\cos^{-1}(}{75a^2}$$

$$= \frac{1088x^3}{16875a^2} + \frac{24x^5}{3125} + \frac{16576\sqrt{1-a^2x^2}\cos^{-1}(ax)}{5625a^5} + \frac{1088x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)}{5625a^3} + \frac{24x^4\sqrt{1-a^2x^2}\,\mathrm{c}}{625a}$$

$$= \frac{16576x}{5625a^4} + \frac{1088x^3}{16875a^2} + \frac{24x^5}{3125} + \frac{16576\sqrt{1-a^2x^2}\cos^{-1}(ax)}{5625a^5} + \frac{1088x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)}{5625a^3} + \frac{24x^4\sqrt{}}{}$$

Mathematica [A] time = 0.0840269, size = 150, normalized size = 0.6

$$\frac{8ax\left(81a^4x^4+680a^2x^2+31080\right)+16875a^5x^5\cos^{-1}(ax)^4-900ax\left(9a^4x^4+20a^2x^2+120\right)\cos^{-1}(ax)^2-4500\sqrt{1-a^2x^2}}{84375a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcCos[a*x]^4,x]
```

```
[Out] (8*a*x*(31080 + 680*a^2*x^2 + 81*a^4*x^4) + 120*Sqrt[1 - a^2*x^2]*(2072 + 1
36*a^2*x^2 + 27*a^4*x^4)*ArcCos[a*x] - 900*a*x*(120 + 20*a^2*x^2 + 9*a^4*x^
4)*ArcCos[a*x]^2 - 4500*Sqrt[1 - a^2*x^2]*(8 + 4*a^2*x^2 + 3*a^4*x^4)*ArcCo
s[a*x]^3 + 16875*a^5*x^5*ArcCos[a*x]^4)/(84375*a^5)
```

Maple [A] time = 0.058, size = 197, normalized size = 0.8

$$\frac{1}{a^5}\left(\frac{a^5x^5(\arccos(ax))^4}{5} - \frac{4(\arccos(ax))^3\left(3a^4x^4+4a^2x^2+8\right)}{75}\sqrt{-a^2x^2+1} - \frac{32ax(\arccos(ax))^2}{25} + \frac{16576ax}{5625} + \frac{64\text{ a}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccos(a*x)^4,x)
```

```
[Out] 1/a^5*(1/5*a^5*x^5*arccos(a*x)^4-4/75*arccos(a*x)^3*(3*a^4*x^4+4*a^2*x^2+8)
*(-a^2*x^2+1)^(1/2)-32/25*a*x*arccos(a*x)^2+16576/5625*a*x+64/25*arccos(a*x
)*(-a^2*x^2+1)^(1/2)-12/125*arccos(a*x)^2*a^5*x^5+8/625*arccos(a*x)*(3*a^4*
x^4+4*a^2*x^2+8)*(-a^2*x^2+1)^(1/2)+24/3125*a^5*x^5+1088/16875*a^3*x^3-16/7
5*a^3*x^3*arccos(a*x)^2+32/225*arccos(a*x)*(a^2*x^2+2)*(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.52494, size = 278, normalized size = 1.11

$$\frac{1}{5}\,x^5\arccos(ax)^4-\frac{4}{75}\left(\frac{3\sqrt{-a^2x^2+1}x^4}{a^2}+\frac{4\sqrt{-a^2x^2+1}x^2}{a^4}+\frac{8\sqrt{-a^2x^2+1}}{a^6}\right)a\arccos(ax)^3+\frac{4}{84375}\left(2\,a\left(\frac{15\left(27\sqrt{-a^2x^2}\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arccos(a*x)^4 - 4/75*(3*sqrt(-a^2*x^2 + 1)*x^4/a^2 + 4*sqrt(-a^2*x^
2 + 1)*x^2/a^4 + 8*sqrt(-a^2*x^2 + 1)/a^6)*a*arccos(a*x)^3 + 4/84375*(2*a*(
15*(27*sqrt(-a^2*x^2 + 1)*a^2*x^4 + 136*sqrt(-a^2*x^2 + 1)*x^2 + 2072*sqrt(
-a^2*x^2 + 1)/a^2)*arccos(a*x)/a^5 + (81*a^4*x^5 + 680*a^2*x^3 + 31080*x)/a
^6) - 225*(9*a^4*x^5 + 20*a^2*x^3 + 120*x)*arccos(a*x)^2/a^5)*a
```

Fricas [A] time = 2.33717, size = 352, normalized size = 1.41

$$\frac{16875\,a^5x^5\arccos(ax)^4+648\,a^5x^5+5440\,a^3x^3-900\left(9\,a^5x^5+20\,a^3x^3+120\,ax\right)\arccos(ax)^2+248640\,ax-60\sqrt{-a^2x}}{84375\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccos(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/84375*(16875*a^5*x^5*arccos(a*x)^4 + 648*a^5*x^5 + 5440*a^3*x^3 - 900*(9*
a^5*x^5 + 20*a^3*x^3 + 120*a*x)*arccos(a*x)^2 + 248640*a*x - 60*sqrt(-a^2*x
^2 + 1)*(75*(3*a^4*x^4 + 4*a^2*x^2 + 8)*arccos(a*x)^3 - 2*(27*a^4*x^4 + 136
*a^2*x^2 + 2072)*arccos(a*x)))/a^5
```

Sympy [A] time = 13.673, size = 248, normalized size = 0.99

$$\begin{cases} \frac{x^5\operatorname{acos}^4(ax)}{5} - \frac{12x^5\operatorname{acos}^2(ax)}{125} + \frac{24x^5}{3125} - \frac{4x^4\sqrt{-a^2x^2+1}\operatorname{acos}^3(ax)}{25a} + \frac{24x^4\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{625a} - \frac{16x^3\operatorname{acos}^2(ax)}{75a^2} + \frac{1088x^3}{16875a^2} - \frac{16x^2\sqrt{-a^2x^2+1}\operatorname{acc}}{75a^3} \\ \frac{\pi^4x^5}{80} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acos(a*x)**4,x)
```

```
[Out] Piecewise((x**5*acos(a*x)**4/5 - 12*x**5*acos(a*x)**2/125 + 24*x**5/3125 -
4*x**4*sqrt(-a**2*x**2 + 1)*acos(a*x)**3/(25*a) + 24*x**4*sqrt(-a**2*x**2 +
 1)*acos(a*x)/(625*a) - 16*x**3*acos(a*x)**2/(75*a**2) + 1088*x**3/(16875*a
**2) - 16*x**2*sqrt(-a**2*x**2 + 1)*acos(a*x)**3/(75*a**3) + 1088*x**2*sqrt
(-a**2*x**2 + 1)*acos(a*x)/(5625*a**3) - 32*x*acos(a*x)**2/(25*a**4) + 1657
6*x/(5625*a**4) - 32*sqrt(-a**2*x**2 + 1)*acos(a*x)**3/(75*a**5) + 16576*sq
rt(-a**2*x**2 + 1)*acos(a*x)/(5625*a**5), Ne(a, 0)), (pi**4*x**5/80, True))
```

Giac [A] time = 1.16928, size = 286, normalized size = 1.14

$$\frac{1}{5}x^5\arccos(ax)^4 - \frac{12}{125}x^5\arccos(ax)^2 - \frac{4\sqrt{-a^2x^2+1}x^4\arccos(ax)^3}{25a} + \frac{24}{3125}x^5 + \frac{24\sqrt{-a^2x^2+1}x^4\arccos(ax)}{625a} - \frac{16}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] 1/5*x^5*arccos(a*x)^4 - 12/125*x^5*arccos(a*x)^2 - 4/25*sqrt(-a^2*x^2 + 1)*
x^4*arccos(a*x)^3/a + 24/3125*x^5 + 24/625*sqrt(-a^2*x^2 + 1)*x^4*arccos(a*
x)/a - 16/75*x^3*arccos(a*x)^2/a^2 - 16/75*sqrt(-a^2*x^2 + 1)*x^2*arccos(a*
x)^3/a^3 + 1088/16875*x^3/a^2 + 1088/5625*sqrt(-a^2*x^2 + 1)*x^2*arccos(a*x
)/a^3 - 32/25*x*arccos(a*x)^2/a^4 - 32/75*sqrt(-a^2*x^2 + 1)*arccos(a*x)^3/
a^5 + 16576/5625*x/a^4 + 16576/5625*sqrt(-a^2*x^2 + 1)*arccos(a*x)/a^5
```

3.34 $\int x^3 \cos^{-1}(ax)^4 \, dx$

Optimal. Leaf size=198

$$\frac{45x^2}{128a^2} - \frac{x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{4a} + \frac{3x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)}{32a} - \frac{9x^2\cos^{-1}(ax)^2}{16a^2} - \frac{3x\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{8a^3} + \frac{45x\sqrt{1-a^2x^2}}{64a}$$

```
[Out] (45*x^2)/(128*a^2) + (3*x^4)/128 + (45*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(64
*a^3) + (3*x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(32*a) + (45*ArcCos[a*x]^2)/(
128*a^4) - (9*x^2*ArcCos[a*x]^2)/(16*a^2) - (3*x^4*ArcCos[a*x]^2)/16 - (3*x
*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/(8*a^3) - (x^3*Sqrt[1 - a^2*x^2]*ArcCos[a
*x]^3)/(4*a) - (3*ArcCos[a*x]^4)/(32*a^4) + (x^4*ArcCos[a*x]^4)/4
```

Rubi [A] time = 0.519584, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4628, 4708, 4642, 30}

$$\frac{45x^2}{128a^2} - \frac{x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{4a} + \frac{3x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)}{32a} - \frac{9x^2\cos^{-1}(ax)^2}{16a^2} - \frac{3x\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{8a^3} + \frac{45x\sqrt{1-a^2x^2}}{64a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCos[a*x]^4,x]
```

```
[Out] (45*x^2)/(128*a^2) + (3*x^4)/128 + (45*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(64
*a^3) + (3*x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(32*a) + (45*ArcCos[a*x]^2)/(
128*a^4) - (9*x^2*ArcCos[a*x]^2)/(16*a^2) - (3*x^4*ArcCos[a*x]^2)/16 - (3*x
*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/(8*a^3) - (x^3*Sqrt[1 - a^2*x^2]*ArcCos[a
*x]^3)/(4*a) - (3*ArcCos[a*x]^4)/(32*a^4) + (x^4*ArcCos[a*x]^4)/4
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
```

```
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^3\cos^{-1}(ax)^4\,dx = \frac{1}{4}x^4\cos^{-1}(ax)^4 + a\int\frac{x^4\cos^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{4a} + \frac{1}{4}x^4\cos^{-1}(ax)^4 - \frac{3}{4}\int x^3\cos^{-1}(ax)^2\,dx + \frac{3\int\frac{x^2\cos^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx}{4a}$$

$$= -\frac{3}{16}x^4\cos^{-1}(ax)^2 - \frac{3x\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{8a^3} - \frac{x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{4a} + \frac{1}{4}x^4\cos^{-1}(ax)^4 + \frac{3\int\ldots}{\ldots}$$

$$= \frac{3x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)}{32a} - \frac{9x^2\cos^{-1}(ax)^2}{16a^2} - \frac{3}{16}x^4\cos^{-1}(ax)^2 - \frac{3x\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{8a^3} - \frac{x^3\sqrt{1-\ldots}}{\ldots}$$

$$= \frac{3x^4}{128} + \frac{45x\sqrt{1-a^2x^2}\cos^{-1}(ax)}{64a^3} + \frac{3x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)}{32a} - \frac{9x^2\cos^{-1}(ax)^2}{16a^2} - \frac{3}{16}x^4\cos^{-1}(ax)^2 - \ldots$$

$$= \frac{45x^2}{128a^2} + \frac{3x^4}{128} + \frac{45x\sqrt{1-a^2x^2}\cos^{-1}(ax)}{64a^3} + \frac{3x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)}{32a} + \frac{45\cos^{-1}(ax)^2}{128a^4} - \frac{9x^2\cos^{-1}\ldots}{16a^2}$$

Mathematica [A] time = 0.066978, size = 135, normalized size = 0.68

$$\frac{3a^2x^2\left(a^2x^2+15\right)+4\left(8a^4x^4-3\right)\cos^{-1}(ax)^4-16ax\sqrt{1-a^2x^2}\left(2a^2x^2+3\right)\cos^{-1}(ax)^3-3\left(8a^4x^4+24a^2x^2-15\right)\cos^{-1}(\ldots}{128a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCos[a*x]^4,x]
```

```
[Out] (3*a^2*x^2*(15 + a^2*x^2) + 6*a*x*Sqrt[1 - a^2*x^2]*(15 + 2*a^2*x^2)*ArcCos
[a*x] - 3*(-15 + 24*a^2*x^2 + 8*a^4*x^4)*ArcCos[a*x]^2 - 16*a*x*Sqrt[1 - a^
2*x^2]*(3 + 2*a^2*x^2)*ArcCos[a*x]^3 + 4*(-3 + 8*a^4*x^4)*ArcCos[a*x]^4)/(1
28*a^4)
```

Maple [A] time = 0.06, size = 207, normalized size = 1.1

$$\frac{1}{a^4}\left(\frac{a^4x^4(\arccos(ax))^4}{4}-\frac{(\arccos(ax))^3}{8}\left(2a^3x^3\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}+3\arccos(ax)\right)-\frac{3a^4x^4(\arccos(ax))^2}{16}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccos(a*x)^4,x)
```

```
[Out] 1/a^4*(1/4*a^4*x^4*arccos(a*x)^4-1/8*arccos(a*x)^3*(2*a^3*x^3*(-a^2*x^2+1)^
(1/2)+3*a*x*(-a^2*x^2+1)^(1/2)+3*arccos(a*x))-3/16*a^4*x^4*arccos(a*x)^2+3/
64*arccos(a*x)*(2*a^3*x^3*(-a^2*x^2+1)^(1/2)+3*a*x*(-a^2*x^2+1)^(1/2)+3*arc
cos(a*x))-45/128*arccos(a*x)^2+3/128*a^4*x^4+45/128*a^2*x^2-9/16*a^2*x^2*ar
ccos(a*x)^2+9/16*arccos(a*x)*(a*x*(-a^2*x^2+1)^(1/2)+arccos(a*x))-9/32+9/32
*arccos(a*x)^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}x^4\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^4-a\int\frac{\sqrt{ax+1}\sqrt{-ax+1}x^4\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^3}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^4 - a*integrate(sqrt(a*x
 + 1)*sqrt(-a*x + 1)*x^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3/(a^2*
x^2 - 1), x)
```

Fricas [A] time = 2.32852, size = 292, normalized size = 1.47

$$\frac{3\,a^4x^4 + 4\left(8\,a^4x^4 - 3\right)\arccos\left(ax\right)^4 + 45\,a^2x^2 - 3\left(8\,a^4x^4 + 24\,a^2x^2 - 15\right)\arccos\left(ax\right)^2 - 2\,\sqrt{-a^2x^2+1}\left(8\left(2\,a^3x^3 + 3\,ax\right.\right.}{128\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(3*a^4*x^4 + 4*(8*a^4*x^4 - 3)*arccos(a*x)^4 + 45*a^2*x^2 - 3*(8*a^4*
x^4 + 24*a^2*x^2 - 15)*arccos(a*x)^2 - 2*sqrt(-a^2*x^2 + 1)*(8*(2*a^3*x^3 +
 3*a*x)*arccos(a*x)^3 - 3*(2*a^3*x^3 + 15*a*x)*arccos(a*x)))/a^4
```

Sympy [A] time = 8.28441, size = 197, normalized size = 0.99

$$\begin{cases} \frac{x^4\operatorname{acos}^4(ax)}{4} - \frac{3x^4\operatorname{acos}^2(ax)}{16} + \frac{3x^4}{128} - \frac{x^3\sqrt{-a^2x^2+1}\operatorname{acos}^3(ax)}{4a} + \frac{3x^3\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{32a} - \frac{9x^2\operatorname{acos}^2(ax)}{16a^2} + \frac{45x^2}{128a^2} - \frac{3x\sqrt{-a^2x^2+1}\operatorname{acos}^3(ax)}{8a^3} + \\ \frac{\pi^4x^4}{64} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acos(a*x)**4,x)
```

```
[Out] Piecewise((x**4*acos(a*x)**4/4 - 3*x**4*acos(a*x)**2/16 + 3*x**4/128 - x**3
*sqrt(-a**2*x**2 + 1)*acos(a*x)**3/(4*a) + 3*x**3*sqrt(-a**2*x**2 + 1)*acos
(a*x)/(32*a) - 9*x**2*acos(a*x)**2/(16*a**2) + 45*x**2/(128*a**2) - 3*x*sqr
t(-a**2*x**2 + 1)*acos(a*x)**3/(8*a**3) + 45*x*sqrt(-a**2*x**2 + 1)*acos(a*
x)/(64*a**3) - 3*acos(a*x)**4/(32*a**4) + 45*acos(a*x)**2/(128*a**4), Ne(a,
 0)), (pi**4*x**4/64, True))
```

Giac [A] time = 1.20515, size = 234, normalized size = 1.18

$$\frac{1}{4}\,x^4\arccos\left(ax\right)^4 - \frac{3}{16}\,x^4\arccos\left(ax\right)^2 - \frac{\sqrt{-a^2x^2+1}x^3\arccos\left(ax\right)^3}{4\,a} + \frac{3}{128}\,x^4 + \frac{3\,\sqrt{-a^2x^2+1}x^3\arccos\left(ax\right)}{32\,a} - \frac{9\,x^2\,\text{arc}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] 1/4*x^4*arccos(a*x)^4 - 3/16*x^4*arccos(a*x)^2 - 1/4*sqrt(-a^2*x^2 + 1)*x^3
*arccos(a*x)^3/a + 3/128*x^4 + 3/32*sqrt(-a^2*x^2 + 1)*x^3*arccos(a*x)/a -
9/16*x^2*arccos(a*x)^2/a^2 - 3/8*sqrt(-a^2*x^2 + 1)*x*arccos(a*x)^3/a^3 + 4
5/128*x^2/a^2 - 3/32*arccos(a*x)^4/a^4 + 45/64*sqrt(-a^2*x^2 + 1)*x*arccos(
a*x)/a^3 + 45/128*arccos(a*x)^2/a^4 - 189/1024/a^4
```

3.35 $\int x^2 \cos^{-1}(ax)^4 \, dx$

Optimal. Leaf size=166

$$-\frac{4x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{9a}-\frac{8\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{9a^3}+\frac{8x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)}{27a}+\frac{160\sqrt{1-a^2x^2}\cos^{-1}(ax)}{27a^3}+\frac{160x}{27a^2}-8x$$

```
[Out] (160*x)/(27*a^2) + (8*x^3)/81 + (160*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(27*a^3
) + (8*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(27*a) - (8*x*ArcCos[a*x]^2)/(3*a
^2) - (4*x^3*ArcCos[a*x]^2)/9 - (8*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/(9*a^3)
 - (4*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/(9*a) + (x^3*ArcCos[a*x]^4)/3
```

Rubi [A] time = 0.365734, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4628, 4708, 4678, 4620, 8, 30}

$$-\frac{4x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{9a}-\frac{8\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{9a^3}+\frac{8x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)}{27a}+\frac{160\sqrt{1-a^2x^2}\cos^{-1}(ax)}{27a^3}+\frac{160x}{27a^2}-8x$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCos[a*x]^4,x]
```

```
[Out] (160*x)/(27*a^2) + (8*x^3)/81 + (160*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(27*a^3
) + (8*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(27*a) - (8*x*ArcCos[a*x]^2)/(3*a
^2) - (4*x^3*ArcCos[a*x]^2)/9 - (8*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/(9*a^3)
 - (4*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/(9*a) + (x^3*ArcCos[a*x]^4)/3
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
```

```
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2 \cos^{-1}(ax)^4\,dx = \frac{1}{3}x^3\cos^{-1}(ax)^4 + \frac{1}{3}(4a)\int \frac{x^3\cos^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{4x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{9a} + \frac{1}{3}x^3\cos^{-1}(ax)^4 - \frac{4}{3}\int x^2\cos^{-1}(ax)^2\,dx + \frac{8\int \frac{x\cos^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx}{9a}$$
$$= -\frac{4}{9}x^3\cos^{-1}(ax)^2 - \frac{8\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{9a^3} - \frac{4x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{9a} + \frac{1}{3}x^3\cos^{-1}(ax)^4 - \frac{8\int \mathrm{co}$$
$$= \frac{8x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)}{27a} - \frac{8x\cos^{-1}(ax)^2}{3a^2} - \frac{4}{9}x^3\cos^{-1}(ax)^2 - \frac{8\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{9a^3} - \frac{4x^2\sqrt{1-a}$$
$$= \frac{8x^3}{81} + \frac{160\sqrt{1-a^2x^2}\cos^{-1}(ax)}{27a^3} + \frac{8x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)}{27a} - \frac{8x\cos^{-1}(ax)^2}{3a^2} - \frac{4}{9}x^3\cos^{-1}(ax)^2 - \frac{8}{}$$
$$= \frac{160x}{27a^2} + \frac{8x^3}{81} + \frac{160\sqrt{1-a^2x^2}\cos^{-1}(ax)}{27a^3} + \frac{8x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)}{27a} - \frac{8x\cos^{-1}(ax)^2}{3a^2} - \frac{4}{9}x^3\cos^{-1}($$

Mathematica [A] time = 0.0722178, size = 114, normalized size = 0.69

$$\frac{8ax\left(a^2x^2+60\right) + 27a^3x^3\cos^{-1}(ax)^4 - 36\sqrt{1-a^2x^2}\left(a^2x^2+2\right)\cos^{-1}(ax)^3 - 36ax\left(a^2x^2+6\right)\cos^{-1}(ax)^2 + 24\sqrt{1-a^2x^2}}{81a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCos[a*x]^4,x]
```

```
[Out] (8*a*x*(60 + a^2*x^2) + 24*Sqrt[1 - a^2*x^2]*(20 + a^2*x^2)*ArcCos[a*x] - 3
6*a*x*(6 + a^2*x^2)*ArcCos[a*x]^2 - 36*Sqrt[1 - a^2*x^2]*(2 + a^2*x^2)*ArcC
os[a*x]^3 + 27*a^3*x^3*ArcCos[a*x]^4)/(81*a^3)
```

Maple [A] time = 0.052, size = 130, normalized size = 0.8

$$\frac{1}{a^3}\left(\frac{a^3x^3(\arccos(ax))^4}{3} - \frac{4(\arccos(ax))^3\left(a^2x^2+2\right)}{9}\sqrt{-a^2x^2+1} - \frac{8ax(\arccos(ax))^2}{3} + \frac{160ax}{27} + \frac{16\arccos(ax)}{3}\sqrt{-a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccos(a*x)^4,x)
```

```
[Out] 1/a^3*(1/3*a^3*x^3*arccos(a*x)^4-4/9*arccos(a*x)^3*(a^2*x^2+2)*(-a^2*x^2+1)
^(1/2)-8/3*a*x*arccos(a*x)^2+160/27*a*x+16/3*arccos(a*x)*(-a^2*x^2+1)^(1/2)
-4/9*a^3*x^3*arccos(a*x)^2+8/27*arccos(a*x)*(a^2*x^2+2)*(-a^2*x^2+1)^(1/2)+
8/81*a^3*x^3)
```

Maxima [A] time = 1.52923, size = 197, normalized size = 1.19

$$\frac{1}{3}x^3\arccos(ax)^4-\frac{4}{9}a\left(\frac{\sqrt{-a^2x^2+1}x^2}{a^2}+\frac{2\sqrt{-a^2x^2+1}}{a^4}\right)\arccos(ax)^3+\frac{4}{81}\left(2a\left(\frac{3\left(\sqrt{-a^2x^2+1}x^2+\frac{20\sqrt{-a^2x^2+1}}{a^2}\right)\arccos(}{a^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccos(a*x)^4 - 4/9*a*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^2
 + 1)/a^4)*arccos(a*x)^3 + 4/81*(2*a*(3*(sqrt(-a^2*x^2 + 1)*x^2 + 20*sqrt(-
a^2*x^2 + 1)/a^2)*arccos(a*x)/a^3 + (a^2*x^3 + 60*x)/a^4) - 9*(a^2*x^3 + 6*
x)*arccos(a*x)^2/a^3)*a
```

Fricas [A] time = 2.42694, size = 247, normalized size = 1.49

$$\frac{27a^3x^3\arccos(ax)^4+8a^3x^3-36\left(a^3x^3+6ax\right)\arccos(ax)^2+480ax-12\sqrt{-a^2x^2+1}\left(3\left(a^2x^2+2\right)\arccos(ax)^3-2\left(a^2\right.\right.}{81a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/81*(27*a^3*x^3*arccos(a*x)^4 + 8*a^3*x^3 - 36*(a^3*x^3 + 6*a*x)*arccos(a*
x)^2 + 480*a*x - 12*sqrt(-a^2*x^2 + 1)*(3*(a^2*x^2 + 2)*arccos(a*x)^3 - 2*(
a^2*x^2 + 20)*arccos(a*x)))/a^3
```

Sympy [A] time = 4.37318, size = 165, normalized size = 0.99

$$\begin{cases}\frac{x^3\operatorname{acos}^4(ax)}{3}-\frac{4x^3\operatorname{acos}^2(ax)}{9}+\frac{8x^3}{81}-\frac{4x^2\sqrt{-a^2x^2+1}\operatorname{acos}^3(ax)}{9a}+\frac{8x^2\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{27a}-\frac{8x\operatorname{acos}^2(ax)}{3a^2}+\frac{160x}{27a^2}-\frac{8\sqrt{-a^2x^2+1}\operatorname{acos}^3(ax)}{9a^3}+160 \\ \frac{\pi^4x^3}{48}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acos(a*x)**4,x)
```

```
[Out] Piecewise((x**3*acos(a*x)**4/3 - 4*x**3*acos(a*x)**2/9 + 8*x**3/81 - 4*x**2
*sqrt(-a**2*x**2 + 1)*acos(a*x)**3/(9*a) + 8*x**2*sqrt(-a**2*x**2 + 1)*acos
(a*x)/(27*a) - 8*x*acos(a*x)**2/(3*a**2) + 160*x/(27*a**2) - 8*sqrt(-a**2*x
**2 + 1)*acos(a*x)**3/(9*a**3) + 160*sqrt(-a**2*x**2 + 1)*acos(a*x)/(27*a**
3), Ne(a, 0)), (pi**4*x**3/48, True))
```

Giac [A] time = 1.17803, size = 189, normalized size = 1.14

$$\frac{1}{3}x^3\arccos(ax)^4 - \frac{4}{9}x^3\arccos(ax)^2 - \frac{4\sqrt{-a^2x^2+1}x^2\arccos(ax)^3}{9a} + \frac{8}{81}x^3 + \frac{8\sqrt{-a^2x^2+1}x^2\arccos(ax)}{27a} - \frac{8x\,\mathrm{arcc}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] 1/3*x^3*arccos(a*x)^4 - 4/9*x^3*arccos(a*x)^2 - 4/9*sqrt(-a^2*x^2 + 1)*x^2*
arccos(a*x)^3/a + 8/81*x^3 + 8/27*sqrt(-a^2*x^2 + 1)*x^2*arccos(a*x)/a - 8/
3*x*arccos(a*x)^2/a^2 - 8/9*sqrt(-a^2*x^2 + 1)*arccos(a*x)^3/a^3 + 160/27*x
/a^2 + 160/27*sqrt(-a^2*x^2 + 1)*arccos(a*x)/a^3
```

3.36 $\int x\cos^{-1}(ax)^4\,dx$

Optimal. Leaf size=112

$$-\frac{x\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{a}+\frac{3x\sqrt{1-a^2x^2}\cos^{-1}(ax)}{2a}-\frac{\cos^{-1}(ax)^4}{4a^2}+\frac{3\cos^{-1}(ax)^2}{4a^2}+\frac{1}{2}x^2\cos^{-1}(ax)^4-\frac{3}{2}x^2\cos^{-1}(ax)^2+\frac{3x}{4}$$

```
[Out] (3*x^2)/4 + (3*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(2*a) + (3*ArcCos[a*x]^2)/(
4*a^2) - (3*x^2*ArcCos[a*x]^2)/2 - (x*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/a -
ArcCos[a*x]^4/(4*a^2) + (x^2*ArcCos[a*x]^4)/2
```

Rubi [A] time = 0.237046, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4628, 4708, 4642, 30}

$$-\frac{x\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{a}+\frac{3x\sqrt{1-a^2x^2}\cos^{-1}(ax)}{2a}-\frac{\cos^{-1}(ax)^4}{4a^2}+\frac{3\cos^{-1}(ax)^2}{4a^2}+\frac{1}{2}x^2\cos^{-1}(ax)^4-\frac{3}{2}x^2\cos^{-1}(ax)^2+\frac{3x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCos[a*x]^4,x]
```

```
[Out] (3*x^2)/4 + (3*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(2*a) + (3*ArcCos[a*x]^2)/(
4*a^2) - (3*x^2*ArcCos[a*x]^2)/2 - (x*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/a -
ArcCos[a*x]^4/(4*a^2) + (x^2*ArcCos[a*x]^4)/2
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

```
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x\cos^{-1}(ax)^4\,dx = \frac{1}{2}x^2\cos^{-1}(ax)^4 + (2a)\int \frac{x^2\cos^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{x\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{a} + \frac{1}{2}x^2\cos^{-1}(ax)^4 - 3\int x\cos^{-1}(ax)^2\,dx + \frac{\int \frac{\cos^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx}{a}$$

$$= -\frac{3}{2}x^2\cos^{-1}(ax)^2 - \frac{x\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{a} - \frac{\cos^{-1}(ax)^4}{4a^2} + \frac{1}{2}x^2\cos^{-1}(ax)^4 - (3a)\int \frac{x^2\cos^{-1}(ax)}{\sqrt{1-a^2x^2}}\,d$$

$$= \frac{3x\sqrt{1-a^2x^2}\cos^{-1}(ax)}{2a} - \frac{3}{2}x^2\cos^{-1}(ax)^2 - \frac{x\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{a} - \frac{\cos^{-1}(ax)^4}{4a^2} + \frac{1}{2}x^2\cos^{-1}(ax)^4$$

$$= \frac{3x^2}{4} + \frac{3x\sqrt{1-a^2x^2}\cos^{-1}(ax)}{2a} + \frac{3\cos^{-1}(ax)^2}{4a^2} - \frac{3}{2}x^2\cos^{-1}(ax)^2 - \frac{x\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{a} - \frac{\cos^{-1}(}{4a}$$

Mathematica [A] time = 0.0381788, size = 96, normalized size = 0.86

$$\frac{3a^2x^2 + \left(2a^2x^2-1\right)\cos^{-1}(ax)^4 - 4ax\sqrt{1-a^2x^2}\cos^{-1}(ax)^3 + \left(3-6a^2x^2\right)\cos^{-1}(ax)^2 + 6ax\sqrt{1-a^2x^2}\cos^{-1}(ax)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCos[a*x]^4,x]
```

```
[Out] (3*a^2*x^2 + 6*a*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x] + (3 - 6*a^2*x^2)*ArcCos[a
*x]^2 - 4*a*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3 + (-1 + 2*a^2*x^2)*ArcCos[a*x
]^4)/(4*a^2)
```

Maple [A] time = 0.053, size = 113, normalized size = 1.

$$\frac{1}{a^2}\left(\frac{a^2x^2(\arccos(ax))^4}{2}-(\arccos(ax))^3\left(ax\sqrt{-a^2x^2+1}+\arccos(ax)\right)-\frac{3a^2x^2(\arccos(ax))^2}{2}+\frac{3\arccos(ax)}{2}\left(ax\sqrt{-a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccos(a*x)^4,x)
```

```
[Out] 1/a^2*(1/2*a^2*x^2*arccos(a*x)^4-arccos(a*x)^3*(a*x*(-a^2*x^2+1)^(1/2)+arcc
os(a*x))-3/2*a^2*x^2*arccos(a*x)^2+3/2*arccos(a*x)*(a*x*(-a^2*x^2+1)^(1/2)+
arccos(a*x))-3/4*arccos(a*x)^2+3/4*a^2*x^2-3/4+3/4*arccos(a*x)^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x^2\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^4-2a\int\frac{\sqrt{ax+1}\sqrt{-ax+1}x^2\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^3}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^4 - 2*a*integrate(sqrt(a
*x + 1)*sqrt(-a*x + 1)*x^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3/(a^
2*x^2 - 1), x)
```

Fricas [A] time = 2.32759, size = 205, normalized size = 1.83

$$\frac{\left(2a^2x^2-1\right)\arccos(ax)^4+3a^2x^2-3\left(2a^2x^2-1\right)\arccos(ax)^2-2\left(2ax\arccos(ax)^3-3ax\arccos(ax)\right)\sqrt{-a^2x^2+1}}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2*x^2 - 1)*arccos(a*x)^4 + 3*a^2*x^2 - 3*(2*a^2*x^2 - 1)*arccos(a
*x)^2 - 2*(2*a*x*arccos(a*x)^3 - 3*a*x*arccos(a*x))*sqrt(-a^2*x^2 + 1))/a^2
```

Sympy [A] time = 2.34343, size = 110, normalized size = 0.98

$$\begin{cases} \frac{x^2\operatorname{acos}^4(ax)}{2} - \frac{3x^2\operatorname{acos}^2(ax)}{2} + \frac{3x^2}{4} - \frac{x\sqrt{-a^2x^2+1}\operatorname{acos}^3(ax)}{a} + \frac{3x\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{2a} - \frac{\operatorname{acos}^4(ax)}{4a^2} + \frac{3\operatorname{acos}^2(ax)}{4a^2} & \text{for } a \neq 0 \\ \frac{\pi^4x^2}{32} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acos(a*x)**4,x)
```

```
[Out] Piecewise((x**2*acos(a*x)**4/2 - 3*x**2*acos(a*x)**2/2 + 3*x**2/4 - x*sqrt(
-a**2*x**2 + 1)*acos(a*x)**3/a + 3*x*sqrt(-a**2*x**2 + 1)*acos(a*x)/(2*a) -
 acos(a*x)**4/(4*a**2) + 3*acos(a*x)**2/(4*a**2), Ne(a, 0)), (pi**4*x**2/32
, True))
```

Giac [A] time = 1.16109, size = 136, normalized size = 1.21

$$\frac{1}{2}x^2\arccos(ax)^4 - \frac{3}{2}x^2\arccos(ax)^2 - \frac{\sqrt{-a^2x^2+1}x\arccos(ax)^3}{a} + \frac{3}{4}x^2 - \frac{\arccos(ax)^4}{4a^2} + \frac{3\sqrt{-a^2x^2+1}x\arccos(ax)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] 1/2*x^2*arccos(a*x)^4 - 3/2*x^2*arccos(a*x)^2 - sqrt(-a^2*x^2 + 1)*x*arccos
(a*x)^3/a + 3/4*x^2 - 1/4*arccos(a*x)^4/a^2 + 3/2*sqrt(-a^2*x^2 + 1)*x*arcc
os(a*x)/a + 3/4*arccos(a*x)^2/a^2 - 3/8/a^2
```

3.37 $\int \cos^{-1}(ax)^4\,dx$

Optimal. Leaf size=69

$$-\frac{4\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{a}+\frac{24\sqrt{1-a^2x^2}\cos^{-1}(ax)}{a}+x\cos^{-1}(ax)^4-12x\cos^{-1}(ax)^2+24x$$

```
[Out] 24*x + (24*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a - 12*x*ArcCos[a*x]^2 - (4*Sqrt[
1 - a^2*x^2]*ArcCos[a*x]^3)/a + x*ArcCos[a*x]^4
```

Rubi [A] time = 0.120369, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4620, 4678, 8}

$$-\frac{4\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{a}+\frac{24\sqrt{1-a^2x^2}\cos^{-1}(ax)}{a}+x\cos^{-1}(ax)^4-12x\cos^{-1}(ax)^2+24x$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^4,x]
```

```
[Out] 24*x + (24*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a - 12*x*ArcCos[a*x]^2 - (4*Sqrt[
1 - a^2*x^2]*ArcCos[a*x]^3)/a + x*ArcCos[a*x]^4
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^{-1}(ax)^4\,dx = x\cos^{-1}(ax)^4 + (4a)\int \frac{x\cos^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{4\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{a} + x\cos^{-1}(ax)^4 - 12\int \cos^{-1}(ax)^2\,dx$$
$$= -12x\cos^{-1}(ax)^2 - \frac{4\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{a} + x\cos^{-1}(ax)^4 - (24a)\int \frac{x\cos^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{24\sqrt{1-a^2x^2}\cos^{-1}(ax)}{a} - 12x\cos^{-1}(ax)^2 - \frac{4\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{a} + x\cos^{-1}(ax)^4 + 24\int 1\,dx$$
$$= 24x + \frac{24\sqrt{1-a^2x^2}\cos^{-1}(ax)}{a} - 12x\cos^{-1}(ax)^2 - \frac{4\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{a} + x\cos^{-1}(ax)^4$$

Mathematica [A] time = 0.0216089, size = 69, normalized size = 1.

$$-\frac{4\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{a} + \frac{24\sqrt{1-a^2x^2}\cos^{-1}(ax)}{a} + x\cos^{-1}(ax)^4 - 12x\cos^{-1}(ax)^2 + 24x$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x]^4,x]
```

```
[Out] 24*x + (24*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a - 12*x*ArcCos[a*x]^2 - (4*Sqrt[
1 - a^2*x^2]*ArcCos[a*x]^3)/a + x*ArcCos[a*x]^4
```

Maple [A] time = 0.047, size = 67, normalized size = 1.

$$\frac{1}{a}\left(ax\left(\arccos(ax)\right)^4 - 4\left(\arccos(ax)\right)^3\sqrt{-a^2x^2+1} - 12\,ax\left(\arccos(ax)\right)^2 + 24\,ax + 24\arccos(ax)\sqrt{-a^2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^4,x)
```

```
[Out] 1/a*(a*x*arccos(a*x)^4-4*arccos(a*x)^3*(-a^2*x^2+1)^(1/2)-12*a*x*arccos(a*x
)^2+24*a*x+24*arccos(a*x)*(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.48534, size = 100, normalized size = 1.45

$$x\arccos(ax)^4 - \frac{4\sqrt{-a^2x^2+1}\arccos(ax)^3}{a} - 12\left(\frac{x\arccos(ax)^2}{a} - \frac{2\left(x + \frac{\sqrt{-a^2x^2+1}\arccos(ax)}{a}\right)}{a}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] x*arccos(a*x)^4 - 4*sqrt(-a^2*x^2 + 1)*arccos(a*x)^3/a - 12*(x*arccos(a*x)^
2/a - 2*(x + sqrt(-a^2*x^2 + 1)*arccos(a*x)/a)/a)*a
```

Fricas [A] time = 2.38187, size = 149, normalized size = 2.16

$$\frac{ax\arccos(ax)^4 - 12\,ax\arccos(ax)^2 + 24\,ax - 4\sqrt{-a^2x^2+1}\left(\arccos(ax)^3 - 6\,\arccos(ax)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^4,x, algorithm="fricas")
```

```
[Out] (a*x*arccos(a*x)^4 - 12*a*x*arccos(a*x)^2 + 24*a*x - 4*sqrt(-a^2*x^2 + 1)*(
arccos(a*x)^3 - 6*arccos(a*x)))/a
```

Sympy [A] time = 1.10413, size = 70, normalized size = 1.01

$$\begin{cases} x\operatorname{acos}^4(ax) - 12x\operatorname{acos}^2(ax) + 24x - \frac{4\sqrt{-a^2x^2+1}\operatorname{acos}^3(ax)}{a} + \frac{24\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{a} & \text{for } a \neq 0 \\ \frac{\pi^4 x}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**4,x)
```

```
[Out] Piecewise((x*acos(a*x)**4 - 12*x*acos(a*x)**2 + 24*x - 4*sqrt(-a**2*x**2 +
1)*acos(a*x)**3/a + 24*sqrt(-a**2*x**2 + 1)*acos(a*x)/a, Ne(a, 0)), (pi**4*
```

```
x/16, True))
```

Giac [A] time = 1.14171, size = 88, normalized size = 1.28

$$x \arccos(ax)^4 - 12\, x \arccos(ax)^2 - \frac{4\, \sqrt{-a^2x^2+1} \arccos(ax)^3}{a} + 24\, x + \frac{24\, \sqrt{-a^2x^2+1} \arccos(ax)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] x*arccos(a*x)^4 - 12*x*arccos(a*x)^2 - 4*sqrt(-a^2*x^2 + 1)*arccos(a*x)^3/a
 + 24*x + 24*sqrt(-a^2*x^2 + 1)*arccos(a*x)/a
```

3.38 $\int \frac{\cos^{-1}(ax)^4}{x} dx$

Optimal. Leaf size=119

$$-2i\cos^{-1}(ax)^3\text{PolyLog}\left(2,-e^{2i\cos^{-1}(ax)}\right)+3\cos^{-1}(ax)^2\text{PolyLog}\left(3,-e^{2i\cos^{-1}(ax)}\right)+3i\cos^{-1}(ax)\text{PolyLog}\left(4,-e^{2i\cos^{-1}(ax)}\right.$$

```
[Out] (-I/5)*ArcCos[a*x]^5 + ArcCos[a*x]^4*Log[1 + E^((2*I)*ArcCos[a*x])] - (2*I)
*ArcCos[a*x]^3*PolyLog[2, -E^((2*I)*ArcCos[a*x])] + 3*ArcCos[a*x]^2*PolyLog
[3, -E^((2*I)*ArcCos[a*x])] + (3*I)*ArcCos[a*x]*PolyLog[4, -E^((2*I)*ArcCos
[a*x])] - (3*PolyLog[5, -E^((2*I)*ArcCos[a*x])])/2
```

Rubi [A] time = 0.125483, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {4626, 3719, 2190, 2531, 6609, 2282, 6589}

$$-2i\cos^{-1}(ax)^3\text{PolyLog}\left(2,-e^{2i\cos^{-1}(ax)}\right)+3\cos^{-1}(ax)^2\text{PolyLog}\left(3,-e^{2i\cos^{-1}(ax)}\right)+3i\cos^{-1}(ax)\text{PolyLog}\left(4,-e^{2i\cos^{-1}(ax)}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^4/x,x]
```


```
[Out] (-I/5)*ArcCos[a*x]^5 + ArcCos[a*x]^4*Log[1 + E^((2*I)*ArcCos[a*x])] - (2*I)
*ArcCos[a*x]^3*PolyLog[2, -E^((2*I)*ArcCos[a*x])] + 3*ArcCos[a*x]^2*PolyLog
[3, -E^((2*I)*ArcCos[a*x])] + (3*I)*ArcCos[a*x]*PolyLog[4, -E^((2*I)*ArcCos
[a*x])] - (3*PolyLog[5, -E^((2*I)*ArcCos[a*x])])/2
```


Rule 4626

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n/Cot[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cos^{-1}(ax)^4}{x}\,dx = -\,\text{Subst}\left(\int x^4\tan(x)\,dx, x, \cos^{-1}(ax)\right)$$
$$= -\frac{1}{5}i\cos^{-1}(ax)^5 + 2i\,\text{Subst}\left(\int \frac{e^{2ix}x^4}{1+e^{2ix}}\,dx, x, \cos^{-1}(ax)\right)$$
$$= -\frac{1}{5}i\cos^{-1}(ax)^5 + \cos^{-1}(ax)^4\log\left(1+e^{2i\cos^{-1}(ax)}\right) - 4\,\text{Subst}\left(\int x^3\log\left(1+e^{2ix}\right)dx, x, \cos^{-1}(ax)\right)$$
$$= -\frac{1}{5}i\cos^{-1}(ax)^5 + \cos^{-1}(ax)^4\log\left(1+e^{2i\cos^{-1}(ax)}\right) - 2i\cos^{-1}(ax)^3\text{Li}_2\left(-e^{2i\cos^{-1}(ax)}\right) + 6i\,\text{Subst}\left(\int x^2\right.$$
$$= -\frac{1}{5}i\cos^{-1}(ax)^5 + \cos^{-1}(ax)^4\log\left(1+e^{2i\cos^{-1}(ax)}\right) - 2i\cos^{-1}(ax)^3\text{Li}_2\left(-e^{2i\cos^{-1}(ax)}\right) + 3\cos^{-1}(ax)^2\text{Li}_3$$
$$= -\frac{1}{5}i\cos^{-1}(ax)^5 + \cos^{-1}(ax)^4\log\left(1+e^{2i\cos^{-1}(ax)}\right) - 2i\cos^{-1}(ax)^3\text{Li}_2\left(-e^{2i\cos^{-1}(ax)}\right) + 3\cos^{-1}(ax)^2\text{Li}_3$$
$$= -\frac{1}{5}i\cos^{-1}(ax)^5 + \cos^{-1}(ax)^4\log\left(1+e^{2i\cos^{-1}(ax)}\right) - 2i\cos^{-1}(ax)^3\text{Li}_2\left(-e^{2i\cos^{-1}(ax)}\right) + 3\cos^{-1}(ax)^2\text{Li}_3$$
$$= -\frac{1}{5}i\cos^{-1}(ax)^5 + \cos^{-1}(ax)^4\log\left(1+e^{2i\cos^{-1}(ax)}\right) - 2i\cos^{-1}(ax)^3\text{Li}_2\left(-e^{2i\cos^{-1}(ax)}\right) + 3\cos^{-1}(ax)^2\text{Li}_3$$

Mathematica [A] time = 0.0210952, size = 119, normalized size = 1.

$$-2i\cos^{-1}(ax)^3\text{PolyLog}\left(2,-e^{2i\cos^{-1}(ax)}\right) + 3\cos^{-1}(ax)^2\text{PolyLog}\left(3,-e^{2i\cos^{-1}(ax)}\right) + 3i\cos^{-1}(ax)\text{PolyLog}\left(4,-e^{2i\cos^{-1}(ax)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x]^4/x,x]
```

```
[Out] (-I/5)*ArcCos[a*x]^5 + ArcCos[a*x]^4*Log[1 + E^((2*I)*ArcCos[a*x])] - (2*I)
*ArcCos[a*x]^3*PolyLog[2, -E^((2*I)*ArcCos[a*x])] + 3*ArcCos[a*x]^2*PolyLog
[3, -E^((2*I)*ArcCos[a*x])] + (3*I)*ArcCos[a*x]*PolyLog[4, -E^((2*I)*ArcCos
[a*x])] - (3*PolyLog[5, -E^((2*I)*ArcCos[a*x])])/2
```

Maple [A] time = 0.069, size = 168, normalized size = 1.4

$$-\frac{i}{5}(\arccos(ax))^5 + (\arccos(ax))^4\ln\left(1+\left(i\sqrt{-a^2x^2+1}+ax\right)^2\right) - 2i(\arccos(ax))^3\,\text{polylog}\left(2,-\left(i\sqrt{-a^2x^2+1}+ax\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^4/x,x)
```

```
[Out] -1/5*I*arccos(a*x)^5+arccos(a*x)^4*ln(1+(I*(-a^2*x^2+1)^(1/2)+a*x)^2)-2*I*a
rccos(a*x)^3*polylog(2,-(I*(-a^2*x^2+1)^(1/2)+a*x)^2)+3*arccos(a*x)^2*polyl
og(3,-(I*(-a^2*x^2+1)^(1/2)+a*x)^2)+3*I*arccos(a*x)*polylog(4,-(I*(-a^2*x^2
+1)^(1/2)+a*x)^2)-3/2*polylog(5,-(I*(-a^2*x^2+1)^(1/2)+a*x)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(ax)^4}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^4/x,x, algorithm="maxima")
```

```
[Out] integrate(arccos(a*x)^4/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arccos(ax)^4}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^4/x,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)^4/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acos}^4(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**4/x,x)
```

```
[Out] Integral(acos(a*x)**4/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(ax)^4}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^4/x,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^4/x, x)
```

3.39 $\int \frac{\cos^{-1}(ax)^4}{x^2}\,dx$

Optimal. Leaf size=176

$$12ia\cos^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{i\cos^{-1}(ax)}\right)-12ia\cos^{-1}(ax)^2\text{PolyLog}\left(2,ie^{i\cos^{-1}(ax)}\right)-24a\cos^{-1}(ax)\text{PolyLog}\left(3,-ie^{i\cos}\right.$$

```
[Out] -(ArcCos[a*x]^4/x) - (8*I)*a*ArcCos[a*x]^3*ArcTan[E^(I*ArcCos[a*x])] + (12*
I)*a*ArcCos[a*x]^2*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] - (12*I)*a*ArcCos[a*x
]^2*PolyLog[2, I*E^(I*ArcCos[a*x])] - 24*a*ArcCos[a*x]*PolyLog[3, (-I)*E^(I
*ArcCos[a*x])] + 24*a*ArcCos[a*x]*PolyLog[3, I*E^(I*ArcCos[a*x])] - (24*I)*
a*PolyLog[4, (-I)*E^(I*ArcCos[a*x])] + (24*I)*a*PolyLog[4, I*E^(I*ArcCos[a*
x])]
```

Rubi [A] time = 0.204032, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {4628, 4710, 4181, 2531, 6609, 2282, 6589}

$$12ia\cos^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{i\cos^{-1}(ax)}\right)-12ia\cos^{-1}(ax)^2\text{PolyLog}\left(2,ie^{i\cos^{-1}(ax)}\right)-24a\cos^{-1}(ax)\text{PolyLog}\left(3,-ie^{i\cos}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^4/x^2,x]
```

```
[Out] -(ArcCos[a*x]^4/x) - (8*I)*a*ArcCos[a*x]^3*ArcTan[E^(I*ArcCos[a*x])] + (12*
I)*a*ArcCos[a*x]^2*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] - (12*I)*a*ArcCos[a*x
]^2*PolyLog[2, I*E^(I*ArcCos[a*x])] - 24*a*ArcCos[a*x]*PolyLog[3, (-I)*E^(I
*ArcCos[a*x])] + 24*a*ArcCos[a*x]*PolyLog[3, I*E^(I*ArcCos[a*x])] - (24*I)*
a*PolyLog[4, (-I)*E^(I*ArcCos[a*x])] + (24*I)*a*PolyLog[4, I*E^(I*ArcCos[a*
x])]
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4710

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> -Dist[(c^(m + 1)*Sqrt[d])^(-1), Subst[Int[(a + b*x)^n
*Cos[x]^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}(ax)^4}{x^2}\,dx &= -\frac{\cos^{-1}(ax)^4}{x} - (4a)\int \frac{\cos^{-1}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\cos^{-1}(ax)^4}{x} + (4a)\,\mathrm{Subst}\left(\int x^3\sec(x)\,dx, x, \cos^{-1}(ax)\right) \\
&= -\frac{\cos^{-1}(ax)^4}{x} - 8ia\cos^{-1}(ax)^3\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right) - (12a)\,\mathrm{Subst}\left(\int x^2\log\left(1-ie^{ix}\right)\,dx, x, \cos^{-1}(ax)\right) \\
&= -\frac{\cos^{-1}(ax)^4}{x} - 8ia\cos^{-1}(ax)^3\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right) + 12ia\cos^{-1}(ax)^2\mathrm{Li}_2\left(-ie^{i\cos^{-1}(ax)}\right) - 12ia\cos^{-1}(ax \\
&= -\frac{\cos^{-1}(ax)^4}{x} - 8ia\cos^{-1}(ax)^3\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right) + 12ia\cos^{-1}(ax)^2\mathrm{Li}_2\left(-ie^{i\cos^{-1}(ax)}\right) - 12ia\cos^{-1}(ax \\
&= -\frac{\cos^{-1}(ax)^4}{x} - 8ia\cos^{-1}(ax)^3\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right) + 12ia\cos^{-1}(ax)^2\mathrm{Li}_2\left(-ie^{i\cos^{-1}(ax)}\right) - 12ia\cos^{-1}(ax \\
&= -\frac{\cos^{-1}(ax)^4}{x} - 8ia\cos^{-1}(ax)^3\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right) + 12ia\cos^{-1}(ax)^2\mathrm{Li}_2\left(-ie^{i\cos^{-1}(ax)}\right) - 12ia\cos^{-1}(ax
\end{aligned}$$

Mathematica [B] time = 1.08086, size = 549, normalized size = 3.12

$$a\Big(12i\cos^{-1}(ax)^2\mathrm{PolyLog}\left(2,-ie^{-i\cos^{-1}(ax)}\right) + 12i\cos^{-1}(ax)^2\mathrm{PolyLog}\left(2,-ie^{i\cos^{-1}(ax)}\right) - 12i\pi\cos^{-1}(ax)\mathrm{PolyLog}\left(2,-ie\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCos[a*x]^4/x^2,x]
```

```
[Out] a*(((-7*I)/16)*Pi^4 - (I/2)*Pi^3*ArcCos[a*x] + ((3*I)/2)*Pi^2*ArcCos[a*x]^2
 - (2*I)*Pi*ArcCos[a*x]^3 + I*ArcCos[a*x]^4 - ArcCos[a*x]^4/(a*x) + 3*Pi^2*
ArcCos[a*x]*Log[1 - I/E^(I*ArcCos[a*x])] - 6*Pi*ArcCos[a*x]^2*Log[1 - I/E^(
I*ArcCos[a*x])] - (Pi^3*Log[1 + I/E^(I*ArcCos[a*x])])/2 + 4*ArcCos[a*x]^3*L
og[1 + I/E^(I*ArcCos[a*x])] + (Pi^3*Log[1 + I*E^(I*ArcCos[a*x])])/2 - 3*Pi^
2*ArcCos[a*x]*Log[1 + I*E^(I*ArcCos[a*x])] + 6*Pi*ArcCos[a*x]^2*Log[1 + I*E
^(I*ArcCos[a*x])] - 4*ArcCos[a*x]^3*Log[1 + I*E^(I*ArcCos[a*x])] + (Pi^3*Lo
g[Tan[(Pi + 2*ArcCos[a*x])/4]])/2 + (12*I)*ArcCos[a*x]^2*PolyLog[2, (-I)/E^
(I*ArcCos[a*x])] + (3*I)*Pi*(Pi - 4*ArcCos[a*x])*PolyLog[2, I/E^(I*ArcCos[a
*x])] + (3*I)*Pi^2*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] - (12*I)*Pi*ArcCos[a*
x]*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] + (12*I)*ArcCos[a*x]^2*PolyLog[2, (-I
)*E^(I*ArcCos[a*x])] + 24*ArcCos[a*x]*PolyLog[3, (-I)/E^(I*ArcCos[a*x])] -
12*Pi*PolyLog[3, I/E^(I*ArcCos[a*x])] + 12*Pi*PolyLog[3, (-I)*E^(I*ArcCos[a
*x])] - 24*ArcCos[a*x]*PolyLog[3, (-I)*E^(I*ArcCos[a*x])] - (24*I)*PolyLog[
4, (-I)/E^(I*ArcCos[a*x])] - (24*I)*PolyLog[4, (-I)*E^(I*ArcCos[a*x])])
```

Maple [F] time = 0.206, size = 0, normalized size = 0.

$$\int \frac{(\arccos (ax))^4}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^4/x^2,x)
```

```
[Out] int(arccos(a*x)^4/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},\,ax\right)^4 - 4\,ax\int \frac{\sqrt{-ax+1}\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^3}{\sqrt{ax+1}(ax-1)x}\,dx}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^4/x^2,x, algorithm="maxima")
```

```
[Out] -(arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^4 - 4*a*x*integrate(sqrt(a*x +
 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3/(a^2*x^3 -
x), x))/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arccos (ax)^4}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^4/x^2,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)^4/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acos}^4(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**4/x**2,x)
```

```
[Out] Integral(acos(a*x)**4/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(ax)^4}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^4/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^4/x^2, x)
```

3.40 $\int \frac{\cos^{-1}(ax)^4}{x^3}\,dx$

Optimal. Leaf size=121

$$-6ia^2\cos^{-1}(ax)\text{PolyLog}\left(2,-e^{2i\cos^{-1}(ax)}\right)+3a^2\text{PolyLog}\left(3,-e^{2i\cos^{-1}(ax)}\right)+\frac{2a\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{x}-2ia^2\cos^{-1}(ax)^3+$$

```
[Out] (-2*I)*a^2*ArcCos[a*x]^3 + (2*a*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/x - ArcCos
[a*x]^4/(2*x^2) + 6*a^2*ArcCos[a*x]^2*Log[1 + E^((2*I)*ArcCos[a*x])] - (6*I
)*a^2*ArcCos[a*x]*PolyLog[2, -E^((2*I)*ArcCos[a*x])] + 3*a^2*PolyLog[3, -E^
((2*I)*ArcCos[a*x])]
```

Rubi [A] time = 0.213345, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {4628, 4682, 4626, 3719, 2190, 2531, 2282, 6589}

$$-6ia^2\cos^{-1}(ax)\text{PolyLog}\left(2,-e^{2i\cos^{-1}(ax)}\right)+3a^2\text{PolyLog}\left(3,-e^{2i\cos^{-1}(ax)}\right)+\frac{2a\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{x}-2ia^2\cos^{-1}(ax)^3+$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^4/x^3,x]
```

```
[Out] (-2*I)*a^2*ArcCos[a*x]^3 + (2*a*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/x - ArcCos
[a*x]^4/(2*x^2) + 6*a^2*ArcCos[a*x]^2*Log[1 + E^((2*I)*ArcCos[a*x])] - (6*I
)*a^2*ArcCos[a*x]*PolyLog[2, -E^((2*I)*ArcCos[a*x])] + 3*a^2*PolyLog[3, -E^
((2*I)*ArcCos[a*x])]
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4682

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b
*ArcCos[c*x])^n)/(d*f*(m + 1)), x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^F
racPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 - c
```

```
^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &
& NeQ[m, -1]
```

Rule 4626

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n/Cot[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cos^{-1}(ax)^4}{x^3}\,dx = -\frac{\cos^{-1}(ax)^4}{2x^2} - (2a)\int \frac{\cos^{-1}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{2a\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{x} - \frac{\cos^{-1}(ax)^4}{2x^2} + \left(6a^2\right)\int \frac{\cos^{-1}(ax)^2}{x}\,dx$$
$$= \frac{2a\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{x} - \frac{\cos^{-1}(ax)^4}{2x^2} - \left(6a^2\right)\text{Subst}\left(\int x^2\tan(x)\,dx, x, \cos^{-1}(ax)\right)$$
$$= -2ia^2\cos^{-1}(ax)^3 + \frac{2a\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{x} - \frac{\cos^{-1}(ax)^4}{2x^2} + \left(12ia^2\right)\text{Subst}\left(\int \frac{e^{2ix}x^2}{1+e^{2ix}}\,dx, x, \cos^{-1}(a\right.$$
$$= -2ia^2\cos^{-1}(ax)^3 + \frac{2a\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{x} - \frac{\cos^{-1}(ax)^4}{2x^2} + 6a^2\cos^{-1}(ax)^2\log\left(1+e^{2i\cos^{-1}(ax)}\right) - (12$$
$$= -2ia^2\cos^{-1}(ax)^3 + \frac{2a\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{x} - \frac{\cos^{-1}(ax)^4}{2x^2} + 6a^2\cos^{-1}(ax)^2\log\left(1+e^{2i\cos^{-1}(ax)}\right) - 6ia$$
$$= -2ia^2\cos^{-1}(ax)^3 + \frac{2a\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{x} - \frac{\cos^{-1}(ax)^4}{2x^2} + 6a^2\cos^{-1}(ax)^2\log\left(1+e^{2i\cos^{-1}(ax)}\right) - 6ia$$
$$= -2ia^2\cos^{-1}(ax)^3 + \frac{2a\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{x} - \frac{\cos^{-1}(ax)^4}{2x^2} + 6a^2\cos^{-1}(ax)^2\log\left(1+e^{2i\cos^{-1}(ax)}\right) - 6ia$$

Mathematica [A] time = 0.35785, size = 115, normalized size = 0.95

$$-\frac{\cos^{-1}(ax)^4}{2x^2} - a^2\left(6i\cos^{-1}(ax)\text{PolyLog}\left(2, -e^{2i\cos^{-1}(ax)}\right) - 3\text{PolyLog}\left(3, -e^{2i\cos^{-1}(ax)}\right) - 2\cos^{-1}(ax)^2\left(\frac{\sqrt{1-a^2x^2}\cos^{-1}(a}{ax}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x]^4/x^3,x]
```

```
[Out] -ArcCos[a*x]^4/(2*x^2) - a^2*(-2*ArcCos[a*x]^2*((-I)*ArcCos[a*x] + (Sqrt[1
- a^2*x^2]*ArcCos[a*x])/(a*x) + 3*Log[1 + E^((2*I)*ArcCos[a*x])]) + (6*I)*A
rcCos[a*x]*PolyLog[2, -E^((2*I)*ArcCos[a*x])] - 3*PolyLog[3, -E^((2*I)*ArcC
os[a*x])])
```

Maple [A] time = 0.124, size = 149, normalized size = 1.2

$$-2\,ia^{2}\left(\arccos\left(ax\right)\right)^{3}-\frac{\left(\arccos\left(ax\right)\right)^{4}}{2\,x^{2}}+6\,a^{2}\left(\arccos\left(ax\right)\right)^{2}\ln\left(1+\left(i\sqrt{-a^{2}x^{2}+1}+ax\right)^{2}\right)-6\,ia^{2}\arccos\left(ax\right)\mathrm{polylog}\left(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^4/x^3,x)
```

```
[Out] -2*I*a^2*arccos(a*x)^3-1/2*arccos(a*x)^4/x^2+6*a^2*arccos(a*x)^2*ln(1+(I*(-
a^2*x^2+1)^(1/2)+a*x)^2)-6*I*a^2*arccos(a*x)*polylog(2,-(I*(-a^2*x^2+1)^(1/
2)+a*x)^2)+3*a^2*polylog(3,-(I*(-a^2*x^2+1)^(1/2)+a*x)^2)+2*a*arccos(a*x)^3
*(-a^2*x^2+1)^(1/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^{4}-\frac{1}{2}\left(\sqrt{ax+1}\sqrt{-ax+1}\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^{3}+8\,x\int\frac{7\,\sqrt{ax+1}\sqrt{-ax+1}\arctan(\sqrt{}}{}\right.}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^4/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^4 - 4*a*x^2*integrate(sqrt
(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3/(a^2*
x^4 - x^2), x))/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arccos\left(ax\right)^{4}}{x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^4/x^3,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)^4/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{acos}^4\left(ax\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**4/x**3,x)
```

```
[Out] Integral(acos(a*x)**4/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos\left(ax\right)^4}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^4/x^3,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^4/x^3, x)
```

3.41 $\int \frac{\cos^{-1}(ax)^4}{x^4}\,dx$

Optimal. Leaf size=304

$$2ia^3\cos^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{i\cos^{-1}(ax)}\right)-2ia^3\cos^{-1}(ax)^2\text{PolyLog}\left(2,ie^{i\cos^{-1}(ax)}\right)-4a^3\cos^{-1}(ax)\text{PolyLog}\left(3,-ie^{i\cos^{-1}}\right.$$

```
[Out] (-2*a^2*ArcCos[a*x]^2)/x + (2*a*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/(3*x^2) -
ArcCos[a*x]^4/(3*x^3) - (8*I)*a^3*ArcCos[a*x]*ArcTan[E^(I*ArcCos[a*x])] - (
(4*I)/3)*a^3*ArcCos[a*x]^3*ArcTan[E^(I*ArcCos[a*x])] + (4*I)*a^3*PolyLog[2,
 (-I)*E^(I*ArcCos[a*x])] + (2*I)*a^3*ArcCos[a*x]^2*PolyLog[2, (-I)*E^(I*Arc
Cos[a*x])] - (4*I)*a^3*PolyLog[2, I*E^(I*ArcCos[a*x])] - (2*I)*a^3*ArcCos[a
*x]^2*PolyLog[2, I*E^(I*ArcCos[a*x])] - 4*a^3*ArcCos[a*x]*PolyLog[3, (-I)*E
^(I*ArcCos[a*x])] + 4*a^3*ArcCos[a*x]*PolyLog[3, I*E^(I*ArcCos[a*x])] - (4*
I)*a^3*PolyLog[4, (-I)*E^(I*ArcCos[a*x])] + (4*I)*a^3*PolyLog[4, I*E^(I*Arc
Cos[a*x])]
```

Rubi [A] time = 0.420771, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {4628, 4702, 4710, 4181, 2531, 6609, 2282, 6589, 2279, 2391}

$$2ia^3\cos^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{i\cos^{-1}(ax)}\right)-2ia^3\cos^{-1}(ax)^2\text{PolyLog}\left(2,ie^{i\cos^{-1}(ax)}\right)-4a^3\cos^{-1}(ax)\text{PolyLog}\left(3,-ie^{i\cos^{-1}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^4/x^4,x]
```


```
[Out] (-2*a^2*ArcCos[a*x]^2)/x + (2*a*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/(3*x^2) -
ArcCos[a*x]^4/(3*x^3) - (8*I)*a^3*ArcCos[a*x]*ArcTan[E^(I*ArcCos[a*x])] - (
(4*I)/3)*a^3*ArcCos[a*x]^3*ArcTan[E^(I*ArcCos[a*x])] + (4*I)*a^3*PolyLog[2,
 (-I)*E^(I*ArcCos[a*x])] + (2*I)*a^3*ArcCos[a*x]^2*PolyLog[2, (-I)*E^(I*Arc
Cos[a*x])] - (4*I)*a^3*PolyLog[2, I*E^(I*ArcCos[a*x])] - (2*I)*a^3*ArcCos[a
*x]^2*PolyLog[2, I*E^(I*ArcCos[a*x])] - 4*a^3*ArcCos[a*x]*PolyLog[3, (-I)*E
^(I*ArcCos[a*x])] + 4*a^3*ArcCos[a*x]*PolyLog[3, I*E^(I*ArcCos[a*x])] - (4*
I)*a^3*PolyLog[4, (-I)*E^(I*ArcCos[a*x])] + (4*I)*a^3*PolyLog[4, I*E^(I*Arc
Cos[a*x])]
```


Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
```

```
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4702

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b
*ArcCos[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)
), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart
[p]), Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 4710

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> -Dist[(c^(m + 1)*Sqrt[d])^(-1), Subst[Int[(a + b*x)^n
*Cos[x]^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\cos^{-1}(ax)^4}{x^4}\,dx = -\frac{\cos^{-1}(ax)^4}{3x^3} - \frac{1}{3}(4a)\int \frac{\cos^{-1}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx$$

$$= \frac{2a\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{3x^2} - \frac{\cos^{-1}(ax)^4}{3x^3} + \left(2a^2\right)\int \frac{\cos^{-1}(ax)^2}{x^2}\,dx - \frac{1}{3}\left(2a^3\right)\int \frac{\cos^{-1}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{2a^2\cos^{-1}(ax)^2}{x} + \frac{2a\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{3x^2} - \frac{\cos^{-1}(ax)^4}{3x^3} + \frac{1}{3}\left(2a^3\right)\text{Subst}\left(\int x^3\sec(x)\,dx, x, \cos^{-1}(\right.$$

$$= -\frac{2a^2\cos^{-1}(ax)^2}{x} + \frac{2a\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{3x^2} - \frac{\cos^{-1}(ax)^4}{3x^3} - \frac{4}{3}ia^3\cos^{-1}(ax)^3\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right) - \left(2a^3\right.$$

$$= -\frac{2a^2\cos^{-1}(ax)^2}{x} + \frac{2a\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{3x^2} - \frac{\cos^{-1}(ax)^4}{3x^3} - 8ia^3\cos^{-1}(ax)\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right) - \frac{4}{3}ia^3$$

$$= -\frac{2a^2\cos^{-1}(ax)^2}{x} + \frac{2a\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{3x^2} - \frac{\cos^{-1}(ax)^4}{3x^3} - 8ia^3\cos^{-1}(ax)\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right) - \frac{4}{3}ia^3$$

$$= -\frac{2a^2\cos^{-1}(ax)^2}{x} + \frac{2a\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{3x^2} - \frac{\cos^{-1}(ax)^4}{3x^3} - 8ia^3\cos^{-1}(ax)\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right) - \frac{4}{3}ia^3$$

$$= -\frac{2a^2\cos^{-1}(ax)^2}{x} + \frac{2a\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}{3x^2} - \frac{\cos^{-1}(ax)^4}{3x^3} - 8ia^3\cos^{-1}(ax)\tan^{-1}\left(e^{i\cos^{-1}(ax)}\right) - \frac{4}{3}ia^3$$

Mathematica [B] time = 12.067, size = 1475, normalized size = 4.85

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCos[a*x]^4/x^4,x]
```

```
[Out] a^3*(-(ArcCos[a*x]^2*(12 + ArcCos[a*x]^2))/6 + 4*(ArcCos[a*x]*(Log[1 - I*E^
(I*ArcCos[a*x])] - Log[1 + I*E^(I*ArcCos[a*x])]) + I*(PolyLog[2, (-I)*E^(I*
ArcCos[a*x])] - PolyLog[2, I*E^(I*ArcCos[a*x])])) + (2*((Pi^3*Log[Cot[(Pi/2
 - ArcCos[a*x])/2]])/8 + (3*Pi^2*((Pi/2 - ArcCos[a*x])*(Log[1 - E^(I*(Pi/2
- ArcCos[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcCos[a*x]))]) + I*(PolyLog[2, -E^
(I*(Pi/2 - ArcCos[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcCos[a*x]))])))/4 - (
3*Pi*((Pi/2 - ArcCos[a*x])^2*(Log[1 - E^(I*(Pi/2 - ArcCos[a*x]))] - Log[1 +
 E^(I*(Pi/2 - ArcCos[a*x]))]) + (2*I)*(Pi/2 - ArcCos[a*x])*(PolyLog[2, -E^(
I*(Pi/2 - ArcCos[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcCos[a*x]))]) + 2*(-Po
lyLog[3, -E^(I*(Pi/2 - ArcCos[a*x]))] + PolyLog[3, E^(I*(Pi/2 - ArcCos[a*x]
))])))/2 + 8*((I/64)*(Pi/2 - ArcCos[a*x])^4 + (I/4)*(Pi/2 + (-Pi/2 + ArcCos
[a*x])/2)^4 - ((Pi/2 - ArcCos[a*x])^3*Log[1 + E^(I*(Pi/2 - ArcCos[a*x]))])/
8 - (Pi^3*(I*(Pi/2 + (-Pi/2 + ArcCos[a*x])/2) - Log[1 + E^((2*I)*(Pi/2 + (-
Pi/2 + ArcCos[a*x])/2))]))/8 - (Pi/2 + (-Pi/2 + ArcCos[a*x])/2)^3*Log[1 + E
```

```
^((2*I)*(Pi/2 + (-Pi/2 + ArcCos[a*x])/2))] + ((3*I)/8)*(Pi/2 - ArcCos[a*x])
^2*PolyLog[2, -E^(I*(Pi/2 - ArcCos[a*x]))] + (3*Pi^2*((I/2)*(Pi/2 + (-Pi/2
+ ArcCos[a*x])/2)^2 - (Pi/2 + (-Pi/2 + ArcCos[a*x])/2)*Log[1 + E^((2*I)*(Pi
/2 + (-Pi/2 + ArcCos[a*x])/2))] + (I/2)*PolyLog[2, -E^((2*I)*(Pi/2 + (-Pi/2
 + ArcCos[a*x])/2))]))/4 + ((3*I)/2)*(Pi/2 + (-Pi/2 + ArcCos[a*x])/2)^2*Pol
yLog[2, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcCos[a*x])/2))] - (3*(Pi/2 - ArcCos[a*
x])*PolyLog[3, -E^(I*(Pi/2 - ArcCos[a*x]))])/4 - (3*Pi*((I/3)*(Pi/2 + (-Pi/
2 + ArcCos[a*x])/2)^3 - (Pi/2 + (-Pi/2 + ArcCos[a*x])/2)^2*Log[1 + E^((2*I)
*(Pi/2 + (-Pi/2 + ArcCos[a*x])/2))] + I*(Pi/2 + (-Pi/2 + ArcCos[a*x])/2)*Po
lyLog[2, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcCos[a*x])/2))] - PolyLog[3, -E^((2*I
)*(Pi/2 + (-Pi/2 + ArcCos[a*x])/2))]/2))/2 - (3*(Pi/2 + (-Pi/2 + ArcCos[a*x
])/2)*PolyLog[3, -E^((2*I)*(Pi/2 + (-Pi/2 + ArcCos[a*x])/2))])/2 - ((3*I)/4
)*PolyLog[4, -E^(I*(Pi/2 - ArcCos[a*x]))] - ((3*I)/4)*PolyLog[4, -E^((2*I)*
(Pi/2 + (-Pi/2 + ArcCos[a*x])/2))])))/3 - (-4*ArcCos[a*x]^3 + ArcCos[a*x]^4
)/(12*(Cos[ArcCos[a*x]/2] - Sin[ArcCos[a*x]/2])^2) - (ArcCos[a*x]^4*Sin[Arc
Cos[a*x]/2])/(6*(Cos[ArcCos[a*x]/2] - Sin[ArcCos[a*x]/2])^3) + (ArcCos[a*x]
^4*Sin[ArcCos[a*x]/2])/(6*(Cos[ArcCos[a*x]/2] + Sin[ArcCos[a*x]/2])^3) - (4
*ArcCos[a*x]^3 + ArcCos[a*x]^4)/(12*(Cos[ArcCos[a*x]/2] + Sin[ArcCos[a*x]/2
])^2) - (-12*ArcCos[a*x]^2*Sin[ArcCos[a*x]/2] - ArcCos[a*x]^4*Sin[ArcCos[a*
x]/2])/(6*(Cos[ArcCos[a*x]/2] + Sin[ArcCos[a*x]/2])) - (12*ArcCos[a*x]^2*Si
n[ArcCos[a*x]/2] + ArcCos[a*x]^4*Sin[ArcCos[a*x]/2])/(6*(Cos[ArcCos[a*x]/2]
 - Sin[ArcCos[a*x]/2])))
```

Maple [A] time = 0.184, size = 451, normalized size = 1.5

$$\frac{2\,a\,(\arccos(ax))^3}{3\,x^2}\sqrt{-a^2x^2+1}-2\,\frac{a^2\,(\arccos(ax))^2}{x}-\frac{(\arccos(ax))^4}{3\,x^3}-\frac{2\,a^3\,(\arccos(ax))^3}{3}\ln\left(1+i\left(i\sqrt{-a^2x^2+1}+ax\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^4/x^4,x)
```

```
[Out] 2/3*a*arccos(a*x)^3*(-a^2*x^2+1)^(1/2)/x^2-2*a^2*arccos(a*x)^2/x-1/3*arccos
(a*x)^4/x^3-2/3*a^3*arccos(a*x)^3*ln(1+I*(I*(-a^2*x^2+1)^(1/2)+a*x))+2*I*a^
3*arccos(a*x)^2*polylog(2,-I*(I*(-a^2*x^2+1)^(1/2)+a*x))-4*a^3*arccos(a*x)*
polylog(3,-I*(I*(-a^2*x^2+1)^(1/2)+a*x))-4*I*a^3*polylog(4,-I*(I*(-a^2*x^2+
1)^(1/2)+a*x))+2/3*a^3*arccos(a*x)^3*ln(1-I*(I*(-a^2*x^2+1)^(1/2)+a*x))-2*I
*a^3*arccos(a*x)^2*polylog(2,I*(I*(-a^2*x^2+1)^(1/2)+a*x))+4*a^3*arccos(a*x
)*polylog(3,I*(I*(-a^2*x^2+1)^(1/2)+a*x))+4*I*a^3*polylog(4,I*(I*(-a^2*x^2+
1)^(1/2)+a*x))-4*a^3*arccos(a*x)*ln(1+I*(I*(-a^2*x^2+1)^(1/2)+a*x))+4*a^3*a
rccos(a*x)*ln(1-I*(I*(-a^2*x^2+1)^(1/2)+a*x))+4*I*a^3*dilog(1+I*(I*(-a^2*x^
2+1)^(1/2)+a*x))-4*I*a^3*dilog(1-I*(I*(-a^2*x^2+1)^(1/2)+a*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{4\,ax^3\int\frac{\sqrt{ax+1}\sqrt{-ax+1}\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^3}{a^2x^5-x^3}\,dx-\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^4}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^4/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(12*a*x^3*integrate(1/3*sqrt(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x +
 1)*sqrt(-a*x + 1), a*x)^3/(a^2*x^5 - x^3), x) - arctan2(sqrt(a*x + 1)*sqrt
(-a*x + 1), a*x)^4)/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\arccos\left(ax\right)^4}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^4/x^4,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)^4/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{acos}^4\left(ax\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**4/x**4,x)
```

```
[Out] Integral(acos(a*x)**4/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos{(ax)}^4}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^4/x^4,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^4/x^4, x)
```

3.42 $\int \frac{x^6}{\cos^{-1}(ax)}\,dx$

Optimal. Leaf size=55

$$-\frac{5\operatorname{Si}\left(\cos^{-1}(ax)\right)}{64a^7}-\frac{9\operatorname{Si}\left(3\cos^{-1}(ax)\right)}{64a^7}-\frac{5\operatorname{Si}\left(5\cos^{-1}(ax)\right)}{64a^7}-\frac{\operatorname{Si}\left(7\cos^{-1}(ax)\right)}{64a^7}$$

```
[Out] (-5*SinIntegral[ArcCos[a*x]])/(64*a^7) - (9*SinIntegral[3*ArcCos[a*x]])/(64
*a^7) - (5*SinIntegral[5*ArcCos[a*x]])/(64*a^7) - SinIntegral[7*ArcCos[a*x]
]/(64*a^7)
```

Rubi [A] time = 0.0907464, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4636, 4406, 3299}

$$-\frac{5\operatorname{Si}\left(\cos^{-1}(ax)\right)}{64a^7}-\frac{9\operatorname{Si}\left(3\cos^{-1}(ax)\right)}{64a^7}-\frac{5\operatorname{Si}\left(5\cos^{-1}(ax)\right)}{64a^7}-\frac{\operatorname{Si}\left(7\cos^{-1}(ax)\right)}{64a^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/ArcCos[a*x],x]
```

```
[Out] (-5*SinIntegral[ArcCos[a*x]])/(64*a^7) - (9*SinIntegral[3*ArcCos[a*x]])/(64
*a^7) - (5*SinIntegral[5*ArcCos[a*x]])/(64*a^7) - SinIntegral[7*ArcCos[a*x]
]/(64*a^7)
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6}{\cos^{-1}(ax)}\,dx &= -\frac{\text{Subst}\left(\int \frac{\cos^6(x)\sin(x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{a^7}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{5\sin(x)}{64x}+\frac{9\sin(3x)}{64x}+\frac{5\sin(5x)}{64x}+\frac{\sin(7x)}{64x}\right) dx, x, \cos^{-1}(ax)\right)}{a^7}\\
&= -\frac{\text{Subst}\left(\int \frac{\sin(7x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{64a^7}-\frac{5\,\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{64a^7}-\frac{5\,\text{Subst}\left(\int \frac{\sin(5x)}{x}\,dx, x, c\right.}{64a^7}\\
&= -\frac{5\text{Si}\left(\cos^{-1}(ax)\right)}{64a^7}-\frac{9\text{Si}\left(3\cos^{-1}(ax)\right)}{64a^7}-\frac{5\text{Si}\left(5\cos^{-1}(ax)\right)}{64a^7}-\frac{\text{Si}\left(7\cos^{-1}(ax)\right)}{64a^7}
\end{aligned}
$$

Mathematica [A] time = 0.102637, size = 40, normalized size = 0.73

$$
-\frac{5\text{Si}\left(\cos^{-1}(ax)\right)+9\text{Si}\left(3\cos^{-1}(ax)\right)+5\text{Si}\left(5\cos^{-1}(ax)\right)+\text{Si}\left(7\cos^{-1}(ax)\right)}{64a^7}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/ArcCos[a*x],x]
```

```
[Out] -(5*SinIntegral[ArcCos[a*x]] + 9*SinIntegral[3*ArcCos[a*x]] + 5*SinIntegral
[5*ArcCos[a*x]] + SinIntegral[7*ArcCos[a*x]])/(64*a^7)
```

Maple [A] time = 0.058, size = 40, normalized size = 0.7

$$
\frac{1}{a^7}\left(-\frac{9\,\text{Si}\,(3\,\arccos\,(ax))}{64}-\frac{5\,\text{Si}\,(5\,\arccos\,(ax))}{64}-\frac{\text{Si}\,(7\,\arccos\,(ax))}{64}-\frac{5\,\text{Si}\,(\arccos\,(ax))}{64}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/arccos(a*x),x)
```

```
[Out] 1/a^7*(-9/64*Si(3*arccos(a*x))-5/64*Si(5*arccos(a*x))-1/64*Si(7*arccos(a*x)
)-5/64*Si(arccos(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\arccos(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/arccos(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^6/arccos(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6}{\arccos(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/arccos(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^6/arccos(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\operatorname{acos}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/acos(a*x),x)
```

```
[Out] Integral(x**6/acos(a*x), x)
```

Giac [A] time = 1.15947, size = 63, normalized size = 1.15

$$-\frac{\operatorname{Si}\left(7\arccos\left(ax\right)\right)}{64\,a^{7}}-\frac{5\,\operatorname{Si}\left(5\arccos\left(ax\right)\right)}{64\,a^{7}}-\frac{9\,\operatorname{Si}\left(3\arccos\left(ax\right)\right)}{64\,a^{7}}-\frac{5\,\operatorname{Si}\left(\arccos\left(ax\right)\right)}{64\,a^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/arccos(a*x),x, algorithm="giac")
```

```
[Out] -1/64*sin_integral(7*arccos(a*x))/a^7 - 5/64*sin_integral(5*arccos(a*x))/a^
7 - 9/64*sin_integral(3*arccos(a*x))/a^7 - 5/64*sin_integral(arccos(a*x))/a
^7
```

3.43 $\int \frac{x^5}{\cos^{-1}(ax)} dx$

Optimal. Leaf size=43

$$-\frac{5\text{Si}\left(2\cos^{-1}(ax)\right)}{32a^6}-\frac{\text{Si}\left(4\cos^{-1}(ax)\right)}{8a^6}-\frac{\text{Si}\left(6\cos^{-1}(ax)\right)}{32a^6}$$

```
[Out] (-5*SinIntegral[2*ArcCos[a*x]])/(32*a^6) - SinIntegral[4*ArcCos[a*x]]/(8*a^
6) - SinIntegral[6*ArcCos[a*x]]/(32*a^6)
```

Rubi [A] time = 0.0788047, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4636, 4406, 3299}

$$-\frac{5\text{Si}\left(2\cos^{-1}(ax)\right)}{32a^6}-\frac{\text{Si}\left(4\cos^{-1}(ax)\right)}{8a^6}-\frac{\text{Si}\left(6\cos^{-1}(ax)\right)}{32a^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/ArcCos[a*x],x]
```

```
[Out] (-5*SinIntegral[2*ArcCos[a*x]])/(32*a^6) - SinIntegral[4*ArcCos[a*x]]/(8*a^
6) - SinIntegral[6*ArcCos[a*x]]/(32*a^6)
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{x^5}{\cos^{-1}(ax)}\,dx = -\frac{\text{Subst}\left(\int \frac{\cos^5(x)\sin(x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{a^6}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{5\sin(2x)}{32x} + \frac{\sin(4x)}{8x} + \frac{\sin(6x)}{32x}\right) dx, x, \cos^{-1}(ax)\right)}{a^6}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sin(6x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{32a^6} - \frac{\text{Subst}\left(\int \frac{\sin(4x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{8a^6} - \frac{5\,\text{Subst}\left(\int \frac{\sin(2x)}{x}\,dx, x, \text{co}\right.}{32a^6}$$
$$= -\frac{5\text{Si}\left(2\cos^{-1}(ax)\right)}{32a^6} - \frac{\text{Si}\left(4\cos^{-1}(ax)\right)}{8a^6} - \frac{\text{Si}\left(6\cos^{-1}(ax)\right)}{32a^6}$$

Mathematica [A] time = 0.087031, size = 33, normalized size = 0.77

$$-\frac{5\text{Si}\left(2\cos^{-1}(ax)\right) + 4\text{Si}\left(4\cos^{-1}(ax)\right) + \text{Si}\left(6\cos^{-1}(ax)\right)}{32a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/ArcCos[a*x],x]
```

```
[Out] -(5*SinIntegral[2*ArcCos[a*x]] + 4*SinIntegral[4*ArcCos[a*x]] + SinIntegral
[6*ArcCos[a*x]])/(32*a^6)
```

Maple [A] time = 0.055, size = 33, normalized size = 0.8

$$\frac{1}{a^6}\left(-\frac{5\,\text{Si}\,(2\,\arccos\,(ax))}{32} - \frac{\text{Si}\,(4\,\arccos\,(ax))}{8} - \frac{\text{Si}\,(6\,\arccos\,(ax))}{32}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/arccos(a*x),x)
```

```
[Out] 1/a^6*(-5/32*Si(2*arccos(a*x))-1/8*Si(4*arccos(a*x))-1/32*Si(6*arccos(a*x))
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\arccos(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arccos(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^5/arccos(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^5}{\arccos(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arccos(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^5/arccos(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\operatorname{acos}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/acos(a*x),x)
```

```
[Out] Integral(x**5/acos(a*x), x)
```

Giac [A] time = 1.15649, size = 50, normalized size = 1.16

$$-\frac{\operatorname{Si}(6\arccos(ax))}{32\,a^6} - \frac{\operatorname{Si}(4\arccos(ax))}{8\,a^6} - \frac{5\operatorname{Si}(2\arccos(ax))}{32\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arccos(a*x),x, algorithm="giac")
```

```
[Out] -1/32*sin_integral(6*arccos(a*x))/a^6 - 1/8*sin_integral(4*arccos(a*x))/a^6
 - 5/32*sin_integral(2*arccos(a*x))/a^6
```

3.44 $\int \frac{x^4}{\cos^{-1}(ax)}\,dx$

Optimal. Leaf size=41

$$-\frac{\text{Si}\left(\cos^{-1}(ax)\right)}{8a^5}-\frac{3\text{Si}\left(3\cos^{-1}(ax)\right)}{16a^5}-\frac{\text{Si}\left(5\cos^{-1}(ax)\right)}{16a^5}$$

```
[Out] -SinIntegral[ArcCos[a*x]]/(8*a^5) - (3*SinIntegral[3*ArcCos[a*x]])/(16*a^5)
 - SinIntegral[5*ArcCos[a*x]]/(16*a^5)
```

Rubi [A] time = 0.0748271, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4636, 4406, 3299}

$$-\frac{\text{Si}\left(\cos^{-1}(ax)\right)}{8a^5}-\frac{3\text{Si}\left(3\cos^{-1}(ax)\right)}{16a^5}-\frac{\text{Si}\left(5\cos^{-1}(ax)\right)}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcCos[a*x],x]
```

```
[Out] -SinIntegral[ArcCos[a*x]]/(8*a^5) - (3*SinIntegral[3*ArcCos[a*x]])/(16*a^5)
 - SinIntegral[5*ArcCos[a*x]]/(16*a^5)
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{x^4}{\cos^{-1}(ax)}\,dx = -\frac{\text{Subst}\left(\int \frac{\cos^4(x)\sin(x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{a^5}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{\sin(x)}{8x} + \frac{3\sin(3x)}{16x} + \frac{\sin(5x)}{16x}\right) dx, x, \cos^{-1}(ax)\right)}{a^5}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sin(5x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{16a^5} - \frac{\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{8a^5} - \frac{3\,\text{Subst}\left(\int \frac{\sin(3x)}{x}\,dx, x, \cos\right.}{16a^5}$$
$$= -\frac{\text{Si}\left(\cos^{-1}(ax)\right)}{8a^5} - \frac{3\text{Si}\left(3\cos^{-1}(ax)\right)}{16a^5} - \frac{\text{Si}\left(5\cos^{-1}(ax)\right)}{16a^5}$$

Mathematica [A] time = 0.0741457, size = 31, normalized size = 0.76

$$-\frac{2\text{Si}\left(\cos^{-1}(ax)\right) + 3\text{Si}\left(3\cos^{-1}(ax)\right) + \text{Si}\left(5\cos^{-1}(ax)\right)}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcCos[a*x],x]
```

```
[Out] -(2*SinIntegral[ArcCos[a*x]] + 3*SinIntegral[3*ArcCos[a*x]] + SinIntegral[5
*ArcCos[a*x]])/(16*a^5)
```

Maple [A] time = 0.047, size = 31, normalized size = 0.8

$$\frac{1}{a^5}\left(-\frac{3\,\text{Si}\,(3\,\arccos(ax))}{16} - \frac{\text{Si}\,(5\,\arccos(ax))}{16} - \frac{\text{Si}\,(\arccos(ax))}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arccos(a*x),x)
```

```
[Out] 1/a^5*(-3/16*Si(3*arccos(a*x))-1/16*Si(5*arccos(a*x))-1/8*Si(arccos(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\arccos(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccos(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4/arccos(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{\arccos(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccos(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^4/arccos(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{acos}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/acos(a*x),x)
```

```
[Out] Integral(x**4/acos(a*x), x)
```

Giac [A] time = 1.12244, size = 47, normalized size = 1.15

$$-\frac{\operatorname{Si}(5\arccos(ax))}{16a^5} - \frac{3\operatorname{Si}(3\arccos(ax))}{16a^5} - \frac{\operatorname{Si}(\arccos(ax))}{8a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccos(a*x),x, algorithm="giac")
```

```
[Out] -1/16*sin_integral(5*arccos(a*x))/a^5 - 3/16*sin_integral(3*arccos(a*x))/a^
5 - 1/8*sin_integral(arccos(a*x))/a^5
```

3.45 $\int \frac{x^3}{\cos^{-1}(ax)} \, dx$

Optimal. Leaf size=29

$$-\frac{\text{Si}\left(2\cos^{-1}(ax)\right)}{4a^4}-\frac{\text{Si}\left(4\cos^{-1}(ax)\right)}{8a^4}$$

```
[Out] -SinIntegral[2*ArcCos[a*x]]/(4*a^4) - SinIntegral[4*ArcCos[a*x]]/(8*a^4)
```

Rubi [A] time = 0.06382, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4636, 4406, 3299}

$$-\frac{\text{Si}\left(2\cos^{-1}(ax)\right)}{4a^4}-\frac{\text{Si}\left(4\cos^{-1}(ax)\right)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcCos[a*x],x]
```

```
[Out] -SinIntegral[2*ArcCos[a*x]]/(4*a^4) - SinIntegral[4*ArcCos[a*x]]/(8*a^4)
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{x^3}{\cos^{-1}(ax)}\,dx = -\frac{\text{Subst}\left(\int \frac{\cos^3(x)\sin(x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{a^4}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{\sin(2x)}{4x} + \frac{\sin(4x)}{8x}\right)dx, x, \cos^{-1}(ax)\right)}{a^4}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sin(4x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{8a^4} - \frac{\text{Subst}\left(\int \frac{\sin(2x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{4a^4}$$
$$= -\frac{\text{Si}\left(2\cos^{-1}(ax)\right)}{4a^4} - \frac{\text{Si}\left(4\cos^{-1}(ax)\right)}{8a^4}$$

Mathematica [A] time = 0.0623818, size = 24, normalized size = 0.83

$$-\frac{2\text{Si}\left(2\cos^{-1}(ax)\right) + \text{Si}\left(4\cos^{-1}(ax)\right)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcCos[a*x],x]
```

```
[Out] -(2*SinIntegral[2*ArcCos[a*x]] + SinIntegral[4*ArcCos[a*x]])/(8*a^4)
```

Maple [A] time = 0.046, size = 24, normalized size = 0.8

$$\frac{1}{a^4}\left(-\frac{\text{Si}\left(2\arccos(ax)\right)}{4} - \frac{\text{Si}\left(4\arccos(ax)\right)}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccos(a*x),x)
```

```
[Out] 1/a^4*(-1/4*Si(2*arccos(a*x))-1/8*Si(4*arccos(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\arccos(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccos(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3/arccos(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3}{\arccos(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccos(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^3/arccos(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{acos}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acos(a*x),x)
```

```
[Out] Integral(x**3/acos(a*x), x)
```

Giac [A] time = 1.17723, size = 34, normalized size = 1.17

$$-\frac{\operatorname{Si}(4\arccos(ax))}{8a^4} - \frac{\operatorname{Si}(2\arccos(ax))}{4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccos(a*x),x, algorithm="giac")
```

```
[Out] -1/8*sin_integral(4*arccos(a*x))/a^4 - 1/4*sin_integral(2*arccos(a*x))/a^4
```

3.46 $\int \frac{x^2}{\cos^{-1}(ax)} dx$

Optimal. Leaf size=27

$$-\frac{\text{Si}\left(\cos^{-1}(ax)\right)}{4a^3}-\frac{\text{Si}\left(3\cos^{-1}(ax)\right)}{4a^3}$$

```
[Out] -SinIntegral[ArcCos[a*x]]/(4*a^3) - SinIntegral[3*ArcCos[a*x]]/(4*a^3)
```

Rubi [A] time = 0.0600118, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4636, 4406, 3299}

$$-\frac{\text{Si}\left(\cos^{-1}(ax)\right)}{4a^3}-\frac{\text{Si}\left(3\cos^{-1}(ax)\right)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcCos[a*x],x]
```

```
[Out] -SinIntegral[ArcCos[a*x]]/(4*a^3) - SinIntegral[3*ArcCos[a*x]]/(4*a^3)
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{x^2}{\cos^{-1}(ax)}\,dx = -\frac{\text{Subst}\left(\int \frac{\cos^2(x)\sin(x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{a^3}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{\sin(x)}{4x}+\frac{\sin(3x)}{4x}\right)dx, x, \cos^{-1}(ax)\right)}{a^3}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{4a^3} - \frac{\text{Subst}\left(\int \frac{\sin(3x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{4a^3}$$
$$= -\frac{\text{Si}\left(\cos^{-1}(ax)\right)}{4a^3} - \frac{\text{Si}\left(3\cos^{-1}(ax)\right)}{4a^3}$$

Mathematica [A] time = 0.0527136, size = 20, normalized size = 0.74

$$-\frac{\text{Si}\left(\cos^{-1}(ax)\right)+\text{Si}\left(3\cos^{-1}(ax)\right)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcCos[a*x],x]
```

```
[Out] -(SinIntegral[ArcCos[a*x]] + SinIntegral[3*ArcCos[a*x]])/(4*a^3)
```

Maple [A] time = 0.045, size = 22, normalized size = 0.8

$$\frac{1}{a^3}\left(-\frac{\text{Si}\left(3\arccos(ax)\right)}{4}-\frac{\text{Si}\left(\arccos(ax)\right)}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccos(a*x),x)
```

```
[Out] 1/a^3*(-1/4*Si(3*arccos(a*x))-1/4*Si(arccos(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\arccos(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccos(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2/arccos(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\arccos(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccos(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^2/arccos(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{acos}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acos(a*x),x)
```

```
[Out] Integral(x**2/acos(a*x), x)
```

Giac [A] time = 1.17207, size = 31, normalized size = 1.15

$$-\frac{\operatorname{Si}(3\arccos(ax))}{4a^3} - \frac{\operatorname{Si}(\arccos(ax))}{4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccos(a*x),x, algorithm="giac")
```

```
[Out] -1/4*sin_integral(3*arccos(a*x))/a^3 - 1/4*sin_integral(arccos(a*x))/a^3
```

3.47 $\int \frac{x}{\cos^{-1}(ax)}\,dx$

Optimal. Leaf size=14

$$-\frac{\text{Si}\left(2\cos^{-1}(ax)\right)}{2a^2}$$

```
[Out] -SinIntegral[2*ArcCos[a*x]]/(2*a^2)
```

Rubi [A] time = 0.0348895, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4636, 4406, 12, 3299}

$$-\frac{\text{Si}\left(2\cos^{-1}(ax)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcCos[a*x],x]
```

```
[Out] -SinIntegral[2*ArcCos[a*x]]/(2*a^2)
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\cos^{-1}(ax)}\,dx &= -\frac{\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{a^2}\\
&= -\frac{\text{Subst}\left(\int \frac{\sin(2x)}{2x}\,dx, x, \cos^{-1}(ax)\right)}{a^2}\\
&= -\frac{\text{Subst}\left(\int \frac{\sin(2x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{2a^2}\\
&= -\frac{\text{Si}\left(2\cos^{-1}(ax)\right)}{2a^2}
\end{aligned}
$$

Mathematica [A] time = 0.0187512, size = 14, normalized size = 1.

$$-\frac{\text{Si}\left(2\cos^{-1}(ax)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcCos[a*x],x]
```

```
[Out] -SinIntegral[2*ArcCos[a*x]]/(2*a^2)
```

Maple [A] time = 0.043, size = 13, normalized size = 0.9

$$-\frac{\text{Si}\left(2\arccos(ax)\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccos(a*x),x)
```

```
[Out] -1/2*Si(2*arccos(a*x))/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\arccos{(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/arccos(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\arccos{(ax)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(a*x),x, algorithm="fricas")
```

```
[Out] integral(x/arccos(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{acos}{(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acos(a*x),x)
```

```
[Out] Integral(x/acos(a*x), x)
```

Giac [A] time = 1.16111, size = 16, normalized size = 1.14

$$-\frac{\operatorname{Si}{(2\arccos{(ax)})}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(a*x),x, algorithm="giac")
```

```
[Out] -1/2*sin_integral(2*arccos(a*x))/a^2
```

3.48 $\int \frac{1}{\cos^{-1}(ax)} dx$

Optimal. Leaf size=10

$$-\frac{\text{Si}\left(\cos^{-1}(ax)\right)}{a}$$

[Out] -(SinIntegral[ArcCos[a*x]]/a)

Rubi [A] time = 0.0153486, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4624, 3299}

$$-\frac{\text{Si}\left(\cos^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a*x]^(-1),x]

[Out] -(SinIntegral[ArcCos[a*x]]/a)

Rule 4624

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Sub
st[Int[x^n*Sin[a/b - x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c,
 n}, x]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^{-1}(ax)} dx &= -\frac{\text{Subst}\left(\int \frac{\sin(x)}{x} dx, x, \cos^{-1}(ax)\right)}{a}\\ &= -\frac{\text{Si}\left(\cos^{-1}(ax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.0235791, size = 10, normalized size = 1.

$$-\frac{\text{Si}\left(\cos^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x]^(-1),x]

[Out] -(SinIntegral[ArcCos[a*x]]/a)
```

Maple [A] time = 0.041, size = 11, normalized size = 1.1

$$-\frac{\text{Si}\left(\arccos\left(ax\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos(a*x),x)

[Out] -Si(arccos(a*x))/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\arccos\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(a*x),x, algorithm="maxima")

[Out] integrate(1/arccos(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\arccos\left(ax\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/arccos(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{acos}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acos(a*x),x)
```

```
[Out] Integral(1/acos(a*x), x)
```

Giac [A] time = 1.11767, size = 14, normalized size = 1.4

$$-\frac{\operatorname{Si}\left(\arccos\left(ax\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(a*x),x, algorithm="giac")
```

```
[Out] -sin_integral(arccos(a*x))/a
```

3.49 $\int \frac{1}{x \cos^{-1}(ax)} \, dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x \cos^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable[1/(x*ArcCos[a*x]), x]
```

Rubi [A] time = 0.0134597, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{x \cos^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcCos[a*x]),x]
```

```
[Out] Defer[Int][1/(x*ArcCos[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x \cos^{-1}(ax)} \, dx = \int \frac{1}{x \cos^{-1}(ax)} \, dx$$

Mathematica [A] time = 0.229524, size = 0, normalized size = 0.

$$\int \frac{1}{x \cos^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcCos[a*x]),x]
```

```
[Out] Integrate[1/(x*ArcCos[a*x]), x]
```

Maple [A] time = 0.145, size = 0, normalized size = 0.

$$\int \frac{1}{x \arccos (ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccos(a*x),x)
```

```
[Out] int(1/x/arccos(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \arccos (ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccos(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*arccos(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x \arccos (ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccos(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/(x*arccos(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{acos}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acos(a*x),x)
```

```
[Out] Integral(1/(x*acos(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \arccos (ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccos(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/(x*arccos(a*x)), x)
```

3.50 $\int \frac{1}{x^2 \cos^{-1}(ax)} \, dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x^2 \cos^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable[1/(x^2*ArcCos[a*x]), x]
```

Rubi [A] time = 0.0141143, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2 \cos^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*ArcCos[a*x]),x]
```

```
[Out] Defer[Int][1/(x^2*ArcCos[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x^2 \cos^{-1}(ax)} \, dx = \int \frac{1}{x^2 \cos^{-1}(ax)} \, dx$$

Mathematica [A] time = 0.854648, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \cos^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*ArcCos[a*x]),x]
```

```
[Out] Integrate[1/(x^2*ArcCos[a*x]), x]
```

Maple [A] time = 0.188, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \arccos(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arccos(a*x),x)
```

```
[Out] int(1/x^2/arccos(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \arccos(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccos(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*arccos(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x^2 \arccos(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccos(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/(x^2*arccos(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{acos}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/acos(a*x),x)
```

```
[Out] Integral(1/(x**2*acos(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \arccos(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccos(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*arccos(a*x)), x)
```

3.51 $\int \frac{x^6}{\cos^{-1}(ax)^2} dx$

Optimal. Leaf size=82

$$-\frac{5\text{CosIntegral}\left(\cos^{-1}(ax)\right)}{64a^7}-\frac{27\text{CosIntegral}\left(3\cos^{-1}(ax)\right)}{64a^7}-\frac{25\text{CosIntegral}\left(5\cos^{-1}(ax)\right)}{64a^7}-\frac{7\text{CosIntegral}\left(7\cos^{-}\right.}{64a^7}$$

```
[Out] (x^6*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) - (5*CosIntegral[ArcCos[a*x]])/(64*
a^7) - (27*CosIntegral[3*ArcCos[a*x]])/(64*a^7) - (25*CosIntegral[5*ArcCos[
a*x]])/(64*a^7) - (7*CosIntegral[7*ArcCos[a*x]])/(64*a^7)
```

Rubi [A] time = 0.0789818, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4632, 3302}

$$-\frac{5\text{CosIntegral}\left(\cos^{-1}(ax)\right)}{64a^7}-\frac{27\text{CosIntegral}\left(3\cos^{-1}(ax)\right)}{64a^7}-\frac{25\text{CosIntegral}\left(5\cos^{-1}(ax)\right)}{64a^7}-\frac{7\text{CosIntegral}\left(7\cos^{-}\right.}{64a^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/ArcCos[a*x]^2,x]
```

```
[Out] (x^6*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) - (5*CosIntegral[ArcCos[a*x]])/(64*
a^7) - (27*CosIntegral[3*ArcCos[a*x]])/(64*a^7) - (25*CosIntegral[5*ArcCos[
a*x]])/(64*a^7) - (7*CosIntegral[7*ArcCos[a*x]])/(64*a^7)
```

Rule 4632

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dis
t[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), Co
s[x]^(m - 1)*(m - (m + 1)*Cos[x]^2), x], x], x, ArcCos[c*x]], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x^6}{\cos^{-1}(ax)^2}\,dx = \frac{x^6\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)} + \frac{\text{Subst}\left(\int\left(-\frac{5\cos(x)}{64x}-\frac{27\cos(3x)}{64x}-\frac{25\cos(5x)}{64x}-\frac{7\cos(7x)}{64x}\right)dx, x, \cos^{-1}(ax)\right)}{a^7}$$

$$= \frac{x^6\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)} - \frac{5\,\text{Subst}\left(\int\frac{\cos(x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{64a^7} - \frac{7\,\text{Subst}\left(\int\frac{\cos(7x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{64a^7} - \frac{25\,\text{Subst}}{}$$

$$= \frac{x^6\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)} - \frac{5\text{Ci}\left(\cos^{-1}(ax)\right)}{64a^7} - \frac{27\text{Ci}\left(3\cos^{-1}(ax)\right)}{64a^7} - \frac{25\text{Ci}\left(5\cos^{-1}(ax)\right)}{64a^7} - \frac{7\text{Ci}\left(7\cos^{-1}(ax)\right)}{64a^7}$$

Mathematica [A] time = 0.157698, size = 86, normalized size = 1.05

$$-\frac{-64a^6x^6\sqrt{1-a^2x^2}+5\cos^{-1}(ax)\text{CosIntegral}\left(\cos^{-1}(ax)\right)+27\cos^{-1}(ax)\text{CosIntegral}\left(3\cos^{-1}(ax)\right)+25\cos^{-1}(ax)\text{Co}}{64a^7\cos^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/ArcCos[a*x]^2,x]
```

```
[Out] -(-64*a^6*x^6*Sqrt[1 - a^2*x^2] + 5*ArcCos[a*x]*CosIntegral[ArcCos[a*x]] +
27*ArcCos[a*x]*CosIntegral[3*ArcCos[a*x]] + 25*ArcCos[a*x]*CosIntegral[5*Ar
cCos[a*x]] + 7*ArcCos[a*x]*CosIntegral[7*ArcCos[a*x]])/(64*a^7*ArcCos[a*x])
```

Maple [A] time = 0.062, size = 105, normalized size = 1.3

$$\frac{1}{a^7}\left(\frac{9\sin(3\arccos(ax))}{64\arccos(ax)} - \frac{27\,\text{Ci}(3\arccos(ax))}{64} + \frac{5\sin(5\arccos(ax))}{64\arccos(ax)} - \frac{25\,\text{Ci}(5\arccos(ax))}{64} + \frac{\sin(7\arccos(ax))}{64\arccos(ax)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/arccos(a*x)^2,x)
```

```
[Out] 1/a^7*(9/64/arccos(a*x)*sin(3*arccos(a*x))-27/64*Ci(3*arccos(a*x))+5/64/arc
cos(a*x)*sin(5*arccos(a*x))-25/64*Ci(5*arccos(a*x))+1/64/arccos(a*x)*sin(7*
arccos(a*x))-7/64*Ci(7*arccos(a*x))+5/64*(-a^2*x^2+1)^(1/2)/arccos(a*x)-5/6
4*Ci(arccos(a*x)))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/arccos(a*x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6}{\arccos(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/arccos(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^6/arccos(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\operatorname{acos}^2(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/acos(a*x)**2,x)
```

```
[Out] Integral(x**6/acos(a*x)**2, x)
```

Giac [A] time = 1.18624, size = 97, normalized size = 1.18

$$\frac{\sqrt{-a^2x^2+1}x^6}{a\arccos(ax)} - \frac{7\,\mathrm{Ci}(7\arccos(ax))}{64\,a^7} - \frac{25\,\mathrm{Ci}(5\arccos(ax))}{64\,a^7} - \frac{27\,\mathrm{Ci}(3\arccos(ax))}{64\,a^7} - \frac{5\,\mathrm{Ci}(\arccos(ax))}{64\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] sqrt(-a^2*x^2 + 1)*x^6/(a*arccos(a*x)) - 7/64*cos_integral(7*arccos(a*x))/a
^7 - 25/64*cos_integral(5*arccos(a*x))/a^7 - 27/64*cos_integral(3*arccos(a*
x))/a^7 - 5/64*cos_integral(arccos(a*x))/a^7
```

3.52 $\int \frac{x^5}{\cos^{-1}(ax)^2}\,dx$

Optimal. Leaf size=70

$$-\frac{5\text{CosIntegral}\left(2\cos^{-1}(ax)\right)}{16a^6}-\frac{\text{CosIntegral}\left(4\cos^{-1}(ax)\right)}{2a^6}-\frac{3\text{CosIntegral}\left(6\cos^{-1}(ax)\right)}{16a^6}+\frac{x^5\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)}$$

```
[Out] (x^5*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) - (5*CosIntegral[2*ArcCos[a*x]])/(1
6*a^6) - CosIntegral[4*ArcCos[a*x]]/(2*a^6) - (3*CosIntegral[6*ArcCos[a*x]]
)/(16*a^6)
```

Rubi [A] time = 0.0640758, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4632, 3302}

$$-\frac{5\text{CosIntegral}\left(2\cos^{-1}(ax)\right)}{16a^6}-\frac{\text{CosIntegral}\left(4\cos^{-1}(ax)\right)}{2a^6}-\frac{3\text{CosIntegral}\left(6\cos^{-1}(ax)\right)}{16a^6}+\frac{x^5\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/ArcCos[a*x]^2,x]
```

```
[Out] (x^5*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) - (5*CosIntegral[2*ArcCos[a*x]])/(1
6*a^6) - CosIntegral[4*ArcCos[a*x]]/(2*a^6) - (3*CosIntegral[6*ArcCos[a*x]]
)/(16*a^6)
```

Rule 4632

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dis
t[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), Co
s[x]^(m - 1)*(m - (m + 1)*Cos[x]^2), x], x], x, ArcCos[c*x]], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x^5}{\cos^{-1}(ax)^2}\,dx = \frac{x^5\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)} + \frac{\text{Subst}\left(\int \left(-\frac{5\cos(2x)}{16x} - \frac{\cos(4x)}{2x} - \frac{3\cos(6x)}{16x}\right) dx, x, \cos^{-1}(ax)\right)}{a^6}$$
$$= \frac{x^5\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)} - \frac{3\,\text{Subst}\left(\int \frac{\cos(6x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{16a^6} - \frac{5\,\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{16a^6} - \frac{\text{Subst}\left(\int\right.}{}$$
$$= \frac{x^5\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)} - \frac{5\text{Ci}\left(2\cos^{-1}(ax)\right)}{16a^6} - \frac{\text{Ci}\left(4\cos^{-1}(ax)\right)}{2a^6} - \frac{3\text{Ci}\left(6\cos^{-1}(ax)\right)}{16a^6}$$

Mathematica [A] time = 0.162543, size = 63, normalized size = 0.9

$$-\frac{-\frac{16a^5x^5\sqrt{1-a^2x^2}}{\cos^{-1}(ax)} + 5\text{CosIntegral}\left(2\cos^{-1}(ax)\right) + 8\text{CosIntegral}\left(4\cos^{-1}(ax)\right) + 3\text{CosIntegral}\left(6\cos^{-1}(ax)\right)}{16a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/ArcCos[a*x]^2,x]
```

```
[Out] -((-16*a^5*x^5*Sqrt[1 - a^2*x^2])/ArcCos[a*x] + 5*CosIntegral[2*ArcCos[a*x]
] + 8*CosIntegral[4*ArcCos[a*x]] + 3*CosIntegral[6*ArcCos[a*x]])/(16*a^6)
```

Maple [A] time = 0.056, size = 78, normalized size = 1.1

$$\frac{1}{a^6}\left(\frac{5\sin(2\arccos(ax))}{32\arccos(ax)} - \frac{5\,\text{Ci}(2\arccos(ax))}{16} + \frac{\sin(4\arccos(ax))}{8\arccos(ax)} - \frac{\text{Ci}(4\arccos(ax))}{2} + \frac{\sin(6\arccos(ax))}{32\arccos(ax)} - \frac{3\,\text{C}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/arccos(a*x)^2,x)
```

```
[Out] 1/a^6*(5/32/arccos(a*x)*sin(2*arccos(a*x))-5/16*Ci(2*arccos(a*x))+1/8*sin(4
*arccos(a*x))/arccos(a*x)-1/2*Ci(4*arccos(a*x))+1/32/arccos(a*x)*sin(6*arcc
os(a*x))-3/16*Ci(6*arccos(a*x)))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arccos(a*x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^5}{\arccos(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arccos(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^5/arccos(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\operatorname{acos}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/acos(a*x)**2,x)
```

```
[Out] Integral(x**5/acos(a*x)**2, x)
```

Giac [A] time = 1.16818, size = 84, normalized size = 1.2

$$\frac{\sqrt{-a^2x^2+1}x^5}{a\arccos(ax)} - \frac{3\,\mathrm{Ci}\,(6\,\arccos(ax))}{16\,a^6} - \frac{\mathrm{Ci}\,(4\,\arccos(ax))}{2\,a^6} - \frac{5\,\mathrm{Ci}\,(2\,\arccos(ax))}{16\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] sqrt(-a^2*x^2 + 1)*x^5/(a*arccos(a*x)) - 3/16*cos_integral(6*arccos(a*x))/a
^6 - 1/2*cos_integral(4*arccos(a*x))/a^6 - 5/16*cos_integral(2*arccos(a*x))
/a^6
```

3.53 $\int \frac{x^4}{\cos^{-1}(ax)^2}\,dx$

Optimal. Leaf size=68

$$-\frac{\text{CosIntegral}\left(\cos^{-1}(ax)\right)}{8a^5}-\frac{9\text{CosIntegral}\left(3\cos^{-1}(ax)\right)}{16a^5}-\frac{5\text{CosIntegral}\left(5\cos^{-1}(ax)\right)}{16a^5}+\frac{x^4\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)}$$

```
[Out] (x^4*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) - CosIntegral[ArcCos[a*x]]/(8*a^5)
- (9*CosIntegral[3*ArcCos[a*x]])/(16*a^5) - (5*CosIntegral[5*ArcCos[a*x]])/
(16*a^5)
```

Rubi [A] time = 0.062514, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4632, 3302}

$$-\frac{\text{CosIntegral}\left(\cos^{-1}(ax)\right)}{8a^5}-\frac{9\text{CosIntegral}\left(3\cos^{-1}(ax)\right)}{16a^5}-\frac{5\text{CosIntegral}\left(5\cos^{-1}(ax)\right)}{16a^5}+\frac{x^4\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcCos[a*x]^2,x]
```

```
[Out] (x^4*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) - CosIntegral[ArcCos[a*x]]/(8*a^5)
- (9*CosIntegral[3*ArcCos[a*x]])/(16*a^5) - (5*CosIntegral[5*ArcCos[a*x]])/
(16*a^5)
```

Rule 4632

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dis
t[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), Co
s[x]^(m - 1)*(m - (m + 1)*Cos[x]^2), x], x], x, ArcCos[c*x]], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x^4}{\cos^{-1}(ax)^2}\,dx = \frac{x^4\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)} + \frac{\text{Subst}\left(\int\left(-\frac{\cos(x)}{8x}-\frac{9\cos(3x)}{16x}-\frac{5\cos(5x)}{16x}\right)dx, x, \cos^{-1}(ax)\right)}{a^5}$$
$$= \frac{x^4\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)} - \frac{\text{Subst}\left(\int\frac{\cos(x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{8a^5} - \frac{5\,\text{Subst}\left(\int\frac{\cos(5x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{16a^5} - \frac{9\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{x^4\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)} - \frac{\text{Ci}\left(\cos^{-1}(ax)\right)}{8a^5} - \frac{9\text{Ci}\left(3\cos^{-1}(ax)\right)}{16a^5} - \frac{5\text{Ci}\left(5\cos^{-1}(ax)\right)}{16a^5}$$

Mathematica [A] time = 0.160174, size = 61, normalized size = 0.9

$$-\frac{-\frac{16a^4x^4\sqrt{1-a^2x^2}}{\cos^{-1}(ax)} + 2\text{CosIntegral}\left(\cos^{-1}(ax)\right) + 9\text{CosIntegral}\left(3\cos^{-1}(ax)\right) + 5\text{CosIntegral}\left(5\cos^{-1}(ax)\right)}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcCos[a*x]^2,x]
```

```
[Out] -((-16*a^4*x^4*Sqrt[1 - a^2*x^2])/ArcCos[a*x] + 2*CosIntegral[ArcCos[a*x]]
+ 9*CosIntegral[3*ArcCos[a*x]] + 5*CosIntegral[5*ArcCos[a*x]])/(16*a^5)
```

Maple [A] time = 0.05, size = 81, normalized size = 1.2

$$\frac{1}{a^5}\left(\frac{3\sin(3\arccos(ax))}{16\arccos(ax)} - \frac{9\,\text{Ci}(3\arccos(ax))}{16} + \frac{\sin(5\arccos(ax))}{16\arccos(ax)} - \frac{5\,\text{Ci}(5\arccos(ax))}{16} + \frac{1}{8\arccos(ax)}\sqrt{-a^2x^2+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arccos(a*x)^2,x)
```

```
[Out] 1/a^5*(3/16/arccos(a*x)*sin(3*arccos(a*x))-9/16*Ci(3*arccos(a*x))+1/16/arcc
os(a*x)*sin(5*arccos(a*x))-5/16*Ci(5*arccos(a*x))+1/8*(-a^2*x^2+1)^(1/2)/ar
ccos(a*x)-1/8*Ci(arccos(a*x)))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccos(a*x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{\arccos(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccos(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^4/arccos(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{acos}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/acos(a*x)**2,x)
```

```
[Out] Integral(x**4/acos(a*x)**2, x)
```

Giac [A] time = 1.17492, size = 81, normalized size = 1.19

$$\frac{\sqrt{-a^2x^2+1}x^4}{a\arccos(ax)} - \frac{5\ \text{Ci}(5\ \arccos(ax))}{16\,a^5} - \frac{9\ \text{Ci}(3\ \arccos(ax))}{16\,a^5} - \frac{\text{Ci}(\arccos(ax))}{8\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] sqrt(-a^2*x^2 + 1)*x^4/(a*arccos(a*x)) - 5/16*cos_integral(5*arccos(a*x))/a
^5 - 9/16*cos_integral(3*arccos(a*x))/a^5 - 1/8*cos_integral(arccos(a*x))/a
^5
```

3.54 $\int \frac{x^3}{\cos^{-1}(ax)^2} dx$

Optimal. Leaf size=56

$$-\frac{\text{CosIntegral}\left(2\cos^{-1}(ax)\right)}{2a^4}-\frac{\text{CosIntegral}\left(4\cos^{-1}(ax)\right)}{2a^4}+\frac{x^3\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)}$$

```
[Out] (x^3*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) - CosIntegral[2*ArcCos[a*x]]/(2*a^4
) - CosIntegral[4*ArcCos[a*x]]/(2*a^4)
```

Rubi [A] time = 0.0502264, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4632, 3302}

$$-\frac{\text{CosIntegral}\left(2\cos^{-1}(ax)\right)}{2a^4}-\frac{\text{CosIntegral}\left(4\cos^{-1}(ax)\right)}{2a^4}+\frac{x^3\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcCos[a*x]^2,x]
```


```
[Out] (x^3*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) - CosIntegral[2*ArcCos[a*x]]/(2*a^4
) - CosIntegral[4*ArcCos[a*x]]/(2*a^4)
```


Rule 4632

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dis
t[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), Co
s[x]^(m - 1)*(m - (m + 1)*Cos[x]^2), x], x], x, ArcCos[c*x]], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x^3}{\cos^{-1}(ax)^2}\,dx = \frac{x^3\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)} + \frac{\text{Subst}\left(\int\left(-\frac{\cos(2x)}{2x}-\frac{\cos(4x)}{2x}\right)dx, x, \cos^{-1}(ax)\right)}{a^4}$$
$$= \frac{x^3\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}dx, x, \cos^{-1}(ax)\right)}{2a^4} - \frac{\text{Subst}\left(\int \frac{\cos(4x)}{x}dx, x, \cos^{-1}(ax)\right)}{2a^4}$$
$$= \frac{x^3\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)} - \frac{\text{Ci}\left(2\cos^{-1}(ax)\right)}{2a^4} - \frac{\text{Ci}\left(4\cos^{-1}(ax)\right)}{2a^4}$$

Mathematica [A] time = 0.138831, size = 50, normalized size = 0.89

$$-\frac{-\frac{2a^3x^3\sqrt{1-a^2x^2}}{\cos^{-1}(ax)} + \text{CosIntegral}\left(2\cos^{-1}(ax)\right) + \text{CosIntegral}\left(4\cos^{-1}(ax)\right)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcCos[a*x]^2,x]
```

```
[Out] -((-2*a^3*x^3*Sqrt[1 - a^2*x^2])/ArcCos[a*x] + CosIntegral[2*ArcCos[a*x]] +
 CosIntegral[4*ArcCos[a*x]])/(2*a^4)
```

Maple [A] time = 0.045, size = 54, normalized size = 1.

$$\frac{1}{a^4}\left(\frac{\sin(2\arccos(ax))}{4\arccos(ax)} - \frac{\text{Ci}(2\arccos(ax))}{2} + \frac{\sin(4\arccos(ax))}{8\arccos(ax)} - \frac{\text{Ci}(4\arccos(ax))}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccos(a*x)^2,x)
```

```
[Out] 1/a^4*(1/4/arccos(a*x)*sin(2*arccos(a*x))-1/2*Ci(2*arccos(a*x))+1/8*sin(4*a
rccos(a*x))/arccos(a*x)-1/2*Ci(4*arccos(a*x)))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccos(a*x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3}{\arccos\left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccos(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3/arccos(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{acos}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acos(a*x)**2,x)
```

```
[Out] Integral(x**3/acos(a*x)**2, x)
```

Giac [A] time = 1.18811, size = 68, normalized size = 1.21

$$\frac{\sqrt{-a^2x^2+1}x^3}{a\arccos\left(ax\right)} - \frac{\operatorname{Ci}\left(4\arccos\left(ax\right)\right)}{2\,a^4} - \frac{\operatorname{Ci}\left(2\arccos\left(ax\right)\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] sqrt(-a^2*x^2 + 1)*x^3/(a*arccos(a*x)) - 1/2*cos_integral(4*arccos(a*x))/a^
4 - 1/2*cos_integral(2*arccos(a*x))/a^4
```

3.55 $\int \frac{x^2}{\cos^{-1}(ax)^2}\,dx$

Optimal. Leaf size=54

$$-\frac{\text{CosIntegral}\left(\cos^{-1}(ax)\right)}{4a^3}-\frac{3\text{CosIntegral}\left(3\cos^{-1}(ax)\right)}{4a^3}+\frac{x^2\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)}$$

```
[Out] (x^2*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) - CosIntegral[ArcCos[a*x]]/(4*a^3)
- (3*CosIntegral[3*ArcCos[a*x]])/(4*a^3)
```

Rubi [A] time = 0.0461439, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4632, 3302}

$$-\frac{\text{CosIntegral}\left(\cos^{-1}(ax)\right)}{4a^3}-\frac{3\text{CosIntegral}\left(3\cos^{-1}(ax)\right)}{4a^3}+\frac{x^2\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcCos[a*x]^2,x]
```

```
[Out] (x^2*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) - CosIntegral[ArcCos[a*x]]/(4*a^3)
- (3*CosIntegral[3*ArcCos[a*x]])/(4*a^3)
```

Rule 4632

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dis
t[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), Co
s[x]^(m - 1)*(m - (m + 1)*Cos[x]^2), x], x], x, ArcCos[c*x]], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x^2}{\cos^{-1}(ax)^2}\,dx = \frac{x^2\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)} + \frac{\text{Subst}\left(\int\left(-\frac{\cos(x)}{4x}-\frac{3\cos(3x)}{4x}\right)dx,x,\cos^{-1}(ax)\right)}{a^3}$$
$$= \frac{x^2\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)} - \frac{\text{Subst}\left(\int\frac{\cos(x)}{x}\,dx,x,\cos^{-1}(ax)\right)}{4a^3} - \frac{3\,\text{Subst}\left(\int\frac{\cos(3x)}{x}\,dx,x,\cos^{-1}(ax)\right)}{4a^3}$$
$$= \frac{x^2\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)} - \frac{\text{Ci}\left(\cos^{-1}(ax)\right)}{4a^3} - \frac{3\text{Ci}\left(3\cos^{-1}(ax)\right)}{4a^3}$$

Mathematica [A] time = 0.122994, size = 50, normalized size = 0.93

$$-\frac{-\frac{4a^2x^2\sqrt{1-a^2x^2}}{\cos^{-1}(ax)}+\text{CosIntegral}\left(\cos^{-1}(ax)\right)+3\text{CosIntegral}\left(3\cos^{-1}(ax)\right)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcCos[a*x]^2,x]
```

```
[Out] -((-4*a^2*x^2*Sqrt[1 - a^2*x^2])/ArcCos[a*x] + CosIntegral[ArcCos[a*x]] + 3
*CosIntegral[3*ArcCos[a*x]])/(4*a^3)
```

Maple [A] time = 0.047, size = 57, normalized size = 1.1

$$\frac{1}{a^3}\left(\frac{\sin(3\arccos(ax))}{4\arccos(ax)} - \frac{3\,\text{Ci}(3\arccos(ax))}{4} + \frac{1}{4\arccos(ax)}\sqrt{-a^2x^2+1} - \frac{\text{Ci}(\arccos(ax))}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccos(a*x)^2,x)
```

```
[Out] 1/a^3*(1/4/arccos(a*x)*sin(3*arccos(a*x))-3/4*Ci(3*arccos(a*x))+1/4*(-a^2*x
^2+1)^(1/2)/arccos(a*x)-1/4*Ci(arccos(a*x)))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccos(a*x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\arccos\left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccos(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2/arccos(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{acos}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acos(a*x)**2,x)
```

```
[Out] Integral(x**2/acos(a*x)**2, x)
```

Giac [A] time = 1.17569, size = 65, normalized size = 1.2

$$\frac{\sqrt{-a^2x^2+1}x^2}{a\arccos\left(ax\right)} - \frac{3\ \text{Ci}\left(3\ \arccos\left(ax\right)\right)}{4\,a^3} - \frac{\text{Ci}\left(\arccos\left(ax\right)\right)}{4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] sqrt(-a^2*x^2 + 1)*x^2/(a*arccos(a*x)) - 3/4*cos_integral(3*arccos(a*x))/a^
3 - 1/4*cos_integral(arccos(a*x))/a^3
```

3.56 $\int \frac{x}{\cos^{-1}(ax)^2} \, dx$

Optimal. Leaf size=38

$$\frac{x\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)} - \frac{\text{CosIntegral}\left(2\cos^{-1}(ax)\right)}{a^2}$$

```
[Out] (x*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) - CosIntegral[2*ArcCos[a*x]]/a^2
```

Rubi [A] time = 0.0244747, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4632, 3302}

$$\frac{x\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)} - \frac{\text{CosIntegral}\left(2\cos^{-1}(ax)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcCos[a*x]^2,x]
```

```
[Out] (x*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) - CosIntegral[2*ArcCos[a*x]]/a^2
```

Rule 4632

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dis
t[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), Co
s[x]^(m - 1)*(m - (m + 1)*Cos[x]^2), x], x], x, ArcCos[c*x]], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x}{\cos^{-1}(ax)^2}\,dx = \frac{x\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{a^2}$$
$$= \frac{x\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)} - \frac{\text{Ci}\left(2\cos^{-1}(ax)\right)}{a^2}$$

Mathematica [A] time = 0.0854411, size = 37, normalized size = 0.97

$$\frac{\frac{ax\sqrt{1-a^2x^2}}{\cos^{-1}(ax)} - \text{CosIntegral}\left(2\cos^{-1}(ax)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcCos[a*x]^2,x]
```

```
[Out] ((a*x*Sqrt[1 - a^2*x^2])/ArcCos[a*x] - CosIntegral[2*ArcCos[a*x]])/a^2
```

Maple [A] time = 0.045, size = 30, normalized size = 0.8

$$\frac{1}{a^2}\left(\frac{\sin(2\arccos(ax))}{2\arccos(ax)} - \text{Ci}(2\arccos(ax))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccos(a*x)^2,x)
```

```
[Out] 1/a^2*(1/2/arccos(a*x)*sin(2*arccos(a*x))-Ci(2*arccos(a*x)))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(a*x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\arccos(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x/arccos(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{acos}^2(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acos(a*x)**2,x)
```

```
[Out] Integral(x/acos(a*x)**2, x)
```

Giac [A] time = 1.2191, size = 49, normalized size = 1.29

$$\frac{\sqrt{-a^2x^2+1}x}{a\arccos(ax)} - \frac{\operatorname{Ci}(2\arccos(ax))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] sqrt(-a^2*x^2 + 1)*x/(a*arccos(a*x)) - cos_integral(2*arccos(a*x))/a^2
```

3.57 $\int \frac{1}{\cos^{-1}(ax)^2}\,dx$

Optimal. Leaf size=35

$$\frac{\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)} - \frac{\text{CosIntegral}\left(\cos^{-1}(ax)\right)}{a}$$

```
[Out] Sqrt[1 - a^2*x^2]/(a*ArcCos[a*x]) - CosIntegral[ArcCos[a*x]]/a
```

Rubi [A] time = 0.0811583, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4622, 4724, 3302}

$$\frac{\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)} - \frac{\text{CosIntegral}\left(\cos^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^(-2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(a*ArcCos[a*x]) - CosIntegral[ArcCos[a*x]]/a
```

Rule 4622

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(Sqrt[1 - c
^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1))
, Int[(x*(a + b*ArcCos[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && LtQ[n, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{-1}(ax)^2}\,dx = \frac{\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)} + a\int \frac{x}{\sqrt{1-a^2x^2}\cos^{-1}(ax)}\,dx$$
$$= \frac{\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cos(x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{a}$$
$$= \frac{\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)} - \frac{\text{Ci}\left(\cos^{-1}(ax)\right)}{a}$$

Mathematica [A] time = 0.041016, size = 35, normalized size = 1.

$$\frac{\sqrt{1-a^2x^2}}{a\cos^{-1}(ax)} - \frac{\text{CosIntegral}\left(\cos^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x]^(-2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(a*ArcCos[a*x]) - CosIntegral[ArcCos[a*x]]/a
```

Maple [A] time = 0.046, size = 32, normalized size = 0.9

$$\frac{1}{a}\left(\frac{1}{\arccos(ax)}\sqrt{-a^2x^2+1} - \text{Ci}\left(\arccos(ax)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos(a*x)^2,x)
```

```
[Out] 1/a*((-a^2*x^2+1)^(1/2)/arccos(a*x)-Ci(arccos(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2 \arctan\left(\sqrt{ax+1}\sqrt{-ax+1}, ax\right) \int \frac{\sqrt{-ax+1}x}{\sqrt{ax+1}(ax-1)\arctan\left(\sqrt{ax+1}\sqrt{-ax+1}, ax\right)}\, dx - \sqrt{ax+1}\sqrt{-ax+1}}{a \arctan\left(\sqrt{ax+1}\sqrt{-ax+1}, ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)*integrate(sqrt(a*x + 1)*sq
rt(-a*x + 1)*x/((a^2*x^2 - 1)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)),
x) - sqrt(a*x + 1)*sqrt(-a*x + 1))/(a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1),
 a*x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\arccos\left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)^(-2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{acos}^2\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acos(a*x)**2,x)
```

```
[Out] Integral(acos(a*x)**(-2), x)
```

<u>Giac</u> [A] time = 1.14229, size = 45, normalized size = 1.29

$$-\frac{\mathrm{Ci}\left(\arccos\left(ax\right)\right)}{a}+\frac{\sqrt{-a^{2}x^{2}+1}}{a\arccos\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] -cos_integral(arccos(a*x))/a + sqrt(-a^2*x^2 + 1)/(a*arccos(a*x))
```

3.58 $\int \frac{1}{x\cos^{-1}(ax)^2}\,dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x\cos^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[1/(x*ArcCos[a*x]^2), x]
```

Rubi [A] time = 0.0119504, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\cos^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcCos[a*x]^2),x]
```

```
[Out] Defer[Int][1/(x*ArcCos[a*x]^2), x]
```

Rubi steps

$$\int \frac{1}{x\cos^{-1}(ax)^2}\,dx=\int \frac{1}{x\cos^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 0.960006, size = 0, normalized size = 0.

$$\int \frac{1}{x\cos^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcCos[a*x]^2),x]
```

```
[Out] Integrate[1/(x*ArcCos[a*x]^2), x]
```

Maple [A] time = 0.146, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\arccos\left(ax\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccos(a*x)^2,x)
```

```
[Out] int(1/x/arccos(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{x \arctan\left(\sqrt{ax+1}\sqrt{-ax+1}, ax\right) \int \frac{\sqrt{-ax+1}}{\sqrt{ax+1}ax^{3}\arctan\left(\sqrt{ax+1}\sqrt{-ax+1}, ax\right)-\sqrt{ax+1}x^{2}\arctan\left(\sqrt{ax+1}\sqrt{-ax+1}, ax\right)}\,dx - \sqrt{ax+1}\sqrt{-ax+1}}{ax \arctan\left(\sqrt{ax+1}\sqrt{-ax+1}, ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccos(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a*x*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)*integrate(sqrt(a*x + 1)*sq
rt(-a*x + 1)/((a^3*x^4 - a*x^2)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))
, x) - sqrt(a*x + 1)*sqrt(-a*x + 1))/(a*x*arctan2(sqrt(a*x + 1)*sqrt(-a*x +
 1), a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x \arccos\left(ax\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccos(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(x*arccos(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{acos}^2(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acos(a*x)**2,x)
```

```
[Out] Integral(1/(x*acos(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \arccos(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x*arccos(a*x)^2), x)
```

3.59 $\int \frac{1}{x^2 \cos^{-1}(ax)^2} \, dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x^2 \cos^{-1}(ax)^2}, x\right)$$

```
[Out] Unintegrable[1/(x^2*ArcCos[a*x]^2), x]
```

Rubi [A] time = 0.0134958, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2 \cos^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*ArcCos[a*x]^2),x]
```

```
[Out] Defer[Int][1/(x^2*ArcCos[a*x]^2), x]
```

Rubi steps

$$\int \frac{1}{x^2 \cos^{-1}(ax)^2} \, dx = \int \frac{1}{x^2 \cos^{-1}(ax)^2} \, dx$$

Mathematica [A] time = 16.6432, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \cos^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*ArcCos[a*x]^2),x]
```

```
[Out] Integrate[1/(x^2*ArcCos[a*x]^2), x]
```

Maple [A] time = 0.206, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(\arccos\left(ax\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arccos(a*x)^2,x)
```

```
[Out] int(1/x^2/arccos(a*x)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccos(a*x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x^2 \arccos\left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccos(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(x^2*arccos(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{acos}^2\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/acos(a*x)**2,x)
```

```
[Out] Integral(1/(x**2*acos(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \arccos{(ax)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*arccos(a*x)^2), x)
```

3.60 $\int \frac{x^4}{\cos^{-1}(ax)^3}\,dx$

Optimal. Leaf size=98

$$\frac{\text{Si}\left(\cos^{-1}(ax)\right)}{16a^5}+\frac{27\text{Si}\left(3\cos^{-1}(ax)\right)}{32a^5}+\frac{25\text{Si}\left(5\cos^{-1}(ax)\right)}{32a^5}+\frac{x^4\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2}-\frac{2x^3}{a^2\cos^{-1}(ax)}+\frac{5x^5}{2\cos^{-1}(ax)}$$

```
[Out] (x^4*Sqrt[1 - a^2*x^2])/(2*a*ArcCos[a*x]^2) - (2*x^3)/(a^2*ArcCos[a*x]) + (
5*x^5)/(2*ArcCos[a*x]) + SinIntegral[ArcCos[a*x]]/(16*a^5) + (27*SinIntegra
l[3*ArcCos[a*x]])/(32*a^5) + (25*SinIntegral[5*ArcCos[a*x]])/(32*a^5)
```

Rubi [A] time = 0.344457, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4634, 4720, 4636, 4406, 3299}

$$\frac{\text{Si}\left(\cos^{-1}(ax)\right)}{16a^5}+\frac{27\text{Si}\left(3\cos^{-1}(ax)\right)}{32a^5}+\frac{25\text{Si}\left(5\cos^{-1}(ax)\right)}{32a^5}+\frac{x^4\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2}-\frac{2x^3}{a^2\cos^{-1}(ax)}+\frac{5x^5}{2\cos^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcCos[a*x]^3,x]
```

```
[Out] (x^4*Sqrt[1 - a^2*x^2])/(2*a*ArcCos[a*x]^2) - (2*x^3)/(a^2*ArcCos[a*x]) + (
5*x^5)/(2*ArcCos[a*x]) + SinIntegral[ArcCos[a*x]]/(16*a^5) + (27*SinIntegra
l[3*ArcCos[a*x]])/(32*a^5) + (25*SinIntegral[5*ArcCos[a*x]])/(32*a^5)
```

Rule 4634

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-D
ist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCos[c*x])^(n + 1))/Sq
rt[1 - c^2*x^2], x], x] + Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*ArcCo
s[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[
m, 0] && LtQ[n, -2]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
```

```
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{x^4}{\cos^{-1}(ax)^3}\,dx = \frac{x^4\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2} - \frac{2\int \frac{x^3}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}\,dx}{a} + \frac{1}{2}(5a)\int \frac{x^5}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}\,dx$$

$$= \frac{x^4\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2} - \frac{2x^3}{a^2\cos^{-1}(ax)} + \frac{5x^5}{2\cos^{-1}(ax)} - \frac{25}{2}\int \frac{x^4}{\cos^{-1}(ax)}\,dx + \frac{6\int \frac{x^2}{\cos^{-1}(ax)}\,dx}{a^2}$$

$$= \frac{x^4\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2} - \frac{2x^3}{a^2\cos^{-1}(ax)} + \frac{5x^5}{2\cos^{-1}(ax)} - \frac{6\,\mathrm{Subst}\left(\int \frac{\cos^2(x)\sin(x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{a^5} + \frac{25\,\mathrm{Subst}}{}$$

$$= \frac{x^4\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2} - \frac{2x^3}{a^2\cos^{-1}(ax)} + \frac{5x^5}{2\cos^{-1}(ax)} - \frac{6\,\mathrm{Subst}\left(\int \left(\frac{\sin(x)}{4x} + \frac{\sin(3x)}{4x}\right) dx, x, \cos^{-1}(ax)\right)}{a^5} + \frac{25\,\mathrm{S}}{}$$

$$= \frac{x^4\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2} - \frac{2x^3}{a^2\cos^{-1}(ax)} + \frac{5x^5}{2\cos^{-1}(ax)} + \frac{25\,\mathrm{Subst}\left(\int \frac{\sin(5x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{32a^5} - \frac{3\,\mathrm{Subst}\left(\int \frac{\mathrm{s}}{}\right.}{}$$

$$= \frac{x^4\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2} - \frac{2x^3}{a^2\cos^{-1}(ax)} + \frac{5x^5}{2\cos^{-1}(ax)} + \frac{\mathrm{Si}\left(\cos^{-1}(ax)\right)}{16a^5} + \frac{27\mathrm{Si}\left(3\cos^{-1}(ax)\right)}{32a^5} + \frac{25\mathrm{Si}\left(5\cos^{-1}\right.}{32a^5}$$

Mathematica [A] time = 0.130407, size = 103, normalized size = 1.05

$$\frac{16a^4x^4\sqrt{1-a^2x^2}+80a^5x^5\cos^{-1}(ax)-64a^3x^3\cos^{-1}(ax)+2\cos^{-1}(ax)^2\text{Si}\left(\cos^{-1}(ax)\right)+27\cos^{-1}(ax)^2\text{Si}\left(3\cos^{-1}(ax)\right)+}{32a^5\cos^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcCos[a*x]^3,x]
```

```
[Out] (16*a^4*x^4*Sqrt[1 - a^2*x^2] - 64*a^3*x^3*ArcCos[a*x] + 80*a^5*x^5*ArcCos[
a*x] + 2*ArcCos[a*x]^2*SinIntegral[ArcCos[a*x]] + 27*ArcCos[a*x]^2*SinInteg
ral[3*ArcCos[a*x]] + 25*ArcCos[a*x]^2*SinIntegral[5*ArcCos[a*x]])/(32*a^5*A
rcCos[a*x]^2)
```

Maple [A] time = 0.059, size = 121, normalized size = 1.2

$$\frac{1}{a^5}\left(\frac{3\sin(3\arccos(ax))}{32(\arccos(ax))^2}+\frac{9\cos(3\arccos(ax))}{32\arccos(ax)}+\frac{27\,\text{Si}(3\arccos(ax))}{32}+\frac{\sin(5\arccos(ax))}{32(\arccos(ax))^2}+\frac{5\cos(5\arccos(ax))}{32\arccos(ax)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arccos(a*x)^3,x)
```

```
[Out] 1/a^5*(3/32/arccos(a*x)^2*sin(3*arccos(a*x))+9/32/arccos(a*x)*cos(3*arccos(
a*x))+27/32*Si(3*arccos(a*x))+1/32/arccos(a*x)^2*sin(5*arccos(a*x))+5/32/ar
ccos(a*x)*cos(5*arccos(a*x))+25/32*Si(5*arccos(a*x))+1/16/arccos(a*x)^2*(-a
^2*x^2+1)^(1/2)+1/16*a*x/arccos(a*x)+1/16*Si(arccos(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{ax+1}\sqrt{-ax+1}ax^4-\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^2\int\frac{(25a^2x^2-12)x^2}{\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)}\,dx+\left(5a^2x^5-4x^3\right)\arctan\left(\sqrt{ax+1}\sqrt{-\right.}{2a^2\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccos(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(a*x + 1)*sqrt(-a*x + 1)*a*x^4 - arctan2(sqrt(a*x + 1)*sqrt(-a*x +
 1), a*x)^2*integrate((25*a^2*x^4 - 12*x^2)/arctan2(sqrt(a*x + 1)*sqrt(-a*x
 + 1), a*x), x) + (5*a^2*x^5 - 4*x^3)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1),
 a*x))/(a^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{\arccos(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccos(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^4/arccos(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{acos}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/acos(a*x)**3,x)
```

```
[Out] Integral(x**4/acos(a*x)**3, x)
```

Giac [A] time = 1.23122, size = 116, normalized size = 1.18

$$\frac{5x^5}{2\arccos(ax)} + \frac{\sqrt{-a^2x^2+1}x^4}{2a\arccos(ax)^2} - \frac{2x^3}{a^2\arccos(ax)} + \frac{25\,\text{Si}(5\arccos(ax))}{32a^5} + \frac{27\,\text{Si}(3\arccos(ax))}{32a^5} + \frac{\text{Si}(\arccos(ax))}{16a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccos(a*x)^3,x, algorithm="giac")
```

```
[Out] 5/2*x^5/arccos(a*x) + 1/2*sqrt(-a^2*x^2 + 1)*x^4/(a*arccos(a*x)^2) - 2*x^3/
(a^2*arccos(a*x)) + 25/32*sin_integral(5*arccos(a*x))/a^5 + 27/32*sin_integ
ral(3*arccos(a*x))/a^5 + 1/16*sin_integral(arccos(a*x))/a^5
```

3.61 $\int \frac{x^3}{\cos^{-1}(ax)^3} \, dx$

Optimal. Leaf size=83

$$\frac{\text{Si}\left(2\cos^{-1}(ax)\right)}{2a^4}+\frac{\text{Si}\left(4\cos^{-1}(ax)\right)}{a^4}+\frac{x^3\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2}-\frac{3x^2}{2a^2\cos^{-1}(ax)}+\frac{2x^4}{\cos^{-1}(ax)}$$

```
[Out] (x^3*Sqrt[1 - a^2*x^2])/(2*a*ArcCos[a*x]^2) - (3*x^2)/(2*a^2*ArcCos[a*x]) +
 (2*x^4)/ArcCos[a*x] + SinIntegral[2*ArcCos[a*x]]/(2*a^4) + SinIntegral[4*A
rcCos[a*x]]/a^4
```

Rubi [A] time = 0.304897, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4634, 4720, 4636, 4406, 3299, 12}

$$\frac{\text{Si}\left(2\cos^{-1}(ax)\right)}{2a^4}+\frac{\text{Si}\left(4\cos^{-1}(ax)\right)}{a^4}+\frac{x^3\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2}-\frac{3x^2}{2a^2\cos^{-1}(ax)}+\frac{2x^4}{\cos^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcCos[a*x]^3,x]
```

```
[Out] (x^3*Sqrt[1 - a^2*x^2])/(2*a*ArcCos[a*x]^2) - (3*x^2)/(2*a^2*ArcCos[a*x]) +
 (2*x^4)/ArcCos[a*x] + SinIntegral[2*ArcCos[a*x]]/(2*a^4) + SinIntegral[4*A
rcCos[a*x]]/a^4
```

Rule 4634

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-D
ist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCos[c*x])^(n + 1))/Sq
rt[1 - c^2*x^2], x], x] + Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*ArcCo
s[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[
m, 0] && LtQ[n, -2]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
```

```
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{x^3}{\cos^{-1}(ax)^3}\,dx = \frac{x^3\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2} - \frac{3\int \frac{x^2}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}\,dx}{2a} + (2a)\int \frac{x^4}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}\,dx$$

$$= \frac{x^3\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2} - \frac{3x^2}{2a^2\cos^{-1}(ax)} + \frac{2x^4}{\cos^{-1}(ax)} - 8\int \frac{x^3}{\cos^{-1}(ax)}\,dx + \frac{3\int \frac{x}{\cos^{-1}(ax)}\,dx}{a^2}$$

$$= \frac{x^3\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2} - \frac{3x^2}{2a^2\cos^{-1}(ax)} + \frac{2x^4}{\cos^{-1}(ax)} - \frac{3\,\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{a^4} + \frac{8\,\text{Subst}\Big(}{}$$

$$= \frac{x^3\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2} - \frac{3x^2}{2a^2\cos^{-1}(ax)} + \frac{2x^4}{\cos^{-1}(ax)} - \frac{3\,\text{Subst}\left(\int \frac{\sin(2x)}{2x}\,dx, x, \cos^{-1}(ax)\right)}{a^4} + \frac{8\,\text{Subst}\Big(\int\Big(}{}$$

$$= \frac{x^3\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2} - \frac{3x^2}{2a^2\cos^{-1}(ax)} + \frac{2x^4}{\cos^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sin(4x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{a^4} - \frac{3\,\text{Subst}\Big(\int \frac{\sin(2}{x}}{}$$

$$= \frac{x^3\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2} - \frac{3x^2}{2a^2\cos^{-1}(ax)} + \frac{2x^4}{\cos^{-1}(ax)} + \frac{\text{Si}\left(2\cos^{-1}(ax)\right)}{2a^4} + \frac{\text{Si}\left(4\cos^{-1}(ax)\right)}{a^4}$$

Mathematica [A] time = 0.152084, size = 70, normalized size = 0.84

$$\frac{\frac{a^2x^2\left(ax\sqrt{1-a^2x^2}+\left(4a^2x^2-3\right)\cos^{-1}(ax)\right)}{\cos^{-1}(ax)^2} + \text{Si}\left(2\cos^{-1}(ax)\right) + 2\text{Si}\left(4\cos^{-1}(ax)\right)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcCos[a*x]^3,x]
```

```
[Out] ((a^2*x^2*(a*x*Sqrt[1 - a^2*x^2] + (-3 + 4*a^2*x^2)*ArcCos[a*x]))/ArcCos[a*
x]^2 + SinIntegral[2*ArcCos[a*x]] + 2*SinIntegral[4*ArcCos[a*x]])/(2*a^4)
```

Maple [A] time = 0.055, size = 82, normalized size = 1.

$$\frac{1}{a^4}\left(\frac{\sin(2\arccos(ax))}{8(\arccos(ax))^2} + \frac{\cos(2\arccos(ax))}{4\arccos(ax)} + \frac{\text{Si}(2\arccos(ax))}{2} + \frac{\sin(4\arccos(ax))}{16(\arccos(ax))^2} + \frac{\cos(4\arccos(ax))}{4\arccos(ax)} + \text{Si}(4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccos(a*x)^3,x)
```

```
[Out] 1/a^4*(1/8/arccos(a*x)^2*sin(2*arccos(a*x))+1/4/arccos(a*x)*cos(2*arccos(a*
x))+1/2*Si(2*arccos(a*x))+1/16*sin(4*arccos(a*x))/arccos(a*x)^2+1/4/arccos(
a*x)*cos(4*arccos(a*x))+Si(4*arccos(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{ax+1}\sqrt{-ax+1}ax^3 - 2\arctan\left(\sqrt{ax+1}\sqrt{-ax+1}, ax\right)^2 \int \frac{\left(8a^2x^2-3\right)x}{\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)}\,dx + \left(4a^2x^4 - 3x^2\right)\arctan\left(\sqrt{ax+1}\cdots\right.}{2a^2\arctan\left(\sqrt{ax+1}\sqrt{-ax+1}, ax\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccos(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(a*x + 1)*sqrt(-a*x + 1)*a*x^3 - 2*arctan2(sqrt(a*x + 1)*sqrt(-a*x
 + 1), a*x)^2*integrate((8*a^2*x^3 - 3*x)/arctan2(sqrt(a*x + 1)*sqrt(-a*x +
 1), a*x), x) + (4*a^2*x^4 - 3*x^2)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a
*x))/(a^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3}{\arccos\left(ax\right)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccos(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^3/arccos(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{acos}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acos(a*x)**3,x)
```

```
[Out] Integral(x**3/acos(a*x)**3, x)
```

Giac [A] time = 1.20064, size = 101, normalized size = 1.22

$$\frac{2\,x^4}{\arccos\left(ax\right)} + \frac{\sqrt{-a^2x^2+1}x^3}{2\,a\arccos\left(ax\right)^2} - \frac{3\,x^2}{2\,a^2\arccos\left(ax\right)} + \frac{\operatorname{Si}\left(4\,\arccos\left(ax\right)\right)}{a^4} + \frac{\operatorname{Si}\left(2\,\arccos\left(ax\right)\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccos(a*x)^3,x, algorithm="giac")
```

```
[Out] 2*x^4/arccos(a*x) + 1/2*sqrt(-a^2*x^2 + 1)*x^3/(a*arccos(a*x)^2) - 3/2*x^2/
(a^2*arccos(a*x)) + sin_integral(4*arccos(a*x))/a^4 + 1/2*sin_integral(2*ar
ccos(a*x))/a^4
```

3.62 $\int \frac{x^2}{\cos^{-1}(ax)^3}\,dx$

Optimal. Leaf size=82

$$\frac{\text{Si}\left(\cos^{-1}(ax)\right)}{8a^3}+\frac{9\text{Si}\left(3\cos^{-1}(ax)\right)}{8a^3}+\frac{x^2\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2}-\frac{x}{a^2\cos^{-1}(ax)}+\frac{3x^3}{2\cos^{-1}(ax)}$$

```
[Out] (x^2*Sqrt[1 - a^2*x^2])/(2*a*ArcCos[a*x]^2) - x/(a^2*ArcCos[a*x]) + (3*x^3)
/(2*ArcCos[a*x]) + SinIntegral[ArcCos[a*x]]/(8*a^3) + (9*SinIntegral[3*ArcC
os[a*x]])/(8*a^3)
```

Rubi [A] time = 0.24308, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {4634, 4720, 4636, 4406, 3299, 4624}

$$\frac{\text{Si}\left(\cos^{-1}(ax)\right)}{8a^3}+\frac{9\text{Si}\left(3\cos^{-1}(ax)\right)}{8a^3}+\frac{x^2\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2}-\frac{x}{a^2\cos^{-1}(ax)}+\frac{3x^3}{2\cos^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcCos[a*x]^3,x]
```

```
[Out] (x^2*Sqrt[1 - a^2*x^2])/(2*a*ArcCos[a*x]^2) - x/(a^2*ArcCos[a*x]) + (3*x^3)
/(2*ArcCos[a*x]) + SinIntegral[ArcCos[a*x]]/(8*a^3) + (9*SinIntegral[3*ArcC
os[a*x]])/(8*a^3)
```

Rule 4634

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-D
ist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCos[c*x])^(n + 1))/Sq
rt[1 - c^2*x^2], x], x] + Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*ArcCo
s[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[
m, 0] && LtQ[n, -2]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
```

```
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4624

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Sub
st[Int[x^n*Sin[a/b - x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c,
 n}, x]
```

Rubi steps

$$\int \frac{x^2}{\cos^{-1}(ax)^3}\,dx = \frac{x^2\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2} - \frac{\int \frac{x}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}\,dx}{a} + \frac{1}{2}(3a)\int \frac{x^3}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}\,dx$$

$$= \frac{x^2\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2} - \frac{x}{a^2\cos^{-1}(ax)} + \frac{3x^3}{2\cos^{-1}(ax)} - \frac{9}{2}\int \frac{x^2}{\cos^{-1}(ax)}\,dx + \frac{\int \frac{1}{\cos^{-1}(ax)}\,dx}{a^2}$$

$$= \frac{x^2\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2} - \frac{x}{a^2\cos^{-1}(ax)} + \frac{3x^3}{2\cos^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{a^3} + \frac{9\,\text{Subst}\left(\int \frac{\cos^2(x)}{x}\right.}{}$$

$$= \frac{x^2\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2} - \frac{x}{a^2\cos^{-1}(ax)} + \frac{3x^3}{2\cos^{-1}(ax)} - \frac{\text{Si}\left(\cos^{-1}(ax)\right)}{a^3} + \frac{9\,\text{Subst}\left(\int \left(\frac{\sin(x)}{4x} + \frac{\sin(3x)}{4x}\right)dx, x, \text{c}\right.}{2a^3}$$

$$= \frac{x^2\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2} - \frac{x}{a^2\cos^{-1}(ax)} + \frac{3x^3}{2\cos^{-1}(ax)} - \frac{\text{Si}\left(\cos^{-1}(ax)\right)}{a^3} + \frac{9\,\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{8a^3} +$$

$$= \frac{x^2\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2} - \frac{x}{a^2\cos^{-1}(ax)} + \frac{3x^3}{2\cos^{-1}(ax)} + \frac{\text{Si}\left(\cos^{-1}(ax)\right)}{8a^3} + \frac{9\text{Si}\left(3\cos^{-1}(ax)\right)}{8a^3}$$

Mathematica [A] time = 0.132357, size = 65, normalized size = 0.79

$$\frac{\frac{4ax\left(ax\sqrt{1-a^2x^2}+\left(3a^2x^2-2\right)\cos^{-1}(ax)\right)}{\cos^{-1}(ax)^2} + \text{Si}\left(\cos^{-1}(ax)\right) + 9\text{Si}\left(3\cos^{-1}(ax)\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcCos[a*x]^3,x]
```

```
[Out] ((4*a*x*(a*x*Sqrt[1 - a^2*x^2] + (-2 + 3*a^2*x^2)*ArcCos[a*x]))/ArcCos[a*x]
^2 + SinIntegral[ArcCos[a*x]] + 9*SinIntegral[3*ArcCos[a*x]])/(8*a^3)
```

Maple [A] time = 0.051, size = 82, normalized size = 1.

$$\frac{1}{a^3}\left(\frac{\sin(3\arccos(ax))}{8(\arccos(ax))^2} + \frac{3\cos(3\arccos(ax))}{8\arccos(ax)} + \frac{9\,\text{Si}(3\arccos(ax))}{8} + \frac{1}{8(\arccos(ax))^2}\sqrt{-a^2x^2+1} + \frac{ax}{8\arccos(ax)} +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccos(a*x)^3,x)
```

```
[Out] 1/a^3*(1/8/arccos(a*x)^2*sin(3*arccos(a*x))+3/8/arccos(a*x)*cos(3*arccos(a*
x))+9/8*Si(3*arccos(a*x))+1/8/arccos(a*x)^2*(-a^2*x^2+1)^(1/2)+1/8*a*x/arcc
os(a*x)+1/8*Si(arccos(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{ax+1}\sqrt{-ax+1}ax^2 - \arctan\left(\sqrt{ax+1}\sqrt{-ax+1}, ax\right)^2 \int \frac{9\,a^2x^2-2}{\arctan\left(\sqrt{ax+1}\sqrt{-ax+1}, ax\right)}\,dx + \left(3\,a^2x^3 - 2\,x\right)\arctan\left(\sqrt{ax+1}\sqrt{}\right.}{2\,a^2\arctan\left(\sqrt{ax+1}\sqrt{-ax+1}, ax\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccos(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(a*x + 1)*sqrt(-a*x + 1)*a*x^2 - arctan2(sqrt(a*x + 1)*sqrt(-a*x +
 1), a*x)^2*integrate((9*a^2*x^2 - 2)/arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1),
 a*x), x) + (3*a^2*x^3 - 2*x)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))/(
a^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\arccos\left(ax\right)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccos(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^2/arccos(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{acos}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acos(a*x)**3,x)
```

```
[Out] Integral(x**2/acos(a*x)**3, x)
```

Giac [A] time = 1.21897, size = 97, normalized size = 1.18

$$\frac{3\,x^3}{2\,\arccos\left(ax\right)} + \frac{\sqrt{-a^2x^2+1}x^2}{2\,a\arccos\left(ax\right)^2} - \frac{x}{a^2\arccos\left(ax\right)} + \frac{9\,\mathrm{Si}\left(3\,\arccos\left(ax\right)\right)}{8\,a^3} + \frac{\mathrm{Si}\left(\arccos\left(ax\right)\right)}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccos(a*x)^3,x, algorithm="giac")
```

```
[Out] 3/2*x^3/arccos(a*x) + 1/2*sqrt(-a^2*x^2 + 1)*x^2/(a*arccos(a*x)^2) - x/(a^2
*arccos(a*x)) + 9/8*sin_integral(3*arccos(a*x))/a^3 + 1/8*sin_integral(arcc
os(a*x))/a^3
```

3.63 $\int \frac{x}{\cos^{-1}(ax)^3}\,dx$

Optimal. Leaf size=63

$$\frac{\text{Si}\left(2\cos^{-1}(ax)\right)}{a^2}+\frac{x\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2}-\frac{1}{2a^2\cos^{-1}(ax)}+\frac{x^2}{\cos^{-1}(ax)}$$

```
[Out] (x*Sqrt[1 - a^2*x^2])/(2*a*ArcCos[a*x]^2) - 1/(2*a^2*ArcCos[a*x]) + x^2/Arc
Cos[a*x] + SinIntegral[2*ArcCos[a*x]]/a^2
```

Rubi [A] time = 0.164639, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {4634, 4720, 4636, 4406, 12, 3299, 4642}

$$\frac{\text{Si}\left(2\cos^{-1}(ax)\right)}{a^2}+\frac{x\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2}-\frac{1}{2a^2\cos^{-1}(ax)}+\frac{x^2}{\cos^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcCos[a*x]^3,x]
```

```
[Out] (x*Sqrt[1 - a^2*x^2])/(2*a*ArcCos[a*x]^2) - 1/(2*a^2*ArcCos[a*x]) + x^2/Arc
Cos[a*x] + SinIntegral[2*ArcCos[a*x]]/a^2
```

Rule 4634

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-D
ist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCos[c*x])^(n + 1))/Sq
rt[1 - c^2*x^2], x], x] + Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*ArcCo
s[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[
m, 0] && LtQ[n, -2]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\cos^{-1}(ax)^3}\,dx &= \frac{x\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}\,dx}{2a} + a\int \frac{x^2}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}\,dx \\
&= \frac{x\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2} - \frac{1}{2a^2\cos^{-1}(ax)} + \frac{x^2}{\cos^{-1}(ax)} - 2\int \frac{x}{\cos^{-1}(ax)}\,dx \\
&= \frac{x\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2} - \frac{1}{2a^2\cos^{-1}(ax)} + \frac{x^2}{\cos^{-1}(ax)} + \frac{2\,\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{a^2} \\
&= \frac{x\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2} - \frac{1}{2a^2\cos^{-1}(ax)} + \frac{x^2}{\cos^{-1}(ax)} + \frac{2\,\text{Subst}\left(\int \frac{\sin(2x)}{2x}\,dx, x, \cos^{-1}(ax)\right)}{a^2} \\
&= \frac{x\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2} - \frac{1}{2a^2\cos^{-1}(ax)} + \frac{x^2}{\cos^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sin(2x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{a^2} \\
&= \frac{x\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2} - \frac{1}{2a^2\cos^{-1}(ax)} + \frac{x^2}{\cos^{-1}(ax)} + \frac{\text{Si}\left(2\cos^{-1}(ax)\right)}{a^2}
\end{aligned}
$$

Mathematica [A] time = 0.0386597, size = 63, normalized size = 1.

$$
\frac{\text{Si}\left(2\cos^{-1}(ax)\right)}{a^2} + \frac{x\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2} + \frac{2a^2x^2-1}{2a^2\cos^{-1}(ax)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcCos[a*x]^3,x]
```

```
[Out] (x*Sqrt[1 - a^2*x^2])/(2*a*ArcCos[a*x]^2) + (-1 + 2*a^2*x^2)/(2*a^2*ArcCos[
a*x]) + SinIntegral[2*ArcCos[a*x]]/a^2
```

Maple [A] time = 0.044, size = 43, normalized size = 0.7

$$
\frac{1}{a^2}\left(\frac{\sin(2\arccos(ax))}{4(\arccos(ax))^2} + \frac{\cos(2\arccos(ax))}{2\arccos(ax)} + \text{Si}(2\arccos(ax))\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccos(a*x)^3,x)
```

```
[Out] 1/a^2*(1/4/arccos(a*x)^2*sin(2*arccos(a*x))+1/2/arccos(a*x)*cos(2*arccos(a*
x))+Si(2*arccos(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{4\,a^2\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^2\int\frac{x}{\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)}\,dx-\sqrt{ax+1}\sqrt{-ax+1}ax-\left(2\,a^2x^2-1\right)\arctan\left(\sqrt{ax+1}\sqrt{\right.}\right.}{2\,a^2\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(4*a^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2*integrate(x/arctan
2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x), x) - sqrt(a*x + 1)*sqrt(-a*x + 1)*a*x
 - (2*a^2*x^2 - 1)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))/(a^2*arctan2
(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\arccos\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x/arccos(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\operatorname{acos}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acos(a*x)**3,x)
```

```
[Out] Integral(x/acos(a*x)**3, x)
```

Giac [A] time = 1.17452, size = 77, normalized size = 1.22

$$\frac{x^2}{\arccos(ax)} + \frac{\operatorname{Si}(2\arccos(ax))}{a^2} + \frac{\sqrt{-a^2x^2 + 1}x}{2a\arccos(ax)^2} - \frac{1}{2a^2\arccos(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(a*x)^3,x, algorithm="giac")
```

```
[Out] x^2/arccos(a*x) + sin_integral(2*arccos(a*x))/a^2 + 1/2*sqrt(-a^2*x^2 + 1)*
x/(a*arccos(a*x)^2) - 1/2/(a^2*arccos(a*x))
```

3.64 $\int \frac{1}{\cos^{-1}(ax)^3}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2}+\frac{\text{Si}\left(\cos^{-1}(ax)\right)}{2a}+\frac{x}{2\cos^{-1}(ax)}$$

```
[Out] Sqrt[1 - a^2*x^2]/(2*a*ArcCos[a*x]^2) + x/(2*ArcCos[a*x]) + SinIntegral[Arc
Cos[a*x]]/(2*a)
```

Rubi [A] time = 0.0828363, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4622, 4720, 4624, 3299}

$$\frac{\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2}+\frac{\text{Si}\left(\cos^{-1}(ax)\right)}{2a}+\frac{x}{2\cos^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^(-3),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(2*a*ArcCos[a*x]^2) + x/(2*ArcCos[a*x]) + SinIntegral[Arc
Cos[a*x]]/(2*a)
```

Rule 4622

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(Sqrt[1 - c
^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1))
, Int[(x*(a + b*ArcCos[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && LtQ[n, -1]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4624

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Sub
st[Int[x^n*Sin[a/b - x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c,
 n}, x]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\cos^{-1}(ax)^3}\,dx &= \frac{\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2}+\frac{1}{2}a\int \frac{x}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^2}\,dx\\
&= \frac{\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2}+\frac{x}{2\cos^{-1}(ax)}-\frac{1}{2}\int \frac{1}{\cos^{-1}(ax)}\,dx\\
&= \frac{\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2}+\frac{x}{2\cos^{-1}(ax)}+\frac{\mathrm{Subst}\left(\int \frac{\sin(x)}{x}\,dx,x,\cos^{-1}(ax)\right)}{2a}\\
&= \frac{\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)^2}+\frac{x}{2\cos^{-1}(ax)}+\frac{\mathrm{Si}\left(\cos^{-1}(ax)\right)}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.0306565, size = 47, normalized size = 0.92

$$
\frac{\sqrt{1-a^2x^2}+\cos^{-1}(ax)^2\mathrm{Si}\left(\cos^{-1}(ax)\right)+ax\cos^{-1}(ax)}{2a\cos^{-1}(ax)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x]^(-3),x]
```

```
[Out] (Sqrt[1 - a^2*x^2] + a*x*ArcCos[a*x] + ArcCos[a*x]^2*SinIntegral[ArcCos[a*x
]])/(2*a*ArcCos[a*x]^2)
```

Maple [A] time = 0.045, size = 43, normalized size = 0.8

$$
\frac{1}{a}\left(\frac{1}{2\,(\arccos(ax))^2}\sqrt{-a^2x^2+1}+\frac{ax}{2\,\arccos(ax)}+\frac{\mathrm{Si}\,(\arccos(ax))}{2}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos(a*x)^3,x)
```

```
[Out] 1/a*(1/2/arccos(a*x)^2*(-a^2*x^2+1)^(1/2)+1/2*a*x/arccos(a*x)+1/2*Si(arccos
(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a \arctan\left(\sqrt{ax+1}\sqrt{-ax+1}, ax\right)^2 \int \frac{1}{\arctan\left(\sqrt{ax+1}\sqrt{-ax+1}, ax\right)}\, dx - ax \arctan\left(\sqrt{ax+1}\sqrt{-ax+1}, ax\right) - \sqrt{ax+1}\sqrt{-ax+1}}{2\, a \arctan\left(\sqrt{ax+1}\sqrt{-ax+1}, ax\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2*integrate(1/arctan2(sq
rt(a*x + 1)*sqrt(-a*x + 1), a*x), x) - a*x*arctan2(sqrt(a*x + 1)*sqrt(-a*x
+ 1), a*x) - sqrt(a*x + 1)*sqrt(-a*x + 1))/(a*arctan2(sqrt(a*x + 1)*sqrt(-a
*x + 1), a*x)^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\arccos\left(ax\right)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)^(-3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{acos}^3\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acos(a*x)**3,x)
```

```
[Out] Integral(acos(a*x)**(-3), x)
```

Giac [A] time = 1.15819, size = 58, normalized size = 1.14

$$\frac{x}{2 \arccos(ax)} + \frac{\operatorname{Si}(\arccos(ax))}{2a} + \frac{\sqrt{-a^2x^2+1}}{2a\arccos(ax)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(a*x)^3,x, algorithm="giac")
```

```
[Out] 1/2*x/arccos(a*x) + 1/2*sin_integral(arccos(a*x))/a + 1/2*sqrt(-a^2*x^2 + 1
)/(a*arccos(a*x)^2)
```

3.65 $\int \frac{1}{x \cos^{-1}(ax)^3} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x \cos^{-1}(ax)^3}, x\right)$$

```
[Out] Unintegrable[1/(x*ArcCos[a*x]^3), x]
```

Rubi [A] time = 0.0121795, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x \cos^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcCos[a*x]^3),x]
```

```
[Out] Defer[Int][1/(x*ArcCos[a*x]^3), x]
```

Rubi steps

$$\int \frac{1}{x \cos^{-1}(ax)^3} dx = \int \frac{1}{x \cos^{-1}(ax)^3} dx$$

Mathematica [A] time = 0.708456, size = 0, normalized size = 0.

$$\int \frac{1}{x \cos^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcCos[a*x]^3),x]
```

```
[Out] Integrate[1/(x*ArcCos[a*x]^3), x]
```

Maple [A] time = 0.149, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\arccos\left(ax\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccos(a*x)^3,x)
```

```
[Out] int(1/x/arccos(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\,x^{2}\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},\,ax\right)^{2}\int\frac{1}{x^{3}\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)}\,dx+\sqrt{ax+1}\sqrt{-ax+1}ax+\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},\right.}{2\,a^{2}x^{2}\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},\,ax\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccos(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*x^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2*integrate(1/(x^3*ar
ctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x) + sqrt(a*x + 1)*sqrt(-a*x + 1
)*a*x + arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))/(a^2*x^2*arctan2(sqrt(a
*x + 1)*sqrt(-a*x + 1), a*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x\arccos\left(ax\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccos(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/(x*arccos(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,\mathrm{acos}^3\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acos(a*x)**3,x)
```

```
[Out] Integral(1/(x*acos(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\arccos\,(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccos(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x*arccos(a*x)^3), x)
```

3.66 $\int \frac{1}{x^2 \cos^{-1}(ax)^3} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x^2 \cos^{-1}(ax)^3}, x\right)$$

```
[Out] Unintegrable[1/(x^2*ArcCos[a*x]^3), x]
```

Rubi [A] time = 0.0128526, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2 \cos^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*ArcCos[a*x]^3),x]
```

```
[Out] Defer[Int][1/(x^2*ArcCos[a*x]^3), x]
```

Rubi steps

$$\int \frac{1}{x^2 \cos^{-1}(ax)^3} dx = \int \frac{1}{x^2 \cos^{-1}(ax)^3} dx$$

Mathematica [A] time = 9.13345, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \cos^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*ArcCos[a*x]^3),x]
```

```
[Out] Integrate[1/(x^2*ArcCos[a*x]^3), x]
```

Maple [A] time = 0.207, size = 0, normalized size = 0.

$$\int \frac{1}{x^{2} \left(\arccos \left(a x\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arccos(a*x)^3,x)
```

```
[Out] int(1/x^2/arccos(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{x^{3} \arctan\left(\sqrt{a x+1} \sqrt{-a x+1}, a x\right)^{2} \int \frac{a^{2} x^{2}-6}{x^{4} \arctan\left(\sqrt{a x+1} \sqrt{-a x+1}, a x\right)}\, dx - \sqrt{a x+1} \sqrt{-a x+1} a x + \left(a^{2} x^{2}-2\right) \arctan\left(\sqrt{a x+1} \sqrt{\right.}{2 \, a^{2} x^{3} \arctan\left(\sqrt{a x+1} \sqrt{-a x+1}, a x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccos(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(x^3*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2*integrate((a^2*x^2 -
 6)/(x^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x) - sqrt(a*x + 1)*sq
rt(-a*x + 1)*a*x + (a^2*x^2 - 2)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)
)/(a^2*x^3*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{x^{2} \arccos \left(a x\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccos(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/(x^2*arccos(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{acos}^3(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/acos(a*x)**3,x)
```

```
[Out] Integral(1/(x**2*acos(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \arccos(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccos(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*arccos(a*x)^3), x)
```

3.67 $\int \frac{x^4}{\cos^{-1}(ax)^4}\,dx$

Optimal. Leaf size=158

$$\frac{\text{CosIntegral}\left(\cos^{-1}(ax)\right)}{48a^5}+\frac{27\text{CosIntegral}\left(3\cos^{-1}(ax)\right)}{32a^5}+\frac{125\text{CosIntegral}\left(5\cos^{-1}(ax)\right)}{96a^5}-\frac{25x^4\sqrt{1-a^2x^2}}{6a\cos^{-1}(ax)}+\frac{x^4\sqrt{1-}}{3a\cos^{-}}$$

```
[Out] (x^4*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^3) - (2*x^3)/(3*a^2*ArcCos[a*x]^2)
 + (5*x^5)/(6*ArcCos[a*x]^2) + (2*x^2*Sqrt[1 - a^2*x^2])/(a^3*ArcCos[a*x])
- (25*x^4*Sqrt[1 - a^2*x^2])/(6*a*ArcCos[a*x]) + CosIntegral[ArcCos[a*x]]/(
48*a^5) + (27*CosIntegral[3*ArcCos[a*x]])/(32*a^5) + (125*CosIntegral[5*Arc
Cos[a*x]])/(96*a^5)
```

Rubi [A] time = 0.337903, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4634, 4720, 4632, 3302}

$$\frac{\text{CosIntegral}\left(\cos^{-1}(ax)\right)}{48a^5}+\frac{27\text{CosIntegral}\left(3\cos^{-1}(ax)\right)}{32a^5}+\frac{125\text{CosIntegral}\left(5\cos^{-1}(ax)\right)}{96a^5}-\frac{25x^4\sqrt{1-a^2x^2}}{6a\cos^{-1}(ax)}+\frac{x^4\sqrt{1-}}{3a\cos^{-}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcCos[a*x]^4,x]
```

```
[Out] (x^4*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^3) - (2*x^3)/(3*a^2*ArcCos[a*x]^2)
 + (5*x^5)/(6*ArcCos[a*x]^2) + (2*x^2*Sqrt[1 - a^2*x^2])/(a^3*ArcCos[a*x])
- (25*x^4*Sqrt[1 - a^2*x^2])/(6*a*ArcCos[a*x]) + CosIntegral[ArcCos[a*x]]/(
48*a^5) + (27*CosIntegral[3*ArcCos[a*x]])/(32*a^5) + (125*CosIntegral[5*Arc
Cos[a*x]])/(96*a^5)
```

Rule 4634

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-D
ist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCos[c*x])^(n + 1))/Sq
rt[1 - c^2*x^2], x], x] + Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*ArcCo
s[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[
m, 0] && LtQ[n, -2]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4632

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dis
t[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), Co
s[x]^(m - 1)*(m - (m + 1)*Cos[x]^2), x], x], x, ArcCos[c*x]], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x^4}{\cos^{-1}(ax)^4}\,dx = \frac{x^4\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3} - \frac{4\int \frac{x^3}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}\,dx}{3a} + \frac{1}{3}(5a)\int \frac{x^5}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}\,dx$$

$$= \frac{x^4\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3} - \frac{2x^3}{3a^2\cos^{-1}(ax)^2} + \frac{5x^5}{6\cos^{-1}(ax)^2} - \frac{25}{6}\int \frac{x^4}{\cos^{-1}(ax)^2}\,dx + \frac{2\int \frac{x^2}{\cos^{-1}(ax)^2}\,dx}{a^2}$$

$$= \frac{x^4\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3} - \frac{2x^3}{3a^2\cos^{-1}(ax)^2} + \frac{5x^5}{6\cos^{-1}(ax)^2} + \frac{2x^2\sqrt{1-a^2x^2}}{a^3\cos^{-1}(ax)} - \frac{25x^4\sqrt{1-a^2x^2}}{6a\cos^{-1}(ax)} + 2\,\mathrm{Subst}\Big(\int\Big(-\ \cdots$$

$$= \frac{x^4\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3} - \frac{2x^3}{3a^2\cos^{-1}(ax)^2} + \frac{5x^5}{6\cos^{-1}(ax)^2} + \frac{2x^2\sqrt{1-a^2x^2}}{a^3\cos^{-1}(ax)} - \frac{25x^4\sqrt{1-a^2x^2}}{6a\cos^{-1}(ax)} - \mathrm{Subst}\Big(\int \frac{\cos(\cdots}{x}\ \cdots$$

$$= \frac{x^4\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3} - \frac{2x^3}{3a^2\cos^{-1}(ax)^2} + \frac{5x^5}{6\cos^{-1}(ax)^2} + \frac{2x^2\sqrt{1-a^2x^2}}{a^3\cos^{-1}(ax)} - \frac{25x^4\sqrt{1-a^2x^2}}{6a\cos^{-1}(ax)} + \frac{\mathrm{Ci}\left(\cos^{-1}(ax)\cdots\right.}{48a^5}\ \cdots$$

Mathematica [A] time = 0.169638, size = 159, normalized size = 1.01

$$\frac{32a^4x^4\sqrt{1-a^2x^2} + 80a^5x^5\cos^{-1}(ax) - 400a^4x^4\sqrt{1-a^2x^2}\cos^{-1}(ax)^2 - 64a^3x^3\cos^{-1}(ax) + 192a^2x^2\sqrt{1-a^2x^2}\cos^{-1}(ax\cdots}{96\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcCos[a*x]^4,x]
```

```
[Out] (32*a^4*x^4*Sqrt[1 - a^2*x^2] - 64*a^3*x^3*ArcCos[a*x] + 80*a^5*x^5*ArcCos[
a*x] + 192*a^2*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2 - 400*a^4*x^4*Sqrt[1 - a
^2*x^2]*ArcCos[a*x]^2 + 2*ArcCos[a*x]^3*CosIntegral[ArcCos[a*x]] + 81*ArcCo
s[a*x]^3*CosIntegral[3*ArcCos[a*x]] + 125*ArcCos[a*x]^3*CosIntegral[5*ArcCo
s[a*x]])/(96*a^5*ArcCos[a*x]^3)
```

Maple [A] time = 0.059, size = 171, normalized size = 1.1

$$\frac{1}{a^5}\left(\frac{1}{24\,(\arccos(ax))^3}\sqrt{-a^2x^2+1}+\frac{ax}{48\,(\arccos(ax))^2}-\frac{1}{48\arccos(ax)}\sqrt{-a^2x^2+1}+\frac{\mathrm{Ci}(\arccos(ax))}{48}+\frac{\sin(3\arccos}{16\,(\arccos(a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arccos(a*x)^4,x)
```

```
[Out] 1/a^5*(1/24/arccos(a*x)^3*(-a^2*x^2+1)^(1/2)+1/48*a*x/arccos(a*x)^2-1/48*(-
a^2*x^2+1)^(1/2)/arccos(a*x)+1/48*Ci(arccos(a*x))+1/16/arccos(a*x)^3*sin(3*
arccos(a*x))+3/32/arccos(a*x)^2*cos(3*arccos(a*x))-9/32/arccos(a*x)*sin(3*a
rccos(a*x))+27/32*Ci(3*arccos(a*x))+1/48/arccos(a*x)^3*sin(5*arccos(a*x))+5
/96/arccos(a*x)^2*cos(5*arccos(a*x))-25/96/arccos(a*x)*sin(5*arccos(a*x))+1
25/96*Ci(5*arccos(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^3\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^3\int\frac{\left(125\,a^4x^5-136\,a^2x^3+24\,x\right)\sqrt{ax+1}\sqrt{-ax+1}}{\left(a^5x^2-a^3\right)\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)}\,dx+\left(2\,a^2x^4-\left(25\,a^2x^4-12\,x^2\right)\arctan\left(\sqrt{ax+1}\sqrt{\right.\right.}{6\,a^3\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(6*a^3*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3*integrate(1/6*(125*
a^4*x^5 - 136*a^2*x^3 + 24*x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^5*x^2 - a^3)
*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x) + (2*a^2*x^4 - (25*a^2*x^4
```

```
 - 12*x^2)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)*sqrt(a*x + 1)*sqrt
(-a*x + 1) + (5*a^3*x^5 - 4*a*x^3)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*
x))/(a^3*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{\arccos\left(ax\right)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccos(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(x^4/arccos(a*x)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{acos}^4\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/acos(a*x)**4,x)
```

```
[Out] Integral(x**4/acos(a*x)**4, x)
```

Giac [A] time = 1.23685, size = 186, normalized size = 1.18

$$\frac{5\,x^5}{6\,\arccos\left(ax\right)^2} - \frac{25\,\sqrt{-a^2x^2+1}x^4}{6\,a\arccos\left(ax\right)} + \frac{\sqrt{-a^2x^2+1}x^4}{3\,a\arccos\left(ax\right)^3} - \frac{2\,x^3}{3\,a^2\arccos\left(ax\right)^2} + \frac{2\,\sqrt{-a^2x^2+1}x^2}{a^3\arccos\left(ax\right)} + \frac{125\,\operatorname{Ci}\left(5\,\arccos\left(ax\right)\right)}{96\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] 5/6*x^5/arccos(a*x)^2 - 25/6*sqrt(-a^2*x^2 + 1)*x^4/(a*arccos(a*x)) + 1/3*s
qrt(-a^2*x^2 + 1)*x^4/(a*arccos(a*x)^3) - 2/3*x^3/(a^2*arccos(a*x)^2) + 2*s
```

```
qrt(-a^2*x^2 + 1)*x^2/(a^3*arccos(a*x)) + 125/96*cos_integral(5*arccos(a*x)
)/a^5 + 27/32*cos_integral(3*arccos(a*x))/a^5 + 1/48*cos_integral(arccos(a*
x))/a^5
```

3.68 $\int \frac{x^3}{\cos^{-1}(ax)^4} \, dx$

Optimal. Leaf size=143

$$\frac{\text{CosIntegral}\left(2\cos^{-1}(ax)\right)}{3a^4}+\frac{4\text{CosIntegral}\left(4\cos^{-1}(ax)\right)}{3a^4}-\frac{8x^3\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)}+\frac{x^3\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3}-\frac{x^2}{2a^2\cos^{-1}(ax)^2}+\frac{x\sqrt{1}}{a^3\cos}$$

```
[Out] (x^3*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^3) - x^2/(2*a^2*ArcCos[a*x]^2) + (
2*x^4)/(3*ArcCos[a*x]^2) + (x*Sqrt[1 - a^2*x^2])/(a^3*ArcCos[a*x]) - (8*x^3
*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]) + CosIntegral[2*ArcCos[a*x]]/(3*a^4)
+ (4*CosIntegral[4*ArcCos[a*x]])/(3*a^4)
```

Rubi [A] time = 0.290715, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4634, 4720, 4632, 3302}

$$\frac{\text{CosIntegral}\left(2\cos^{-1}(ax)\right)}{3a^4}+\frac{4\text{CosIntegral}\left(4\cos^{-1}(ax)\right)}{3a^4}-\frac{8x^3\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)}+\frac{x^3\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3}-\frac{x^2}{2a^2\cos^{-1}(ax)^2}+\frac{x\sqrt{1}}{a^3\cos}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcCos[a*x]^4,x]
```


```
[Out] (x^3*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^3) - x^2/(2*a^2*ArcCos[a*x]^2) + (
2*x^4)/(3*ArcCos[a*x]^2) + (x*Sqrt[1 - a^2*x^2])/(a^3*ArcCos[a*x]) - (8*x^3
*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]) + CosIntegral[2*ArcCos[a*x]]/(3*a^4)
+ (4*CosIntegral[4*ArcCos[a*x]])/(3*a^4)
```


Rule 4634

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-D
ist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCos[c*x])^(n + 1))/Sq
rt[1 - c^2*x^2], x], x] + Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*ArcCo
s[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[
m, 0] && LtQ[n, -2]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
```

```
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4632

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dis
t[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), Co
s[x]^(m - 1)*(m - (m + 1)*Cos[x]^2), x], x], x, ArcCos[c*x]], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\cos^{-1}(ax)^4}\,dx &= \frac{x^3\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3} - \frac{\int \frac{x^2}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}\,dx}{a} + \frac{1}{3}(4a)\int \frac{x^4}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}\,dx \\
&= \frac{x^3\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3} - \frac{x^2}{2a^2\cos^{-1}(ax)^2} + \frac{2x^4}{3\cos^{-1}(ax)^2} - \frac{8}{3}\int \frac{x^3}{\cos^{-1}(ax)^2}\,dx + \frac{\int \frac{x}{\cos^{-1}(ax)^2}\,dx}{a^2} \\
&= \frac{x^3\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3} - \frac{x^2}{2a^2\cos^{-1}(ax)^2} + \frac{2x^4}{3\cos^{-1}(ax)^2} + \frac{x\sqrt{1-a^2x^2}}{a^3\cos^{-1}(ax)} - \frac{8x^3\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\,d\right.}{a^4} \\
&= \frac{x^3\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3} - \frac{x^2}{2a^2\cos^{-1}(ax)^2} + \frac{2x^4}{3\cos^{-1}(ax)^2} + \frac{x\sqrt{1-a^2x^2}}{a^3\cos^{-1}(ax)} - \frac{8x^3\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)} - \frac{\text{Ci}\left(2\cos^{-1}(ax)\right)}{a^4} \\
&= \frac{x^3\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3} - \frac{x^2}{2a^2\cos^{-1}(ax)^2} + \frac{2x^4}{3\cos^{-1}(ax)^2} + \frac{x\sqrt{1-a^2x^2}}{a^3\cos^{-1}(ax)} - \frac{8x^3\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)} + \frac{\text{Ci}\left(2\cos^{-1}(ax)\right)}{3a^4}
\end{aligned}$$

Mathematica [A] time = 0.254877, size = 107, normalized size = 0.75

$$\frac{\frac{ax\left(2a^2x^2\sqrt{1-a^2x^2}+ax\left(4a^2x^2-3\right)\cos^{-1}(ax)-2\sqrt{1-a^2x^2}\left(8a^2x^2-3\right)\cos^{-1}(ax)^2\right)}{\cos^{-1}(ax)^3} + 2\text{CosIntegral}\left(2\cos^{-1}(ax)\right) + 8\text{CosIntegral}\left(4\cos^{-1}(ax)\right)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcCos[a*x]^4,x]
```

```
[Out] ((a*x*(2*a^2*x^2*Sqrt[1 - a^2*x^2] + a*x*(-3 + 4*a^2*x^2)*ArcCos[a*x] - 2*S
qrt[1 - a^2*x^2]*(-3 + 8*a^2*x^2)*ArcCos[a*x]^2))/ArcCos[a*x]^3 + 2*CosInte
gral[2*ArcCos[a*x]] + 8*CosIntegral[4*ArcCos[a*x]])/(6*a^4)
```

Maple [A] time = 0.056, size = 114, normalized size = 0.8

$$\frac{1}{a^4}\left(\frac{\sin(2\arccos(ax))}{12(\arccos(ax))^3}+\frac{\cos(2\arccos(ax))}{12(\arccos(ax))^2}-\frac{\sin(2\arccos(ax))}{6\arccos(ax)}+\frac{\mathrm{Ci}(2\arccos(ax))}{3}+\frac{\sin(4\arccos(ax))}{24(\arccos(ax))^3}+\frac{\cos(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccos(a*x)^4,x)
```

```
[Out] 1/a^4*(1/12/arccos(a*x)^3*sin(2*arccos(a*x))+1/12/arccos(a*x)^2*cos(2*arcco
s(a*x))-1/6/arccos(a*x)*sin(2*arccos(a*x))+1/3*Ci(2*arccos(a*x))+1/24/arcco
s(a*x)^3*sin(4*arccos(a*x))+1/12/arccos(a*x)^2*cos(4*arccos(a*x))-1/3*sin(4
*arccos(a*x))/arccos(a*x)+4/3*Ci(4*arccos(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2a^3\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^3\int\frac{\left(32a^4x^4-30a^2x^2+3\right)\sqrt{ax+1}\sqrt{-ax+1}}{\left(a^5x^2-a^3\right)\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)}\,dx+2\left(a^2x^3-\left(8a^2x^3-3x\right)\arctan\left(\sqrt{ax+1}\sqrt{-a\right.\right.}{6a^3\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(6*a^3*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3*integrate(1/3*(32*a
^4*x^4 - 30*a^2*x^2 + 3)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^5*x^2 - a^3)*arct
an2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x) + 2*(a^2*x^3 - (8*a^2*x^3 - 3*x
)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)*sqrt(a*x + 1)*sqrt(-a*x + 1
) + (4*a^3*x^4 - 3*a*x^2)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))/(a^3*
arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^3}{\arccos\left(ax\right)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccos(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(x^3/arccos(a*x)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\mathrm{acos}^4\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acos(a*x)**4,x)
```

```
[Out] Integral(x**3/acos(a*x)**4, x)
```

Giac [A] time = 1.20239, size = 169, normalized size = 1.18

$$\frac{2\,x^4}{3\,\arccos\left(ax\right)^2} - \frac{8\,\sqrt{-a^2x^2+1}x^3}{3\,a\arccos\left(ax\right)} + \frac{\sqrt{-a^2x^2+1}x^3}{3\,a\arccos\left(ax\right)^3} - \frac{x^2}{2\,a^2\arccos\left(ax\right)^2} + \frac{\sqrt{-a^2x^2+1}x}{a^3\arccos\left(ax\right)} + \frac{4\,\mathrm{Ci}\left(4\,\arccos\left(ax\right)\right)}{3\,a^4} + \mathrm{Ci}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] 2/3*x^4/arccos(a*x)^2 - 8/3*sqrt(-a^2*x^2 + 1)*x^3/(a*arccos(a*x)) + 1/3*sq
rt(-a^2*x^2 + 1)*x^3/(a*arccos(a*x)^3) - 1/2*x^2/(a^2*arccos(a*x)^2) + sqrt
(-a^2*x^2 + 1)*x/(a^3*arccos(a*x)) + 4/3*cos_integral(4*arccos(a*x))/a^4 +
1/3*cos_integral(2*arccos(a*x))/a^4
```

3.69 $\int \frac{x^2}{\cos^{-1}(ax)^4} dx$

Optimal. Leaf size=141

$$\frac{\text{CosIntegral}\left(\cos^{-1}(ax)\right)}{24a^3}+\frac{9\text{CosIntegral}\left(3\cos^{-1}(ax)\right)}{8a^3}-\frac{3x^2\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)}+\frac{x^2\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3}+\frac{\sqrt{1-a^2x^2}}{3a^3\cos^{-1}(ax)}-\frac{x}{3a^2\cos^{-}}$$

```
[Out] (x^2*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^3) - x/(3*a^2*ArcCos[a*x]^2) + x^3
/(2*ArcCos[a*x]^2) + Sqrt[1 - a^2*x^2]/(3*a^3*ArcCos[a*x]) - (3*x^2*Sqrt[1
- a^2*x^2])/(2*a*ArcCos[a*x]) + CosIntegral[ArcCos[a*x]]/(24*a^3) + (9*CosI
ntegral[3*ArcCos[a*x]])/(8*a^3)
```

Rubi [A] time = 0.312028, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4634, 4720, 4632, 3302, 4622, 4724}

$$\frac{\text{CosIntegral}\left(\cos^{-1}(ax)\right)}{24a^3}+\frac{9\text{CosIntegral}\left(3\cos^{-1}(ax)\right)}{8a^3}-\frac{3x^2\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)}+\frac{x^2\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3}+\frac{\sqrt{1-a^2x^2}}{3a^3\cos^{-1}(ax)}-\frac{x}{3a^2\cos^{-}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcCos[a*x]^4,x]
```


```
[Out] (x^2*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^3) - x/(3*a^2*ArcCos[a*x]^2) + x^3
/(2*ArcCos[a*x]^2) + Sqrt[1 - a^2*x^2]/(3*a^3*ArcCos[a*x]) - (3*x^2*Sqrt[1
- a^2*x^2])/(2*a*ArcCos[a*x]) + CosIntegral[ArcCos[a*x]]/(24*a^3) + (9*CosI
ntegral[3*ArcCos[a*x]])/(8*a^3)
```


Rule 4634

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-D
ist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCos[c*x])^(n + 1))/Sq
rt[1 - c^2*x^2], x], x] + Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*ArcCo
s[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[
m, 0] && LtQ[n, -2]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
```

```
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4632

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dis
t[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), Co
s[x]^(m - 1)*(m - (m + 1)*Cos[x]^2), x], x], x, ArcCos[c*x]], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4622

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(Sqrt[1 - c
^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1))
, Int[(x*(a + b*ArcCos[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && LtQ[n, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{\cos^{-1}(ax)^4}\,dx = \frac{x^2\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3} - \frac{2\int \frac{x}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}\,dx}{3a} + a\int \frac{x^3}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}\,dx$$

$$= \frac{x^2\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3} - \frac{x}{3a^2\cos^{-1}(ax)^2} + \frac{x^3}{2\cos^{-1}(ax)^2} - \frac{3}{2}\int \frac{x^2}{\cos^{-1}(ax)^2}\,dx + \frac{\int \frac{1}{\cos^{-1}(ax)^2}\,dx}{3a^2}$$

$$= \frac{x^2\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3} - \frac{x}{3a^2\cos^{-1}(ax)^2} + \frac{x^3}{2\cos^{-1}(ax)^2} + \frac{\sqrt{1-a^2x^2}}{3a^3\cos^{-1}(ax)} - \frac{3x^2\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)} - \frac{3\,\mathrm{Subst}\left(\int\left(-\right.\right.}{}$$

$$= \frac{x^2\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3} - \frac{x}{3a^2\cos^{-1}(ax)^2} + \frac{x^3}{2\cos^{-1}(ax)^2} + \frac{\sqrt{1-a^2x^2}}{3a^3\cos^{-1}(ax)} - \frac{3x^2\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)} - \frac{\mathrm{Subst}\left(\int \frac{\cos(x)}{x}\right.}{3}$$

$$= \frac{x^2\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3} - \frac{x}{3a^2\cos^{-1}(ax)^2} + \frac{x^3}{2\cos^{-1}(ax)^2} + \frac{\sqrt{1-a^2x^2}}{3a^3\cos^{-1}(ax)} - \frac{3x^2\sqrt{1-a^2x^2}}{2a\cos^{-1}(ax)} + \frac{\mathbf{Ci}\left(\cos^{-1}(ax)\right)}{24a^3}$$

Mathematica [A] time = 0.119853, size = 129, normalized size = 0.91

$$-\frac{10\text{CosIntegral}\left(\cos^{-1}(ax)\right)}{3a^3} - \frac{9\left(-3\text{CosIntegral}\left(\cos^{-1}(ax)\right) - \text{CosIntegral}\left(3\cos^{-1}(ax)\right)\right)}{8a^3} + \frac{x^2\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3} - \frac{\sqrt{1-}}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcCos[a*x]^4,x]
```

```
[Out] (x^2*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^3) + (-2*x + 3*a^2*x^3)/(6*a^2*Arc
Cos[a*x]^2) - (Sqrt[1 - a^2*x^2]*(-2 + 9*a^2*x^2))/(6*a^3*ArcCos[a*x]) - (1
0*CosIntegral[ArcCos[a*x]])/(3*a^3) - (9*(-3*CosIntegral[ArcCos[a*x]] - Cos
Integral[3*ArcCos[a*x]]))/(8*a^3)
```

Maple [A] time = 0.056, size = 117, normalized size = 0.8

$$\frac{1}{a^3}\left(\frac{1}{12\left(\arccos(ax)\right)^3}\sqrt{-a^2x^2+1} + \frac{ax}{24\left(\arccos(ax)\right)^2} - \frac{1}{24\arccos(ax)}\sqrt{-a^2x^2+1} + \frac{\mathrm{Ci}\left(\arccos(ax)\right)}{24} + \frac{\sin(3\arccos}{12\left(\arccos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccos(a*x)^4,x)
```

```
[Out] 1/a^3*(1/12/arccos(a*x)^3*(-a^2*x^2+1)^(1/2)+1/24*a*x/arccos(a*x)^2-1/24*(-
a^2*x^2+1)^(1/2)/arccos(a*x)+1/24*Ci(arccos(a*x))+1/12/arccos(a*x)^3*sin(3*
arccos(a*x))+1/8/arccos(a*x)^2*cos(3*arccos(a*x))-3/8/arccos(a*x)*sin(3*arc
cos(a*x))+9/8*Ci(3*arccos(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^3 \arctan\left(\sqrt{ax+1}\sqrt{-ax+1}, ax\right)^3 \int \frac{\left(27\,a^2x^3-20\,x\right)\sqrt{ax+1}\sqrt{-ax+1}}{\left(a^3x^2-a\right)\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)}\,dx + \left(2\,a^2x^2 - \left(9\,a^2x^2-2\right)\arctan\left(\sqrt{ax+1}\sqrt{-ax+1}\right.\right.}{6\,a^3 \arctan\left(\sqrt{ax+1}\sqrt{-ax+1}, ax\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(6*a^3*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3*integrate(1/6*(27*a
^2*x^3 - 20*x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^3*x^2 - a)*arctan2(sqrt(a*x
 + 1)*sqrt(-a*x + 1), a*x)), x) + (2*a^2*x^2 - (9*a^2*x^2 - 2)*arctan2(sqrt
(a*x + 1)*sqrt(-a*x + 1), a*x)^2)*sqrt(a*x + 1)*sqrt(-a*x + 1) + (3*a^3*x^3
 - 2*a*x)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))/(a^3*arctan2(sqrt(a*x
 + 1)*sqrt(-a*x + 1), a*x)^3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\arccos(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccos(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(x^2/arccos(a*x)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{acos}^4(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acos(a*x)**4,x)
```

```
[Out] Integral(x**2/acos(a*x)**4, x)
```

Giac [A] time = 1.19292, size = 163, normalized size = 1.16

$$\frac{x^3}{2\arccos(ax)^2} - \frac{3\sqrt{-a^2x^2+1}x^2}{2a\arccos(ax)} + \frac{\sqrt{-a^2x^2+1}x^2}{3a\arccos(ax)^3} + \frac{9\operatorname{Ci}(3\arccos(ax))}{8a^3} + \frac{\operatorname{Ci}(\arccos(ax))}{24a^3} - \frac{x}{3a^2\arccos(ax)^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] 1/2*x^3/arccos(a*x)^2 - 3/2*sqrt(-a^2*x^2 + 1)*x^2/(a*arccos(a*x)) + 1/3*sq
rt(-a^2*x^2 + 1)*x^2/(a*arccos(a*x)^3) + 9/8*cos_integral(3*arccos(a*x))/a^
3 + 1/24*cos_integral(arccos(a*x))/a^3 - 1/3*x/(a^2*arccos(a*x)^2) + 1/3*sq
rt(-a^2*x^2 + 1)/(a^3*arccos(a*x))
```

3.70 $\int \frac{x}{\cos^{-1}(ax)^4}\,dx$

Optimal. Leaf size=97

$$\frac{2\text{CosIntegral}\left(2\cos^{-1}(ax)\right)}{3a^2}-\frac{2x\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)}+\frac{x\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3}-\frac{1}{6a^2\cos^{-1}(ax)^2}+\frac{x^2}{3\cos^{-1}(ax)^2}$$

```
[Out] (x*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^3) - 1/(6*a^2*ArcCos[a*x]^2) + x^2/(
3*ArcCos[a*x]^2) - (2*x*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]) + (2*CosIntegr
al[2*ArcCos[a*x]])/(3*a^2)
```

Rubi [A] time = 0.166116, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4634, 4720, 4632, 3302, 4642}

$$\frac{2\text{CosIntegral}\left(2\cos^{-1}(ax)\right)}{3a^2}-\frac{2x\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)}+\frac{x\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3}-\frac{1}{6a^2\cos^{-1}(ax)^2}+\frac{x^2}{3\cos^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcCos[a*x]^4,x]
```

```
[Out] (x*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^3) - 1/(6*a^2*ArcCos[a*x]^2) + x^2/(
3*ArcCos[a*x]^2) - (2*x*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]) + (2*CosIntegr
al[2*ArcCos[a*x]])/(3*a^2)
```

Rule 4634

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-D
ist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCos[c*x])^(n + 1))/Sq
rt[1 - c^2*x^2], x], x] + Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*ArcCo
s[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[
m, 0] && LtQ[n, -2]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
```

```
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4632

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dis
t[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), Co
s[x]^(m - 1)*(m - (m + 1)*Cos[x]^2), x], x], x, ArcCos[c*x]], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\cos^{-1}(ax)^4}\,dx &= \frac{x\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}\,dx}{3a} + \frac{1}{3}(2a)\int \frac{x^2}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}\,dx \\
&= \frac{x\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3} - \frac{1}{6a^2\cos^{-1}(ax)^2} + \frac{x^2}{3\cos^{-1}(ax)^2} - \frac{2}{3}\int \frac{x}{\cos^{-1}(ax)^2}\,dx \\
&= \frac{x\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3} - \frac{1}{6a^2\cos^{-1}(ax)^2} + \frac{x^2}{3\cos^{-1}(ax)^2} - \frac{2x\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)} + \frac{2\,\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx, x, \cos^{-1}(ax\right.}{3a^2} \\
&= \frac{x\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3} - \frac{1}{6a^2\cos^{-1}(ax)^2} + \frac{x^2}{3\cos^{-1}(ax)^2} - \frac{2x\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)} + \frac{2\text{Ci}\left(2\cos^{-1}(ax)\right)}{3a^2}
\end{aligned}$$

Mathematica [A] time = 0.103208, size = 86, normalized size = 0.89

$$\frac{2ax\sqrt{1-a^2x^2} - 4ax\sqrt{1-a^2x^2}\cos^{-1}(ax)^2 + \left(2a^2x^2-1\right)\cos^{-1}(ax) + 4\cos^{-1}(ax)^3\text{CosIntegral}\left(2\cos^{-1}(ax)\right)}{6a^2\cos^{-1}(ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcCos[a*x]^4,x]
```

```
[Out] (2*a*x*Sqrt[1 - a^2*x^2] + (-1 + 2*a^2*x^2)*ArcCos[a*x] - 4*a*x*Sqrt[1 - a^
2*x^2]*ArcCos[a*x]^2 + 4*ArcCos[a*x]^3*CosIntegral[2*ArcCos[a*x]])/(6*a^2*A
rcCos[a*x]^3)
```

Maple [A] time = 0.046, size = 60, normalized size = 0.6

$$\frac{1}{a^2}\left(\frac{\sin(2\arccos(ax))}{6(\arccos(ax))^3}+\frac{\cos(2\arccos(ax))}{6(\arccos(ax))^2}-\frac{\sin(2\arccos(ax))}{3\arccos(ax)}+\frac{2\,\mathrm{Ci}(2\arccos(ax))}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccos(a*x)^4,x)
```

```
[Out] 1/a^2*(1/6/arccos(a*x)^3*sin(2*arccos(a*x))+1/6/arccos(a*x)^2*cos(2*arccos(
a*x))-1/3/arccos(a*x)*sin(2*arccos(a*x))+2/3*Ci(2*arccos(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{4\,a^2\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^3\int\frac{(2a^2x^2-1)\sqrt{ax+1}\sqrt{-ax+1}}{(a^3x^2-a)\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)}\,dx-2\left(2\,ax\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^2-ax\right)\sqrt{}}{6\,a^2\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(6*a^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3*integrate(2/3*(2*a^
2*x^2 - 1)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^3*x^2 - a)*arctan2(sqrt(a*x + 1
)*sqrt(-a*x + 1), a*x)), x) - 2*(2*a*x*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1)
, a*x)^2 - a*x)*sqrt(a*x + 1)*sqrt(-a*x + 1) + (2*a^2*x^2 - 1)*arctan2(sqrt
(a*x + 1)*sqrt(-a*x + 1), a*x))/(a^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1),
a*x)^3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\arccos\left(ax\right)^{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(x/arccos(a*x)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{acos}^{4}\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acos(a*x)**4,x)
```

```
[Out] Integral(x/acos(a*x)**4, x)
```

Giac [A] time = 1.15565, size = 112, normalized size = 1.15

$$\frac{x^{2}}{3 \arccos\left(ax\right)^{2}} - \frac{2 \sqrt{-a^{2} x^{2} + 1} x}{3 a \arccos\left(ax\right)} + \frac{2 \operatorname{Ci}\left(2 \arccos\left(ax\right)\right)}{3 a^{2}} + \frac{\sqrt{-a^{2} x^{2} + 1} x}{3 a \arccos\left(ax\right)^{3}} - \frac{1}{6 a^{2} \arccos\left(ax\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] 1/3*x^2/arccos(a*x)^2 - 2/3*sqrt(-a^2*x^2 + 1)*x/(a*arccos(a*x)) + 2/3*cos_
integral(2*arccos(a*x))/a^2 + 1/3*sqrt(-a^2*x^2 + 1)*x/(a*arccos(a*x)^3) -
1/6/(a^2*arccos(a*x)^2)
```

3.71 $\int \frac{1}{\cos^{-1}(ax)^4} dx$

Optimal. Leaf size=78

$$-\frac{\sqrt{1-a^2x^2}}{6a\cos^{-1}(ax)}+\frac{\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3}+\frac{\text{CosIntegral}\left(\cos^{-1}(ax)\right)}{6a}+\frac{x}{6\cos^{-1}(ax)^2}$$

```
[Out] Sqrt[1 - a^2*x^2]/(3*a*ArcCos[a*x]^3) + x/(6*ArcCos[a*x]^2) - Sqrt[1 - a^2*
x^2]/(6*a*ArcCos[a*x]) + CosIntegral[ArcCos[a*x]]/(6*a)
```

Rubi [A] time = 0.156435, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4622, 4720, 4724, 3302}

$$-\frac{\sqrt{1-a^2x^2}}{6a\cos^{-1}(ax)}+\frac{\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3}+\frac{\text{CosIntegral}\left(\cos^{-1}(ax)\right)}{6a}+\frac{x}{6\cos^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^(-4),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(3*a*ArcCos[a*x]^3) + x/(6*ArcCos[a*x]^2) - Sqrt[1 - a^2*
x^2]/(6*a*ArcCos[a*x]) + CosIntegral[ArcCos[a*x]]/(6*a)
```

Rule 4622

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(Sqrt[1 - c
^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1))
, Int[(x*(a + b*ArcCos[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && LtQ[n, -1]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\cos^{-1}(ax)^4}\,dx &= \frac{\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3} + \frac{1}{3}a\int \frac{x}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^3}\,dx \\
&= \frac{\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3} + \frac{x}{6\cos^{-1}(ax)^2} - \frac{1}{6}\int \frac{1}{\cos^{-1}(ax)^2}\,dx \\
&= \frac{\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3} + \frac{x}{6\cos^{-1}(ax)^2} - \frac{\sqrt{1-a^2x^2}}{6a\cos^{-1}(ax)} - \frac{1}{6}a\int \frac{x}{\sqrt{1-a^2x^2}\cos^{-1}(ax)}\,dx \\
&= \frac{\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3} + \frac{x}{6\cos^{-1}(ax)^2} - \frac{\sqrt{1-a^2x^2}}{6a\cos^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cos(x)}{x}\,dx, x, \cos^{-1}(ax)\right)}{6a} \\
&= \frac{\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^3} + \frac{x}{6\cos^{-1}(ax)^2} - \frac{\sqrt{1-a^2x^2}}{6a\cos^{-1}(ax)} + \frac{\text{Ci}\left(\cos^{-1}(ax)\right)}{6a}
\end{aligned}$$

Mathematica [A] time = 0.044243, size = 71, normalized size = 0.91

$$\frac{2\sqrt{1-a^2x^2} - \sqrt{1-a^2x^2}\cos^{-1}(ax)^2 + \cos^{-1}(ax)^3\text{CosIntegral}\left(\cos^{-1}(ax)\right) + ax\cos^{-1}(ax)}{6a\cos^{-1}(ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x]^(-4),x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2] + a*x*ArcCos[a*x] - Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2 +
ArcCos[a*x]^3*CosIntegral[ArcCos[a*x]])/(6*a*ArcCos[a*x]^3)
```

Maple [A] time = 0.045, size = 63, normalized size = 0.8

$$\frac{1}{a}\left(\frac{1}{3\left(\arccos\left(ax\right)\right)^{3}}\sqrt{-a^{2}x^{2}+1}+\frac{ax}{6\left(\arccos\left(ax\right)\right)^{2}}-\frac{1}{6\arccos\left(ax\right)}\sqrt{-a^{2}x^{2}+1}+\frac{\mathrm{Ci}\left(\arccos\left(ax\right)\right)}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos(a*x)^4,x)
```

```
[Out] 1/a*(1/3/arccos(a*x)^3*(-a^2*x^2+1)^(1/2)+1/6*a*x/arccos(a*x)^2-1/6*(-a^2*x
^2+1)^(1/2)/arccos(a*x)+1/6*Ci(arccos(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^{2}\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^{3}\int\frac{\sqrt{ax+1}\sqrt{-ax+1}x}{\left(a^{2}x^{2}-1\right)\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)}\,dx+ax\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)-\sqrt{ax+1}\sqrt{-a}}{6\,a\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(6*a^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3*integrate(1/6*sqrt(
a*x + 1)*sqrt(-a*x + 1)*x/((a^2*x^2 - 1)*arctan2(sqrt(a*x + 1)*sqrt(-a*x +
1), a*x)), x) + a*x*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x) - sqrt(a*x +
 1)*sqrt(-a*x + 1)*(arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2 - 2))/(a*a
rctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\arccos\left(ax\right)^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)^(-4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{acos}^4(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acos(a*x)**4,x)
```

```
[Out] Integral(acos(a*x)**(-4), x)
```

Giac [A] time = 1.15204, size = 89, normalized size = 1.14

$$\frac{\operatorname{Ci}(\arccos(ax))}{6\,a} + \frac{x}{6\,\arccos(ax)^2} - \frac{\sqrt{-a^2x^2+1}}{6\,a\arccos(ax)} + \frac{\sqrt{-a^2x^2+1}}{3\,a\arccos(ax)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] 1/6*cos_integral(arccos(a*x))/a + 1/6*x/arccos(a*x)^2 - 1/6*sqrt(-a^2*x^2 +
 1)/(a*arccos(a*x)) + 1/3*sqrt(-a^2*x^2 + 1)/(a*arccos(a*x)^3)
```

3.72 $\int \frac{1}{x \cos^{-1}(ax)^4} \, dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x \cos^{-1}(ax)^4}, x\right)$$

```
[Out] Unintegrable[1/(x*ArcCos[a*x]^4), x]
```

Rubi [A] time = 0.0255599, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x \cos^{-1}(ax)^4} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcCos[a*x]^4),x]
```

```
[Out] Defer[Int][1/(x*ArcCos[a*x]^4), x]
```

Rubi steps

$$\int \frac{1}{x \cos^{-1}(ax)^4} \, dx = \int \frac{1}{x \cos^{-1}(ax)^4} \, dx$$

Mathematica [A] time = 2.79009, size = 0, normalized size = 0.

$$\int \frac{1}{x \cos^{-1}(ax)^4} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcCos[a*x]^4),x]
```

```
[Out] Integrate[1/(x*ArcCos[a*x]^4), x]
```

Maple [A] time = 0.153, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\arccos\left(ax\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccos(a*x)^4,x)
```

```
[Out] int(1/x/arccos(a*x)^4,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\,a^{3}x^{3}\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^{3}\int\frac{\left(2\,a^{2}x^{2}-3\right)\sqrt{ax+1}\sqrt{-ax+1}}{\left(a^{5}x^{6}-a^{3}x^{4}\right)\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)}\,dx+ax\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)+2\left(a\right.}{6\,a^{3}x^{3}\arctan\left(\sqrt{ax+1}\sqrt{-ax+1},ax\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(6*a^3*x^3*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3*integrate(1/3*(
2*a^2*x^2 - 3)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^5*x^6 - a^3*x^4)*arctan2(sq
rt(a*x + 1)*sqrt(-a*x + 1), a*x)), x) + a*x*arctan2(sqrt(a*x + 1)*sqrt(-a*x
 + 1), a*x) + 2*(a^2*x^2 + arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)*sq
rt(a*x + 1)*sqrt(-a*x + 1))/(a^3*x^3*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1),
a*x)^3)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x\arccos\left(ax\right)^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccos(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(1/(x*arccos(a*x)^4), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{acos}^4(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acos(a*x)**4,x)
```

```
[Out] Integral(1/(x*acos(a*x)**4), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \arccos(ax)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(1/(x*arccos(a*x)^4), x)
```

3.73 $\int \frac{1}{x^2 \cos^{-1}(ax)^4} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x^2 \cos^{-1}(ax)^4}, x\right)$$

```
[Out] Unintegrable[1/(x^2*ArcCos[a*x]^4), x]
```

Rubi [A] time = 0.0155747, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2 \cos^{-1}(ax)^4} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*ArcCos[a*x]^4),x]
```

```
[Out] Defer[Int][1/(x^2*ArcCos[a*x]^4), x]
```

Rubi steps

$$\int \frac{1}{x^2 \cos^{-1}(ax)^4} dx = \int \frac{1}{x^2 \cos^{-1}(ax)^4} dx$$

Mathematica [A] time = 27.2556, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \cos^{-1}(ax)^4} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*ArcCos[a*x]^4),x]
```

```
[Out] Integrate[1/(x^2*ArcCos[a*x]^4), x]
```

Maple [A] time = 0.214, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (\arccos (ax))^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arccos(a*x)^4,x)
```

```
[Out] int(1/x^2/arccos(a*x)^4,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3x^4 \arctan \left(\sqrt{ax+1}\sqrt{-ax+1}, ax\right)^3 \int \frac{\left(a^4x^4-20\,a^2x^2+24\right)\sqrt{ax+1}\sqrt{-ax+1}}{\left(a^5x^7-a^3x^5\right)\arctan\left(\sqrt{ax+1}\sqrt{-ax+1}, ax\right)}\, dx - \left(2\,a^2x^2 - \left(a^2x^2-6\right)\arctan\left(\sqrt{ax+1}\sqrt{-ax\right.\right.}{6\,a^3x^4 \arctan \left(\sqrt{ax+1}\sqrt{-ax+1}, ax\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(6*a^3*x^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3*integrate(1/6*
(a^4*x^4 - 20*a^2*x^2 + 24)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^5*x^7 - a^3*x^
5)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x) - (2*a^2*x^2 - (a^2*x^2
- 6)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)*sqrt(a*x + 1)*sqrt(-a*x
+ 1) + (a^3*x^3 - 2*a*x)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))/(a^3*x
^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{x^2 \arccos (ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccos(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(1/(x^2*arccos(a*x)^4), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{acos}^4(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/acos(a*x)**4,x)
```

```
[Out] Integral(1/(x**2*acos(a*x)**4), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \arccos(ax)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*arccos(a*x)^4), x)
```

3.74 $\int x^4\sqrt{\cos^{-1}(ax)}\,dx$

Optimal. Leaf size=121

$$-\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{8a^5}-\frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{16a^5}-\frac{\sqrt{\frac{\pi}{10}}\,\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{80a^5}+\frac{1}{5}x^5\sqrt{\cos^{-1}(a$$

```
[Out] (x^5*Sqrt[ArcCos[a*x]])/5 - (Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x
]]])/(8*a^5) - (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(16*a^5)
 - (Sqrt[Pi/10]*FresnelC[Sqrt[10/Pi]*Sqrt[ArcCos[a*x]]])/(80*a^5)
```

Rubi [A] time = 0.281232, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4630, 4724, 3312, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{8a^5}-\frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{16a^5}-\frac{\sqrt{\frac{\pi}{10}}\,\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{80a^5}+\frac{1}{5}x^5\sqrt{\cos^{-1}(a$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[ArcCos[a*x]],x]
```

```
[Out] (x^5*Sqrt[ArcCos[a*x]])/5 - (Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x
]]])/(8*a^5) - (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(16*a^5)
 - (Sqrt[Pi/10]*FresnelC[Sqrt[10/Pi]*Sqrt[ArcCos[a*x]]])/(80*a^5)
```

Rule 4630

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
x^(m + 1)*(a + b*ArcCos[c*x])^n)/(m + 1), x] + Dist[(b*c*n)/(m + 1), Int[(x
^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int x^4\sqrt{\cos^{-1}(ax)}\,dx = \frac{1}{5}x^5\sqrt{\cos^{-1}(ax)} + \frac{1}{10}a\int \frac{x^5}{\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}\,dx$$

$$= \frac{1}{5}x^5\sqrt{\cos^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cos^5(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{10a^5}$$

$$= \frac{1}{5}x^5\sqrt{\cos^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(\frac{5\cos(x)}{8\sqrt{x}} + \frac{5\cos(3x)}{16\sqrt{x}} + \frac{\cos(5x)}{16\sqrt{x}}\right)dx, x, \cos^{-1}(ax)\right)}{10a^5}$$

$$= \frac{1}{5}x^5\sqrt{\cos^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cos(5x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{160a^5} - \frac{\text{Subst}\left(\int \frac{\cos(3x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{32a^5} - \frac{\text{Subs}}{}$$

$$= \frac{1}{5}x^5\sqrt{\cos^{-1}(ax)} - \frac{\text{Subst}\left(\int \cos\left(5x^2\right)dx, x, \sqrt{\cos^{-1}(ax)}\right)}{80a^5} - \frac{\text{Subst}\left(\int \cos\left(3x^2\right)dx, x, \sqrt{\cos^{-1}(a}\right.}{16a^5}$$

$$= \frac{1}{5}x^5\sqrt{\cos^{-1}(ax)} - \frac{\sqrt{\frac{\pi}{2}}C\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{8a^5} - \frac{\sqrt{\frac{\pi}{6}}C\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{16a^5} - \frac{\sqrt{\frac{\pi}{10}}C\left(\sqrt{\frac{10}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{80a^5}$$

Mathematica [C] time = 0.266099, size = 212, normalized size = 1.75

$$-\frac{25\sqrt{3}\left(-i\cos^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{3}{2}, -3i\cos^{-1}(ax)\right) + 3\sqrt{5}\left(-i\cos^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{3}{2}, -5i\cos^{-1}(ax)\right) - 150\sqrt{\cos^{-1}(}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*Sqrt[ArcCos[a*x]],x]
```

```
[Out] -(-150*Sqrt[I*ArcCos[a*x]]*Sqrt[ArcCos[a*x]^2]*Gamma[3/2, (-I)*ArcCos[a*x]]
 - 150*Sqrt[(-I)*ArcCos[a*x]]*Sqrt[ArcCos[a*x]^2]*Gamma[3/2, I*ArcCos[a*x]]
 + 25*Sqrt[3]*((-I)*ArcCos[a*x])^(3/2)*Gamma[3/2, (-3*I)*ArcCos[a*x]] + 25*
Sqrt[3]*(I*ArcCos[a*x])^(3/2)*Gamma[3/2, (3*I)*ArcCos[a*x]] + 3*Sqrt[5]*((-
I)*ArcCos[a*x])^(3/2)*Gamma[3/2, (-5*I)*ArcCos[a*x]] + 3*Sqrt[5]*(I*ArcCos[
a*x])^(3/2)*Gamma[3/2, (5*I)*ArcCos[a*x]])/(2400*a^5*ArcCos[a*x]^(3/2))
```

Maple [A] time = 0.096, size = 143, normalized size = 1.2

$$\frac{1}{2400\,a^5}\left(-3\,\sqrt{5}\sqrt{2}\sqrt{\arccos(ax)}\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{5}\sqrt{2}\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-25\,\sqrt{3}\sqrt{2}\sqrt{\arccos(ax)}\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{3}\sqrt{2}\sqrt{\mathrm{arc}}}{\sqrt{\pi}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccos(a*x)^(1/2),x)
```

```
[Out] 1/2400/a^5/arccos(a*x)^(1/2)*(-3*5^(1/2)*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)
*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)*arccos(a*x)^(1/2))-25*3^(1/2)*2^(1/2)*ar
ccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*arccos(a*x)^(1/2
))-150*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arccos(
a*x)^(1/2))+300*a*x*arccos(a*x)+150*arccos(a*x)*cos(3*arccos(a*x))+30*arcco
s(a*x)*cos(5*arccos(a*x)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccos(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccos(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\sqrt{\operatorname{acos}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acos(a*x)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(acos(a*x)), x)
```

Giac [B] time = 1.32936, size = 425, normalized size = 3.51

$$\frac{\sqrt{10}\sqrt{\pi}i\,\operatorname{erf}\left(\frac{\sqrt{10}\sqrt{\arccos(ax)}}{i-1}\right)}{1600\,a^5(i-1)}+\frac{\sqrt{6}\sqrt{\pi}i\,\operatorname{erf}\left(\frac{\sqrt{6}\sqrt{\arccos(ax)}}{i-1}\right)}{192\,a^5(i-1)}+\frac{\sqrt{2}\sqrt{\pi}i\,\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\arccos(ax)}}{i-1}\right)}{32\,a^5(i-1)}+\frac{\sqrt{\arccos(ax)}e^{(5\,i\arccos(ax))}}{160\,a^5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccos(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1600*sqrt(10)*sqrt(pi)*i*erf(sqrt(10)*sqrt(arccos(a*x))/(i - 1))/(a^5*(i
- 1)) + 1/192*sqrt(6)*sqrt(pi)*i*erf(sqrt(6)*sqrt(arccos(a*x))/(i - 1))/(a^
5*(i - 1)) + 1/32*sqrt(2)*sqrt(pi)*i*erf(sqrt(2)*sqrt(arccos(a*x))/(i - 1))
/(a^5*(i - 1)) + 1/160*sqrt(arccos(a*x))*e^(5*i*arccos(a*x))/a^5 + 1/32*sqr
t(arccos(a*x))*e^(3*i*arccos(a*x))/a^5 + 1/16*sqrt(arccos(a*x))*e^(i*arccos
(a*x))/a^5 + 1/16*sqrt(arccos(a*x))*e^(-i*arccos(a*x))/a^5 + 1/32*sqrt(arcc
os(a*x))*e^(-3*i*arccos(a*x))/a^5 + 1/160*sqrt(arccos(a*x))*e^(-5*i*arccos(
a*x))/a^5 - 1/1600*sqrt(10)*sqrt(pi)*erf(-sqrt(10)*i*sqrt(arccos(a*x))/(i -
 1))/(a^5*(i - 1)) - 1/192*sqrt(6)*sqrt(pi)*erf(-sqrt(6)*i*sqrt(arccos(a*x)
)/(i - 1))/(a^5*(i - 1)) - 1/32*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*i*sqrt(arccos
(a*x))/(i - 1))/(a^5*(i - 1))
```

3.75 $\int x^3 \sqrt{\cos^{-1}(ax)}\,dx$

Optimal. Leaf size=95

$$-\frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{64a^4}-\frac{\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{16a^4}-\frac{3\sqrt{\cos^{-1}(ax)}}{32a^4}+\frac{1}{4}x^4\sqrt{\cos^{-1}(ax)}$$

```
[Out] (-3*Sqrt[ArcCos[a*x]])/(32*a^4) + (x^4*Sqrt[ArcCos[a*x]])/4 - (Sqrt[Pi/2]*F
resnelC[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(64*a^4) - (Sqrt[Pi]*FresnelC[(2*S
qrt[ArcCos[a*x]])/Sqrt[Pi]])/(16*a^4)
```

Rubi [A] time = 0.194688, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4630, 4724, 3312, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{64a^4}-\frac{\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{16a^4}-\frac{3\sqrt{\cos^{-1}(ax)}}{32a^4}+\frac{1}{4}x^4\sqrt{\cos^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[ArcCos[a*x]],x]
```

```
[Out] (-3*Sqrt[ArcCos[a*x]])/(32*a^4) + (x^4*Sqrt[ArcCos[a*x]])/4 - (Sqrt[Pi/2]*F
resnelC[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(64*a^4) - (Sqrt[Pi]*FresnelC[(2*S
qrt[ArcCos[a*x]])/Sqrt[Pi]])/(16*a^4)
```

Rule 4630

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
x^(m + 1)*(a + b*ArcCos[c*x])^n)/(m + 1), x] + Dist[(b*c*n)/(m + 1), Int[(x
^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int x^3\sqrt{\cos^{-1}(ax)}\,dx = \frac{1}{4}x^4\sqrt{\cos^{-1}(ax)} + \frac{1}{8}a\int \frac{x^4}{\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}\,dx$$

$$= \frac{1}{4}x^4\sqrt{\cos^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cos^4(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{8a^4}$$

$$= \frac{1}{4}x^4\sqrt{\cos^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}} + \frac{\cos(2x)}{2\sqrt{x}} + \frac{\cos(4x)}{8\sqrt{x}}\right)dx, x, \cos^{-1}(ax)\right)}{8a^4}$$

$$= -\frac{3\sqrt{\cos^{-1}(ax)}}{32a^4} + \frac{1}{4}x^4\sqrt{\cos^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cos(4x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{64a^4} - \frac{\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}}\,dx, x, \text{c}\right.}{16a^4}$$

$$= -\frac{3\sqrt{\cos^{-1}(ax)}}{32a^4} + \frac{1}{4}x^4\sqrt{\cos^{-1}(ax)} - \frac{\text{Subst}\left(\int \cos\left(4x^2\right)dx, x, \sqrt{\cos^{-1}(ax)}\right)}{32a^4} - \frac{\text{Subst}\left(\int \cos\left(2x^2\right.\right.}{8}$$

$$= -\frac{3\sqrt{\cos^{-1}(ax)}}{32a^4} + \frac{1}{4}x^4\sqrt{\cos^{-1}(ax)} - \frac{\sqrt{\frac{\pi}{2}}C\left(2\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{64a^4} - \frac{\sqrt{\pi}C\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{16a^4}$$

Mathematica [C] time = 0.109826, size = 125, normalized size = 1.32

$$-\frac{\sqrt{\cos^{-1}(ax)}\left(4i\sqrt{2}\cos^{-1}(ax)\text{Gamma}\left(\frac{3}{2},-2i\cos^{-1}(ax)\right) + 4\sqrt{2}\sqrt{\cos^{-1}(ax)^2}\text{Gamma}\left(\frac{3}{2},2i\cos^{-1}(ax)\right) + i\cos^{-1}(ax)\text{G}\right.}{128a^4\left(-i\cos^{-1}(ax)\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Sqrt[ArcCos[a*x]],x]
```

```
[Out] -(Sqrt[ArcCos[a*x]]*((4*I)*Sqrt[2]*ArcCos[a*x]*Gamma[3/2, (-2*I)*ArcCos[a*x
]] + 4*Sqrt[2]*Sqrt[ArcCos[a*x]^2]*Gamma[3/2, (2*I)*ArcCos[a*x]] + I*ArcCos
[a*x]*Gamma[3/2, (-4*I)*ArcCos[a*x]] + Sqrt[ArcCos[a*x]^2]*Gamma[3/2, (4*I)
*ArcCos[a*x]]))/(128*a^4*((-I)*ArcCos[a*x])^(3/2))
```

Maple [A] time = 0.08, size = 91, normalized size = 1.

$$\frac{1}{128\,a^4}\left(-\sqrt{2}\sqrt{\pi}\sqrt{\arccos(ax)}\operatorname{FresnelC}\left(2\,\frac{\sqrt{2}\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-8\,\sqrt{\pi}\sqrt{\arccos(ax)}\operatorname{FresnelC}\left(2\,\frac{\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+16\,\mathrm{arcc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccos(a*x)^(1/2),x)
```

```
[Out] 1/128/a^4/arccos(a*x)^(1/2)*(-2^(1/2)*Pi^(1/2)*arccos(a*x)^(1/2)*FresnelC(2
*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))-8*Pi^(1/2)*arccos(a*x)^(1/2)*FresnelC(
2*arccos(a*x)^(1/2)/Pi^(1/2))+16*arccos(a*x)*cos(2*arccos(a*x))+4*arccos(a*
x)*cos(4*arccos(a*x)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{\operatorname{acos}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acos(a*x)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(acos(a*x)), x)
```

Giac [B] time = 1.28833, size = 255, normalized size = 2.68

$$\frac{\sqrt{2}\sqrt{\pi}i\operatorname{erf}\left(-\sqrt{2}(i+1)\sqrt{\arccos(ax)}\right)}{256\,a^4(i-1)}+\frac{\sqrt{\pi}i\operatorname{erf}\left(-(i+1)\sqrt{\arccos(ax)}\right)}{32\,a^4(i-1)}+\frac{\sqrt{\arccos(ax)}e^{(4\,i\arccos(ax))}}{64\,a^4}+\frac{\sqrt{\arccos(ax)}e}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/256*sqrt(2)*sqrt(pi)*i*erf(-sqrt(2)*(i + 1)*sqrt(arccos(a*x)))/(a^4*(i -
1)) + 1/32*sqrt(pi)*i*erf(-(i + 1)*sqrt(arccos(a*x)))/(a^4*(i - 1)) + 1/64*
sqrt(arccos(a*x))*e^(4*i*arccos(a*x))/a^4 + 1/16*sqrt(arccos(a*x))*e^(2*i*a
rccos(a*x))/a^4 + 1/16*sqrt(arccos(a*x))*e^(-2*i*arccos(a*x))/a^4 + 1/64*sq
rt(arccos(a*x))*e^(-4*i*arccos(a*x))/a^4 - 1/256*sqrt(2)*sqrt(pi)*erf(sqrt(
2)*(i - 1)*sqrt(arccos(a*x)))/(a^4*(i - 1)) - 1/32*sqrt(pi)*erf((i - 1)*sqr
t(arccos(a*x)))/(a^4*(i - 1))
```

3.76 $\int x^2\sqrt{\cos^{-1}(ax)}\,dx$

Optimal. Leaf size=86

$$-\frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{4a^3}-\frac{\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{12a^3}+\frac{1}{3}x^3\sqrt{\cos^{-1}(ax)}$$

```
[Out] (x^3*Sqrt[ArcCos[a*x]])/3 - (Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x
]]])/(4*a^3) - (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(12*a^3)
```

Rubi [A] time = 0.183938, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4630, 4724, 3312, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{4a^3}-\frac{\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{12a^3}+\frac{1}{3}x^3\sqrt{\cos^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[ArcCos[a*x]],x]
```

```
[Out] (x^3*Sqrt[ArcCos[a*x]])/3 - (Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x
]]])/(4*a^3) - (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(12*a^3)
```

Rule 4630

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
x^(m + 1)*(a + b*ArcCos[c*x])^n)/(m + 1), x] + Dist[(b*c*n)/(m + 1), Int[(x
^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int x^2\sqrt{\cos^{-1}(ax)}\,dx &= \frac{1}{3}x^3\sqrt{\cos^{-1}(ax)}+\frac{1}{6}a\int\frac{x^3}{\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}\,dx\\
&= \frac{1}{3}x^3\sqrt{\cos^{-1}(ax)}-\frac{\operatorname{Subst}\left(\int\frac{\cos^3(x)}{\sqrt{x}}\,dx,x,\cos^{-1}(ax)\right)}{6a^3}\\
&= \frac{1}{3}x^3\sqrt{\cos^{-1}(ax)}-\frac{\operatorname{Subst}\left(\int\left(\frac{3\cos(x)}{4\sqrt{x}}+\frac{\cos(3x)}{4\sqrt{x}}\right)dx,x,\cos^{-1}(ax)\right)}{6a^3}\\
&= \frac{1}{3}x^3\sqrt{\cos^{-1}(ax)}-\frac{\operatorname{Subst}\left(\int\frac{\cos(3x)}{\sqrt{x}}\,dx,x,\cos^{-1}(ax)\right)}{24a^3}-\frac{\operatorname{Subst}\left(\int\frac{\cos(x)}{\sqrt{x}}\,dx,x,\cos^{-1}(ax)\right)}{8a^3}\\
&= \frac{1}{3}x^3\sqrt{\cos^{-1}(ax)}-\frac{\operatorname{Subst}\left(\int\cos\left(3x^2\right)dx,x,\sqrt{\cos^{-1}(ax)}\right)}{12a^3}-\frac{\operatorname{Subst}\left(\int\cos\left(x^2\right)dx,x,\sqrt{\cos^{-1}(ax}\right.}{4a^3}\\
&= \frac{1}{3}x^3\sqrt{\cos^{-1}(ax)}-\frac{\sqrt{\frac{\pi}{2}}C\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{4a^3}-\frac{\sqrt{\frac{\pi}{6}}C\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{12a^3}
\end{aligned}$$

Mathematica [C] time = 0.185464, size = 122, normalized size = 1.42

$$\frac{\sqrt{i\cos^{-1}(ax)}\left(9\sqrt{\cos^{-1}(ax)^2}\operatorname{Gamma}\left(\frac{3}{2},-i\cos^{-1}(ax)\right)-9i\cos^{-1}(ax)\operatorname{Gamma}\left(\frac{3}{2},i\cos^{-1}(ax)\right)+\sqrt{3}\left(\sqrt{\cos^{-1}(ax)^2}\operatorname{Gam}\right.\right.}{72a^3\cos^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[ArcCos[a*x]],x]
```

```
[Out] (Sqrt[I*ArcCos[a*x]]*(9*Sqrt[ArcCos[a*x]^2]*Gamma[3/2, (-I)*ArcCos[a*x]] -
(9*I)*ArcCos[a*x]*Gamma[3/2, I*ArcCos[a*x]] + Sqrt[3]*(Sqrt[ArcCos[a*x]^2]*
Gamma[3/2, (-3*I)*ArcCos[a*x]] - I*ArcCos[a*x]*Gamma[3/2, (3*I)*ArcCos[a*x]
])))/(72*a^3*ArcCos[a*x]^(3/2))
```

Maple [A] time = 0.079, size = 96, normalized size = 1.1

$$\frac{1}{72\,a^3}\left(-\sqrt{3}\sqrt{2}\sqrt{\arccos(ax)}\sqrt{\pi}\mathrm{FresnelC}\left(\frac{\sqrt{3}\sqrt{2}}{\sqrt{\pi}}\sqrt{\arccos(ax)}\right)-9\,\sqrt{2}\sqrt{\arccos(ax)}\sqrt{\pi}\mathrm{FresnelC}\left(\frac{\sqrt{2}\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccos(a*x)^(1/2),x)
```

```
[Out] 1/72/a^3/arccos(a*x)^(1/2)*(-3^(1/2)*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*Fre
snelC(2^(1/2)/Pi^(1/2)*3^(1/2)*arccos(a*x)^(1/2))-9*2^(1/2)*arccos(a*x)^(1/
2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))+18*a*x*arccos(a*x)
+6*arccos(a*x)*cos(3*arccos(a*x)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sqrt{\operatorname{acos}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acos(a*x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(acos(a*x)), x)
```

Giac [B] time = 1.27354, size = 284, normalized size = 3.3

$$\frac{\sqrt{6}\sqrt{\pi} i \operatorname{erf}\left(\frac{\sqrt{6}\sqrt{\arccos(ax)}}{i-1}\right)}{144\, a^3(i-1)} + \frac{\sqrt{2}\sqrt{\pi} i \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\arccos(ax)}}{i-1}\right)}{16\, a^3(i-1)} + \frac{\sqrt{\arccos(ax)} e^{(3\, i \arccos(ax))}}{24\, a^3} + \frac{\sqrt{\arccos(ax)} e^{(i \arccos(ax))}}{8\, a^3} + \sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/144*sqrt(6)*sqrt(pi)*i*erf(sqrt(6)*sqrt(arccos(a*x))/(i - 1))/(a^3*(i - 1
)) + 1/16*sqrt(2)*sqrt(pi)*i*erf(sqrt(2)*sqrt(arccos(a*x))/(i - 1))/(a^3*(i
 - 1)) + 1/24*sqrt(arccos(a*x))*e^(3*i*arccos(a*x))/a^3 + 1/8*sqrt(arccos(a
*x))*e^(i*arccos(a*x))/a^3 + 1/8*sqrt(arccos(a*x))*e^(-i*arccos(a*x))/a^3 +
 1/24*sqrt(arccos(a*x))*e^(-3*i*arccos(a*x))/a^3 - 1/144*sqrt(6)*sqrt(pi)*e
rf(-sqrt(6)*i*sqrt(arccos(a*x))/(i - 1))/(a^3*(i - 1)) - 1/16*sqrt(2)*sqrt(
pi)*erf(-sqrt(2)*i*sqrt(arccos(a*x))/(i - 1))/(a^3*(i - 1))
```

3.77 $\int x\sqrt{\cos^{-1}(ax)}\,dx$

Optimal. Leaf size=59

$$-\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{8a^2}-\frac{\sqrt{\cos^{-1}(ax)}}{4a^2}+\frac{1}{2}x^2\sqrt{\cos^{-1}(ax)}$$

```
[Out] -Sqrt[ArcCos[a*x]]/(4*a^2) + (x^2*Sqrt[ArcCos[a*x]])/2 - (Sqrt[Pi]*FresnelC
[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/(8*a^2)
```

Rubi [A] time = 0.150872, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4630, 4724, 3312, 3304, 3352}

$$-\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{8a^2}-\frac{\sqrt{\cos^{-1}(ax)}}{4a^2}+\frac{1}{2}x^2\sqrt{\cos^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[ArcCos[a*x]],x]
```

```
[Out] -Sqrt[ArcCos[a*x]]/(4*a^2) + (x^2*Sqrt[ArcCos[a*x]])/2 - (Sqrt[Pi]*FresnelC
[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/(8*a^2)
```

Rule 4630

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
x^(m + 1)*(a + b*ArcCos[c*x])^n)/(m + 1), x] + Dist[(b*c*n)/(m + 1), Int[(x
^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{\cos^{-1}(ax)}\,dx &= \frac{1}{2}x^2\sqrt{\cos^{-1}(ax)} + \frac{1}{4}a\int \frac{x^2}{\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}\,dx \\
&= \frac{1}{2}x^2\sqrt{\cos^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cos^2(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{4a^2} \\
&= \frac{1}{2}x^2\sqrt{\cos^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}} + \frac{\cos(2x)}{2\sqrt{x}}\right) dx, x, \cos^{-1}(ax)\right)}{4a^2} \\
&= -\frac{\sqrt{\cos^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\cos^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{8a^2} \\
&= -\frac{\sqrt{\cos^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\cos^{-1}(ax)} - \frac{\text{Subst}\left(\int \cos\left(2x^2\right) dx, x, \sqrt{\cos^{-1}(ax)}\right)}{4a^2} \\
&= -\frac{\sqrt{\cos^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\cos^{-1}(ax)} - \frac{\sqrt{\pi}C\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{8a^2}
\end{aligned}$$

Mathematica [A] time = 0.0364452, size = 49, normalized size = 0.83

$$\frac{\frac{1}{4}\sqrt{\cos^{-1}(ax)}\cos\left(2\cos^{-1}(ax)\right) - \frac{1}{8}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[ArcCos[a*x]],x]
```

```
[Out] ((Sqrt[ArcCos[a*x]]*Cos[2*ArcCos[a*x]])/4 - (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcC
os[a*x]])/Sqrt[Pi]])/8)/a^2
```

Maple [A] time = 0.066, size = 42, normalized size = 0.7

$$-\frac{1}{8\,a^{2}\sqrt{\pi}}\left(\pi\,\mathrm{FresnelC}\left(2\,\frac{\sqrt{\arccos\left(ax\right)}}{\sqrt{\pi}}\right)-2\,\sqrt{\pi}\sqrt{\arccos\left(ax\right)}\cos\left(2\,\arccos\left(ax\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccos(a*x)^(1/2),x)
```

```
[Out] -1/8/a^2/Pi^(1/2)*(Pi*FresnelC(2*arccos(a*x)^(1/2)/Pi^(1/2))-2*Pi^(1/2)*arc
cos(a*x)^(1/2)*cos(2*arccos(a*x)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\operatorname{acos}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acos(a*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(acos(a*x)), x)
```

Giac [B] time = 1.2481, size = 120, normalized size = 2.03

$$\frac{\sqrt{\pi}i\,\mathrm{erf}\left(-(i+1)\sqrt{\arccos(ax)}\right)}{16\,a^2(i-1)}+\frac{\sqrt{\arccos(ax)}e^{(2\,i\arccos(ax))}}{8\,a^2}+\frac{\sqrt{\arccos(ax)}e^{(-2\,i\arccos(ax))}}{8\,a^2}-\frac{\sqrt{\pi}\,\mathrm{erf}\left((i-1)\sqrt{\arccos(\dots}\right.}{16\,a^2(i-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(pi)*i*erf(-(i + 1)*sqrt(arccos(a*x)))/(a^2*(i - 1)) + 1/8*sqrt(ar
ccos(a*x))*e^(2*i*arccos(a*x))/a^2 + 1/8*sqrt(arccos(a*x))*e^(-2*i*arccos(a
*x))/a^2 - 1/16*sqrt(pi)*erf((i - 1)*sqrt(arccos(a*x)))/(a^2*(i - 1))
```

3.78 $\int \sqrt{\cos^{-1}(ax)}\,dx$

Optimal. Leaf size=44

$$x\sqrt{\cos^{-1}(ax)} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a}$$

```
[Out] x*Sqrt[ArcCos[a*x]] - (Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/a
```

Rubi [A] time = 0.0900672, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4620, 4724, 3304, 3352}

$$x\sqrt{\cos^{-1}(ax)} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcCos[a*x]],x]
```

```
[Out] x*Sqrt[ArcCos[a*x]] - (Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/a
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos^{-1}(ax)}\,dx &= x\sqrt{\cos^{-1}(ax)}+\frac{1}{2}a\int \frac{x}{\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}\,dx\\ &= x\sqrt{\cos^{-1}(ax)}-\frac{\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx,x,\cos^{-1}(ax)\right)}{2a}\\ &= x\sqrt{\cos^{-1}(ax)}-\frac{\text{Subst}\left(\int \cos\left(x^2\right)\,dx,x,\sqrt{\cos^{-1}(ax)}\right)}{a}\\ &= x\sqrt{\cos^{-1}(ax)}-\frac{\sqrt{\frac{\pi}{2}}C\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a}\end{aligned}$$

Mathematica [C] time = 0.0339111, size = 76, normalized size = 1.73

$$\frac{\sqrt{\cos^{-1}(ax)}\left(\sqrt{i\cos^{-1}(ax)}\text{Gamma}\left(\frac{3}{2},-i\cos^{-1}(ax)\right)+\sqrt{-i\cos^{-1}(ax)}\text{Gamma}\left(\frac{3}{2},i\cos^{-1}(ax)\right)\right)}{2a\sqrt{\cos^{-1}(ax)^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[ArcCos[a*x]],x]
```

```
[Out] (Sqrt[ArcCos[a*x]]*(Sqrt[I*ArcCos[a*x]]*Gamma[3/2, (-I)*ArcCos[a*x]] + Sqrt
[(-I)*ArcCos[a*x]]*Gamma[3/2, I*ArcCos[a*x]]))/(2*a*Sqrt[ArcCos[a*x]^2])
```

Maple [A] time = 0.065, size = 49, normalized size = 1.1

$$\frac{1}{2\,a}\left(-\sqrt{2}\sqrt{\arccos(ax)}\sqrt{\pi}\text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\sqrt{\arccos(ax)}\right)+2\,ax\arccos(ax)\right)\frac{1}{\sqrt{\arccos(ax)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^(1/2),x)
```

```
[Out] 1/2/a/arccos(a*x)^(1/2)*(-2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/
2)/Pi^(1/2)*arccos(a*x)^(1/2))+2*a*x*arccos(a*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\operatorname{acos}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(acos(a*x)), x)
```

<u>Giac</u> [B] time = 1.24525, size = 142, normalized size = 3.23

$$\frac{\sqrt{2}\sqrt{\pi}i\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{\arccos(ax)}}{i-1}\right)}{4\,a(i-1)}+\frac{\sqrt{\arccos(ax)}e^{(i\arccos(ax))}}{2\,a}+\frac{\sqrt{\arccos(ax)}e^{(-i\arccos(ax))}}{2\,a}-\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{\sqrt{2}i\sqrt{\arccos(ax)}}{i-1}\right)}{4\,a(i-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*sqrt(pi)*i*erf(sqrt(2)*sqrt(arccos(a*x))/(i - 1))/(a*(i - 1)) +
 1/2*sqrt(arccos(a*x))*e^(i*arccos(a*x))/a + 1/2*sqrt(arccos(a*x))*e^(-i*ar
ccos(a*x))/a - 1/4*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*i*sqrt(arccos(a*x))/(i - 1
))/(a*(i - 1))
```

3.79 $\int \frac{\sqrt{\cos^{-1}(ax)}}{x} \, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{\sqrt{\cos^{-1}(ax)}}{x}, x\right)$$

```
[Out] Unintegrable[Sqrt[ArcCos[a*x]]/x, x]
```

Rubi [A] time = 0.0123162, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{\cos^{-1}(ax)}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcCos[a*x]]/x,x]
```

```
[Out] Defer[Int][Sqrt[ArcCos[a*x]]/x, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos^{-1}(ax)}}{x} \, dx = \int \frac{\sqrt{\cos^{-1}(ax)}}{x} \, dx$$

Mathematica [A] time = 0.388462, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos^{-1}(ax)}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcCos[a*x]]/x,x]
```

```
[Out] Integrate[Sqrt[ArcCos[a*x]]/x, x]
```

Maple [A] time = 0.096, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{\arccos(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^(1/2)/x,x)
```

```
[Out] int(arccos(a*x)^(1/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{acos}(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(acos(a*x))/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\arccos{(ax)}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(arccos(a*x))/x, x)
```

3.80 $\int x^4 \cos^{-1}(ax)^{3/2}\, dx$

Optimal. Leaf size=282

$$\frac{2\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{25a^5}+\frac{11\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{400a^5}+\frac{3\sqrt{\frac{3\pi}{2}}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{800a^5}+\frac{\sqrt{\frac{\pi}{6}}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{50a^5}+\frac{3\sqrt{\frac{\pi}{10}}}{}$$

```
[Out] (-4*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])/(25*a^5) - (2*x^2*Sqrt[1 - a^2*x^2
]*Sqrt[ArcCos[a*x]])/(25*a^3) - (3*x^4*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])
/(50*a) + (x^5*ArcCos[a*x]^(3/2))/5 + (11*Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sq
rt[ArcCos[a*x]]])/(400*a^5) + (2*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos
[a*x]]])/(25*a^5) + (Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(50
*a^5) + (3*Sqrt[(3*Pi)/2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(800*a^5)
 + (3*Sqrt[Pi/10]*FresnelS[Sqrt[10/Pi]*Sqrt[ArcCos[a*x]]])/(800*a^5)
```

Rubi [A] time = 0.514535, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4630, 4708, 4678, 4624, 3305, 3351, 4636, 4406}

$$\frac{2\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{25a^5}+\frac{11\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{400a^5}+\frac{3\sqrt{\frac{3\pi}{2}}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{800a^5}+\frac{\sqrt{\frac{\pi}{6}}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{50a^5}+\frac{3\sqrt{\frac{\pi}{10}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCos[a*x]^(3/2),x]
```


```
[Out] (-4*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])/(25*a^5) - (2*x^2*Sqrt[1 - a^2*x^2
]*Sqrt[ArcCos[a*x]])/(25*a^3) - (3*x^4*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])
/(50*a) + (x^5*ArcCos[a*x]^(3/2))/5 + (11*Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sq
rt[ArcCos[a*x]]])/(400*a^5) + (2*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos
[a*x]]])/(25*a^5) + (Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(50
*a^5) + (3*Sqrt[(3*Pi)/2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(800*a^5)
 + (3*Sqrt[Pi/10]*FresnelS[Sqrt[10/Pi]*Sqrt[ArcCos[a*x]]])/(800*a^5)
```


Rule 4630

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
x^(m + 1)*(a + b*ArcCos[c*x])^n)/(m + 1), x] + Dist[(b*c*n)/(m + 1), Int[(x
^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4624

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Sub
st[Int[x^n*Sin[a/b - x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c,
 n}, x]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
```

```
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int x^4 \cos^{-1}(ax)^{3/2}\,dx = \frac{1}{5}x^5\cos^{-1}(ax)^{3/2} + \frac{1}{10}(3a)\int \frac{x^5\sqrt{\cos^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{3x^4\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{50a} + \frac{1}{5}x^5\cos^{-1}(ax)^{3/2} - \frac{3}{100}\int \frac{x^4}{\sqrt{\cos^{-1}(ax)}}\,dx + \frac{6\int \frac{x^3\sqrt{\cos^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx}{25a}$$

$$= -\frac{2x^2\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{50a} + \frac{1}{5}x^5\cos^{-1}(ax)^{3/2} + \frac{3\,\mathrm{Subst}\left(\int \frac{\cos^4(x)}{\sqrt{}}\right.}{1}$$

$$= -\frac{4\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{25a^5} - \frac{2x^2\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{50a} + \frac{1}{5}x^5\cos^{-1}($$

$$= -\frac{4\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{25a^5} - \frac{2x^2\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{50a} + \frac{1}{5}x^5\cos^{-1}($$

$$= -\frac{4\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{25a^5} - \frac{2x^2\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{50a} + \frac{1}{5}x^5\cos^{-1}($$

$$= -\frac{4\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{25a^5} - \frac{2x^2\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{50a} + \frac{1}{5}x^5\cos^{-1}($$

$$= -\frac{4\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{25a^5} - \frac{2x^2\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{50a} + \frac{1}{5}x^5\cos^{-1}($$

Mathematica [C] time = 0.123592, size = 185, normalized size = 0.66

$$-\frac{2250\left(\sqrt{-i\cos^{-1}(ax)}\mathrm{Gamma}\left(\frac{5}{2},-i\cos^{-1}(ax)\right)+\sqrt{i\cos^{-1}(ax)}\mathrm{Gamma}\left(\frac{5}{2},i\cos^{-1}(ax)\right)\right)+125\sqrt{3}\left(\sqrt{-i\cos^{-1}(ax)}\mathrm{Ga}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*ArcCos[a*x]^(3/2),x]
```

```
[Out] -(2250*(Sqrt[(-I)*ArcCos[a*x]]*Gamma[5/2, (-I)*ArcCos[a*x]] + Sqrt[I*ArcCos
[a*x]]*Gamma[5/2, I*ArcCos[a*x]]) + 125*Sqrt[3]*(Sqrt[(-I)*ArcCos[a*x]]*Gam
```

```
ma[5/2, (-3*I)*ArcCos[a*x]] + Sqrt[I*ArcCos[a*x]]*Gamma[5/2, (3*I)*ArcCos[a
*x]]) + 9*Sqrt[5]*(Sqrt[(-I)*ArcCos[a*x]]*Gamma[5/2, (-5*I)*ArcCos[a*x]] +
Sqrt[I*ArcCos[a*x]]*Gamma[5/2, (5*I)*ArcCos[a*x]]))/(36000*a^5*Sqrt[ArcCos[
a*x]])
```

Maple [A] time = 0.104, size = 193, normalized size = 0.7

$$\frac{1}{24000\,a^5}\left(3000\,ax\left(\arccos\left(ax\right)\right)^2+9\,\sqrt{5}\sqrt{2}\sqrt{\arccos\left(ax\right)}\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{\sqrt{5}\sqrt{2}\sqrt{\arccos\left(ax\right)}}{\sqrt{\pi}}\right)+125\,\sqrt{3}\sqrt{2}\sqrt{\arccos\left(ax\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccos(a*x)^(3/2),x)
```

```
[Out] 1/24000/a^5*(3000*a*x*arccos(a*x)^2+9*5^(1/2)*2^(1/2)*arccos(a*x)^(1/2)*Pi^
(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)*arccos(a*x)^(1/2))+125*3^(1/2)*2^(1
/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*arccos(a*x
)^(1/2))+1500*arccos(a*x)^2*cos(3*arccos(a*x))+300*arccos(a*x)^2*cos(5*arcc
os(a*x))+2250*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*
arccos(a*x)^(1/2))-4500*arccos(a*x)*(-a^2*x^2+1)^(1/2)-90*arccos(a*x)*sin(5
*arccos(a*x))-750*arccos(a*x)*sin(3*arccos(a*x)))/arccos(a*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccos(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccos(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acos(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34783, size = 586, normalized size = 2.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccos(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 3/1600*i*sqrt(arccos(a*x))*e^(5*i*arccos(a*x))/a^5 + 1/160*arccos(a*x)^(3/2
)*e^(5*i*arccos(a*x))/a^5 + 1/64*i*sqrt(arccos(a*x))*e^(3*i*arccos(a*x))/a^
5 + 1/32*arccos(a*x)^(3/2)*e^(3*i*arccos(a*x))/a^5 + 3/32*i*sqrt(arccos(a*x
))*e^(i*arccos(a*x))/a^5 + 1/16*arccos(a*x)^(3/2)*e^(i*arccos(a*x))/a^5 - 3
/32*i*sqrt(arccos(a*x))*e^(-i*arccos(a*x))/a^5 + 1/16*arccos(a*x)^(3/2)*e^(
-i*arccos(a*x))/a^5 - 1/64*i*sqrt(arccos(a*x))*e^(-3*i*arccos(a*x))/a^5 + 1
/32*arccos(a*x)^(3/2)*e^(-3*i*arccos(a*x))/a^5 - 3/1600*i*sqrt(arccos(a*x))
*e^(-5*i*arccos(a*x))/a^5 + 1/160*arccos(a*x)^(3/2)*e^(-5*i*arccos(a*x))/a^
5 - 3/16000*sqrt(10)*sqrt(pi)*i*erf(-sqrt(10)*i*sqrt(arccos(a*x))/(i - 1))/
(a^5*(i - 1)) - 1/384*sqrt(6)*sqrt(pi)*i*erf(-sqrt(6)*i*sqrt(arccos(a*x))/(
i - 1))/(a^5*(i - 1)) - 3/64*sqrt(2)*sqrt(pi)*i*erf(-sqrt(2)*i*sqrt(arccos(
a*x))/(i - 1))/(a^5*(i - 1)) + 3/16000*sqrt(10)*sqrt(pi)*erf(sqrt(10)*sqrt(
arccos(a*x))/(i - 1))/(a^5*(i - 1)) + 1/384*sqrt(6)*sqrt(pi)*erf(sqrt(6)*sq
rt(arccos(a*x))/(i - 1))/(a^5*(i - 1)) + 3/64*sqrt(2)*sqrt(pi)*erf(sqrt(2)*
sqrt(arccos(a*x))/(i - 1))/(a^5*(i - 1))
```

3.81 $\int x^3 \cos^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=157

$$\frac{3\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{512a^4}+\frac{3\sqrt{\pi}S\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{64a^4}-\frac{3x^3\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{32a}-\frac{9x\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{64a^3}-\frac{3\cos^{-1}(ax)^{3/2}}{32a^4}$$

```
[Out] (-9*x*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])/(64*a^3) - (3*x^3*Sqrt[1 - a^2*x
^2]*Sqrt[ArcCos[a*x]])/(32*a) - (3*ArcCos[a*x]^(3/2))/(32*a^4) + (x^4*ArcCo
s[a*x]^(3/2))/4 + (3*Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(
512*a^4) + (3*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/(64*a^4)
```

Rubi [A] time = 0.367786, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4630, 4708, 4642, 4636, 4406, 12, 3305, 3351}

$$\frac{3\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{512a^4}+\frac{3\sqrt{\pi}S\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{64a^4}-\frac{3x^3\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{32a}-\frac{9x\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{64a^3}-\frac{3\cos^{-1}(ax)^{3/2}}{32a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCos[a*x]^(3/2),x]
```


```
[Out] (-9*x*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])/(64*a^3) - (3*x^3*Sqrt[1 - a^2*x
^2]*Sqrt[ArcCos[a*x]])/(32*a) - (3*ArcCos[a*x]^(3/2))/(32*a^4) + (x^4*ArcCo
s[a*x]^(3/2))/4 + (3*Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(
512*a^4) + (3*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/(64*a^4)
```


Rule 4630

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
x^(m + 1)*(a + b*ArcCos[c*x])^n)/(m + 1), x] + Dist[(b*c*n)/(m + 1), Int[(x
^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
```

```
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int x^3\cos^{-1}(ax)^{3/2}\,dx = \frac{1}{4}x^4\cos^{-1}(ax)^{3/2} + \frac{1}{8}(3a)\int \frac{x^4\sqrt{\cos^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{3x^3\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{32a} + \frac{1}{4}x^4\cos^{-1}(ax)^{3/2} - \frac{3}{64}\int \frac{x^3}{\sqrt{\cos^{-1}(ax)}}\,dx + \frac{9\int \frac{x^2\sqrt{\cos^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx}{32a}$$

$$= -\frac{9x\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{32a} + \frac{1}{4}x^4\cos^{-1}(ax)^{3/2} + \frac{3\,\mathrm{Subst}\left(\int \frac{\cos^3(x)\sin}{\sqrt{x}}\right.}{64}$$

$$= -\frac{9x\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{32a} - \frac{3\cos^{-1}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4\cos^{-1}(ax)^{3/2} + \frac{3\,\mathrm{S}}{}$$

$$= -\frac{9x\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{32a} - \frac{3\cos^{-1}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4\cos^{-1}(ax)^{3/2} + \frac{3\,\mathrm{S}}{}$$

$$= -\frac{9x\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{32a} - \frac{3\cos^{-1}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4\cos^{-1}(ax)^{3/2} + \frac{3\,\mathrm{S}}{}$$

$$= -\frac{9x\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{32a} - \frac{3\cos^{-1}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4\cos^{-1}(ax)^{3/2} + \frac{3\sqrt{}}{}$$

$$= -\frac{9x\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{32a} - \frac{3\cos^{-1}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4\cos^{-1}(ax)^{3/2} + \frac{3\sqrt{}}{}$$

Mathematica [C] time = 0.0724389, size = 128, normalized size = 0.82

$$-\frac{8\sqrt{2}\sqrt{-i\cos^{-1}(ax)}\mathrm{Gamma}\left(\frac{5}{2},-2i\cos^{-1}(ax)\right) + 8\sqrt{2}\sqrt{i\cos^{-1}(ax)}\mathrm{Gamma}\left(\frac{5}{2},2i\cos^{-1}(ax)\right) + \sqrt{-i\cos^{-1}(ax)}\mathrm{Gamma}}{512a^4\sqrt{\cos^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*ArcCos[a*x]^(3/2),x]
```

```
[Out] -(8*Sqrt[2]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[5/2, (-2*I)*ArcCos[a*x]] + 8*Sqrt[
2]*Sqrt[I*ArcCos[a*x]]*Gamma[5/2, (2*I)*ArcCos[a*x]] + Sqrt[(-I)*ArcCos[a*x
]]*Gamma[5/2, (-4*I)*ArcCos[a*x]] + Sqrt[I*ArcCos[a*x]]*Gamma[5/2, (4*I)*Ar
cCos[a*x]])/(512*a^4*Sqrt[ArcCos[a*x]])
```

Maple [A] time = 0.089, size = 121, normalized size = 0.8

$$\frac{1}{1024\,a^4}\left(3\,\sqrt{2}\sqrt{\pi}\sqrt{\arccos(ax)}\mathrm{FresnelS}\left(2\,\frac{\sqrt{2}\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+128\,(\arccos(ax))^2\cos(2\,\arccos(ax))+32\,(\arccos(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccos(a*x)^(3/2),x)
```

```
[Out] 1/1024/a^4*(3*2^(1/2)*Pi^(1/2)*arccos(a*x)^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2
)*arccos(a*x)^(1/2))+128*arccos(a*x)^2*cos(2*arccos(a*x))+32*arccos(a*x)^2*
cos(4*arccos(a*x))+48*Pi^(1/2)*arccos(a*x)^(1/2)*FresnelS(2*arccos(a*x)^(1/
2)/Pi^(1/2))-12*arccos(a*x)*sin(4*arccos(a*x))-96*arccos(a*x)*sin(2*arccos(
a*x)))/arccos(a*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{acos}^{\frac{3}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acos(a*x)**(3/2),x)
```

```
[Out] Integral(x**3*acos(a*x)**(3/2), x)
```

Giac [B] time = 1.32685, size = 363, normalized size = 2.31

$$\frac{3\,i\sqrt{\arccos\left(ax\right)}e^{\left(4\,i\arccos\left(ax\right)\right)}}{512\,a^4}+\frac{\arccos\left(ax\right)^{\frac{3}{2}}e^{\left(4\,i\arccos\left(ax\right)\right)}}{64\,a^4}+\frac{3\,i\sqrt{\arccos\left(ax\right)}e^{\left(2\,i\arccos\left(ax\right)\right)}}{64\,a^4}+\frac{\arccos\left(ax\right)^{\frac{3}{2}}e^{\left(2\,i\arccos\left(ax\right)\right)}}{16\,a^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 3/512*i*sqrt(arccos(a*x))*e^(4*i*arccos(a*x))/a^4 + 1/64*arccos(a*x)^(3/2)*
e^(4*i*arccos(a*x))/a^4 + 3/64*i*sqrt(arccos(a*x))*e^(2*i*arccos(a*x))/a^4
+ 1/16*arccos(a*x)^(3/2)*e^(2*i*arccos(a*x))/a^4 - 3/64*i*sqrt(arccos(a*x))
*e^(-2*i*arccos(a*x))/a^4 + 1/16*arccos(a*x)^(3/2)*e^(-2*i*arccos(a*x))/a^4
 - 3/512*i*sqrt(arccos(a*x))*e^(-4*i*arccos(a*x))/a^4 + 1/64*arccos(a*x)^(3
/2)*e^(-4*i*arccos(a*x))/a^4 - 3/2048*sqrt(2)*sqrt(pi)*i*erf(sqrt(2)*(i - 1
)*sqrt(arccos(a*x)))/(a^4*(i - 1)) - 3/128*sqrt(pi)*i*erf((i - 1)*sqrt(arcc
os(a*x)))/(a^4*(i - 1)) + 3/2048*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*(i + 1)*sqrt
(arccos(a*x)))/(a^4*(i - 1)) + 3/128*sqrt(pi)*erf(-(i + 1)*sqrt(arccos(a*x)
))/(a^4*(i - 1))
```

3.82 $\int x^2 \cos^{-1}(ax)^{3/2}\, dx$

Optimal. Leaf size=147

$$\frac{3\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{8a^3}+\frac{\sqrt{\frac{\pi}{6}}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{24a^3}-\frac{x^2\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{6a}-\frac{\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{3a^3}+\frac{1}{3}x^3\cos^{-1}(ax)$$

```
[Out] -(Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])/(3*a^3) - (x^2*Sqrt[1 - a^2*x^2]*Sqr
t[ArcCos[a*x]])/(6*a) + (x^3*ArcCos[a*x]^(3/2))/3 + (3*Sqrt[Pi/2]*FresnelS[
Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(8*a^3) + (Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sq
rt[ArcCos[a*x]]])/(24*a^3)
```

Rubi [A] time = 0.302481, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4630, 4708, 4678, 4624, 3305, 3351, 4636, 4406}

$$\frac{3\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{8a^3}+\frac{\sqrt{\frac{\pi}{6}}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{24a^3}-\frac{x^2\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{6a}-\frac{\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{3a^3}+\frac{1}{3}x^3\cos^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCos[a*x]^(3/2),x]
```


```
[Out] -(Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])/(3*a^3) - (x^2*Sqrt[1 - a^2*x^2]*Sqr
t[ArcCos[a*x]])/(6*a) + (x^3*ArcCos[a*x]^(3/2))/3 + (3*Sqrt[Pi/2]*FresnelS[
Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(8*a^3) + (Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sq
rt[ArcCos[a*x]]])/(24*a^3)
```


Rule 4630

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
x^(m + 1)*(a + b*ArcCos[c*x])^n)/(m + 1), x] + Dist[(b*c*n)/(m + 1), Int[(x
^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
```

```
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4624

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Sub
st[Int[x^n*Sin[a/b - x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c,
 n}, x]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int x^2 \cos^{-1}(ax)^{3/2}\,dx = \frac{1}{3}x^3\cos^{-1}(ax)^{3/2} + \frac{1}{2}a\int \frac{x^3\sqrt{\cos^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{x^2\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{6a} + \frac{1}{3}x^3\cos^{-1}(ax)^{3/2} - \frac{1}{12}\int \frac{x^2}{\sqrt{\cos^{-1}(ax)}}\,dx + \frac{\int \frac{x\sqrt{\cos^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx}{3a}$$

$$= -\frac{\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{3a^3} - \frac{x^2\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{6a} + \frac{1}{3}x^3\cos^{-1}(ax)^{3/2} + \frac{\text{Subst}\left(\int \frac{\cos^2(x)\sin(x)}{\sqrt{x}}\,d\right.}{12a^3}$$

$$= -\frac{\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{3a^3} - \frac{x^2\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{6a} + \frac{1}{3}x^3\cos^{-1}(ax)^{3/2} + \frac{\text{Subst}\left(\int\left(\frac{\sin(x)}{4\sqrt{x}} + \frac{\sin(}{4\sqrt{}}\right.\right.}{12a}$$

$$= -\frac{\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{3a^3} - \frac{x^2\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{6a} + \frac{1}{3}x^3\cos^{-1}(ax)^{3/2} + \frac{\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, c\right.}{48a^3}$$

$$= -\frac{\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{3a^3} - \frac{x^2\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{6a} + \frac{1}{3}x^3\cos^{-1}(ax)^{3/2} + \frac{\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right.}{3a^3}$$

$$= -\frac{\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{3a^3} - \frac{x^2\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{6a} + \frac{1}{3}x^3\cos^{-1}(ax)^{3/2} + \frac{3\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax}\right.}{8a^3}$$

Mathematica [C] time = 0.0862844, size = 125, normalized size = 0.85

$$-\frac{27\sqrt{-i\cos^{-1}(ax)}\text{Gamma}\left(\frac{5}{2}, -i\cos^{-1}(ax)\right) + 27\sqrt{i\cos^{-1}(ax)}\text{Gamma}\left(\frac{5}{2}, i\cos^{-1}(ax)\right) + \sqrt{3}\left(\sqrt{-i\cos^{-1}(ax)}\text{Gamma}\right.}{216a^3\sqrt{\cos^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*ArcCos[a*x]^(3/2),x]
```

```
[Out] -(27*Sqrt[(-I)*ArcCos[a*x]]*Gamma[5/2, (-I)*ArcCos[a*x]] + 27*Sqrt[I*ArcCos
[a*x]]*Gamma[5/2, I*ArcCos[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcCos[a*x]]*Gamma[5/
2, (-3*I)*ArcCos[a*x]] + Sqrt[I*ArcCos[a*x]]*Gamma[5/2, (3*I)*ArcCos[a*x]])
)/(216*a^3*Sqrt[ArcCos[a*x]])
```

Maple [A] time = 0.087, size = 130, normalized size = 0.9

$$\frac{1}{144\,a^3}\left(36\,ax\,(\arccos(ax))^2 + \sqrt{3}\sqrt{2}\sqrt{\arccos(ax)}\sqrt{\pi}\text{FresnelS}\left(\frac{\sqrt{3}\sqrt{2}}{\sqrt{\pi}}\sqrt{\arccos(ax)}\right) + 12\,(\arccos(ax))^2\cos(3\,\text{arcc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccos(a*x)^(3/2),x)
```

```
[Out] 1/144/a^3*(36*a*x*arccos(a*x)^2+3^(1/2)*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*
FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*arccos(a*x)^(1/2))+12*arccos(a*x)^2*cos(3
*arccos(a*x))+27*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/
2)*arccos(a*x)^(1/2))-54*arccos(a*x)*(-a^2*x^2+1)^(1/2)-6*arccos(a*x)*sin(3
*arccos(a*x)))/arccos(a*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{acos}^{\frac{3}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acos(a*x)**(3/2),x)
```

```
[Out] Integral(x**2*acos(a*x)**(3/2), x)
```

Giac [B] time = 1.29681, size = 390, normalized size = 2.65

$$\frac{i\sqrt{\arccos(ax)}e^{(3\,i\arccos(ax))}}{48\,a^3}+\frac{\arccos(ax)^{\frac{3}{2}}e^{(3\,i\arccos(ax))}}{24\,a^3}+\frac{3\,i\sqrt{\arccos(ax)}e^{(i\arccos(ax))}}{16\,a^3}+\frac{\arccos(ax)^{\frac{3}{2}}e^{(i\arccos(ax))}}{8\,a^3}-\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*i*sqrt(arccos(a*x))*e^(3*i*arccos(a*x))/a^3 + 1/24*arccos(a*x)^(3/2)*e
^(3*i*arccos(a*x))/a^3 + 3/16*i*sqrt(arccos(a*x))*e^(i*arccos(a*x))/a^3 + 1
/8*arccos(a*x)^(3/2)*e^(i*arccos(a*x))/a^3 - 3/16*i*sqrt(arccos(a*x))*e^(-i
*arccos(a*x))/a^3 + 1/8*arccos(a*x)^(3/2)*e^(-i*arccos(a*x))/a^3 - 1/48*i*s
qrt(arccos(a*x))*e^(-3*i*arccos(a*x))/a^3 + 1/24*arccos(a*x)^(3/2)*e^(-3*i*
arccos(a*x))/a^3 - 1/288*sqrt(6)*sqrt(pi)*i*erf(-sqrt(6)*i*sqrt(arccos(a*x)
)/(i - 1))/(a^3*(i - 1)) - 3/32*sqrt(2)*sqrt(pi)*i*erf(-sqrt(2)*i*sqrt(arcc
os(a*x))/(i - 1))/(a^3*(i - 1)) + 1/288*sqrt(6)*sqrt(pi)*erf(sqrt(6)*sqrt(a
rccos(a*x))/(i - 1))/(a^3*(i - 1)) + 3/32*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt
(arccos(a*x))/(i - 1))/(a^3*(i - 1))
```

3.83 $\int x \cos^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=89

$$\frac{3\sqrt{\pi}S\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{32a^2}-\frac{3x\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{8a}-\frac{\cos^{-1}(ax)^{3/2}}{4a^2}+\frac{1}{2}x^2\cos^{-1}(ax)^{3/2}$$

```
[Out] (-3*x*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])/(8*a) - ArcCos[a*x]^(3/2)/(4*a^2
) + (x^2*ArcCos[a*x]^(3/2))/2 + (3*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/
Sqrt[Pi]])/(32*a^2)
```

Rubi [A] time = 0.17861, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.8$, Rules used = {4630, 4708, 4642, 4636, 4406, 12, 3305, 3351}

$$\frac{3\sqrt{\pi}S\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{32a^2}-\frac{3x\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{8a}-\frac{\cos^{-1}(ax)^{3/2}}{4a^2}+\frac{1}{2}x^2\cos^{-1}(ax)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCos[a*x]^(3/2),x]
```

```
[Out] (-3*x*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])/(8*a) - ArcCos[a*x]^(3/2)/(4*a^2
) + (x^2*ArcCos[a*x]^(3/2))/2 + (3*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/
Sqrt[Pi]])/(32*a^2)
```

Rule 4630

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
x^(m + 1)*(a + b*ArcCos[c*x])^n)/(m + 1), x] + Dist[(b*c*n)/(m + 1), Int[(x
^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

```
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int x\cos^{-1}(ax)^{3/2}\,dx = \frac{1}{2}x^2\cos^{-1}(ax)^{3/2} + \frac{1}{4}(3a)\int \frac{x^2\sqrt{\cos^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{3x\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{8a} + \frac{1}{2}x^2\cos^{-1}(ax)^{3/2} - \frac{3}{16}\int \frac{x}{\sqrt{\cos^{-1}(ax)}}\,dx + \frac{3\int \frac{\sqrt{\cos^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx}{8a}$$

$$= -\frac{3x\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{8a} - \frac{\cos^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\cos^{-1}(ax)^{3/2} + \frac{3\,\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(\right.}{16a^2}$$

$$= -\frac{3x\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{8a} - \frac{\cos^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\cos^{-1}(ax)^{3/2} + \frac{3\,\text{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{16a^2}$$

$$= -\frac{3x\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{8a} - \frac{\cos^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\cos^{-1}(ax)^{3/2} + \frac{3\,\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{32a^2}$$

$$= -\frac{3x\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{8a} - \frac{\cos^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\cos^{-1}(ax)^{3/2} + \frac{3\,\text{Subst}\left(\int \sin\left(2x^2\right)dx, x, \sqrt{\cos^{-}}\right.}{16a^2}$$

$$= -\frac{3x\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{8a} - \frac{\cos^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\cos^{-1}(ax)^{3/2} + \frac{3\sqrt{\pi}S\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{32a^2}$$

Mathematica [A] time = 0.0657384, size = 64, normalized size = 0.72

$$\frac{3\sqrt{\pi}S\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right) - 2\sqrt{\cos^{-1}(ax)}\left(3\sin\left(2\cos^{-1}(ax)\right) - 4\cos^{-1}(ax)\cos\left(2\cos^{-1}(ax)\right)\right)}{32a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCos[a*x]^(3/2),x]
```

```
[Out] (3*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]] - 2*Sqrt[ArcCos[a*x]]*
(-4*ArcCos[a*x]*Cos[2*ArcCos[a*x]] + 3*Sin[2*ArcCos[a*x]]))/(32*a^2)
```

Maple [A] time = 0.076, size = 64, normalized size = 0.7

$$\frac{1}{32\,a^2}\left(8\,(\arccos(ax))^2\cos(2\,\arccos(ax)) + 3\,\sqrt{\pi}\sqrt{\arccos(ax)}\text{FresnelS}\left(2\,\frac{\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right) - 6\,\arccos(ax)\sin(2\,\text{arcco}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccos(a*x)^(3/2),x)
```

```
[Out] 1/32/a^2*(8*arccos(a*x)^2*cos(2*arccos(a*x))+3*Pi^(1/2)*arccos(a*x)^(1/2)*F
resnelS(2*arccos(a*x)^(1/2)/Pi^(1/2))-6*arccos(a*x)*sin(2*arccos(a*x)))/arc
cos(a*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{acos}^{\frac{3}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acos(a*x)**(3/2),x)
```

```
[Out] Integral(x*acos(a*x)**(3/2), x)
```

Giac [A] time = 1.28056, size = 174, normalized size = 1.96

$$\frac{3\,i\sqrt{\arccos\left(ax\right)}e^{\left(2\,i\arccos\left(ax\right)\right)}}{32\,a^2}+\frac{\arccos\left(ax\right)^{\frac{3}{2}}\,e^{\left(2\,i\arccos\left(ax\right)\right)}}{8\,a^2}-\frac{3\,i\sqrt{\arccos\left(ax\right)}e^{\left(-2\,i\arccos\left(ax\right)\right)}}{32\,a^2}+\frac{\arccos\left(ax\right)^{\frac{3}{2}}\,e^{\left(-2\,i\arccos\left(ax\right)\right)}}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 3/32*i*sqrt(arccos(a*x))*e^(2*i*arccos(a*x))/a^2 + 1/8*arccos(a*x)^(3/2)*e^
(2*i*arccos(a*x))/a^2 - 3/32*i*sqrt(arccos(a*x))*e^(-2*i*arccos(a*x))/a^2 +
 1/8*arccos(a*x)^(3/2)*e^(-2*i*arccos(a*x))/a^2 - 3/64*sqrt(pi)*i*erf((i -
1)*sqrt(arccos(a*x)))/(a^2*(i - 1)) + 3/64*sqrt(pi)*erf(-(i + 1)*sqrt(arcco
s(a*x)))/(a^2*(i - 1))
```

3.84 $\int \cos^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=75

$$-\frac{3\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{2a}+\frac{3\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{2a}+x\cos^{-1}(ax)^{3/2}$$

```
[Out] (-3*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])/(2*a) + x*ArcCos[a*x]^(3/2) + (3*S
qrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(2*a)
```

Rubi [A] time = 0.100889, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4620, 4678, 4624, 3305, 3351}

$$-\frac{3\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{2a}+\frac{3\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{2a}+x\cos^{-1}(ax)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^(3/2),x]
```

```
[Out] (-3*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])/(2*a) + x*ArcCos[a*x]^(3/2) + (3*S
qrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(2*a)
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4624

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Sub
st[Int[x^n*Sin[a/b - x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c,
 n}, x]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^{-1}(ax)^{3/2}\,dx &= x\cos^{-1}(ax)^{3/2}+\frac{1}{2}(3a)\int \frac{x\sqrt{\cos^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{3\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{2a}+x\cos^{-1}(ax)^{3/2}-\frac{3}{4}\int \frac{1}{\sqrt{\cos^{-1}(ax)}}\,dx\\
&= -\frac{3\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{2a}+x\cos^{-1}(ax)^{3/2}+\frac{3\,\mathrm{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx,x,\cos^{-1}(ax)\right)}{4a}\\
&= -\frac{3\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{2a}+x\cos^{-1}(ax)^{3/2}+\frac{3\,\mathrm{Subst}\left(\int \sin\left(x^2\right)\,dx,x,\sqrt{\cos^{-1}(ax)}\right)}{2a}\\
&= -\frac{3\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}{2a}+x\cos^{-1}(ax)^{3/2}+\frac{3\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{2a}
\end{aligned}$$

Mathematica [C] time = 0.0259758, size = 66, normalized size = 0.88

$$-\frac{\sqrt{-i\cos^{-1}(ax)}\,\mathrm{Gamma}\left(\frac{5}{2},-i\cos^{-1}(ax)\right)+\sqrt{i\cos^{-1}(ax)}\,\mathrm{Gamma}\left(\frac{5}{2},i\cos^{-1}(ax)\right)}{2a\sqrt{\cos^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCos[a*x]^(3/2),x]
```

```
[Out] -(Sqrt[(-I)*ArcCos[a*x]]*Gamma[5/2, (-I)*ArcCos[a*x]] + Sqrt[I*ArcCos[a*x]]
*Gamma[5/2, I*ArcCos[a*x]])/(2*a*Sqrt[ArcCos[a*x]])
```

Maple [A] time = 0.071, size = 72, normalized size = 1.

$$\frac{\sqrt{2}}{4\,a\sqrt{\pi}}\left(2\,\left(\arccos\left(ax\right)\right)^{3/2}\sqrt{2}\sqrt{\pi}xa-3\,\sqrt{2}\sqrt{\arccos\left(ax\right)}\sqrt{\pi}\sqrt{-a^{2}x^{2}+1}+3\,\pi\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\sqrt{\arccos\left(ax\right)}}{\sqrt{\pi}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^(3/2),x)
```

```
[Out] 1/4/a*2^(1/2)/Pi^(1/2)*(2*arccos(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*x*a-3*2^(1/2)*
arccos(a*x)^(1/2)*Pi^(1/2)*(-a^2*x^2+1)^(1/2)+3*Pi*FresnelS(2^(1/2)/Pi^(1/2
)*arccos(a*x)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acos}^{\frac{3}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**(3/2),x)
```

```
[Out] Integral(acos(a*x)**(3/2), x)
```

Giac [B] time = 1.29515, size = 194, normalized size = 2.59

$$\frac{3\,i\sqrt{\arccos(ax)}e^{(i\arccos(ax))}}{4\,a} + \frac{\arccos(ax)^{\frac{3}{2}}\,e^{(i\arccos(ax))}}{2\,a} - \frac{3\,i\sqrt{\arccos(ax)}e^{(-i\arccos(ax))}}{4\,a} + \frac{\arccos(ax)^{\frac{3}{2}}\,e^{(-i\arccos(ax))}}{2\,a} - \frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 3/4*i*sqrt(arccos(a*x))*e^(i*arccos(a*x))/a + 1/2*arccos(a*x)^(3/2)*e^(i*ar
ccos(a*x))/a - 3/4*i*sqrt(arccos(a*x))*e^(-i*arccos(a*x))/a + 1/2*arccos(a*
x)^(3/2)*e^(-i*arccos(a*x))/a - 3/8*sqrt(2)*sqrt(pi)*i*erf(-sqrt(2)*i*sqrt(
arccos(a*x))/(i - 1))/(a*(i - 1)) + 3/8*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(a
rccos(a*x))/(i - 1))/(a*(i - 1))
```

3.85 $\int \frac{\cos^{-1}(ax)^{3/2}}{x} \, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{\cos^{-1}(ax)^{3/2}}{x}, x\right)$$

```
[Out] Unintegrable[ArcCos[a*x]^(3/2)/x, x]
```

Rubi [A] time = 0.0134277, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos^{-1}(ax)^{3/2}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCos[a*x]^(3/2)/x,x]
```

```
[Out] Defer[Int][ArcCos[a*x]^(3/2)/x, x]
```

Rubi steps

$$\int \frac{\cos^{-1}(ax)^{3/2}}{x} \, dx = \int \frac{\cos^{-1}(ax)^{3/2}}{x} \, dx$$

Mathematica [A] time = 0.399362, size = 0, normalized size = 0.

$$\int \frac{\cos^{-1}(ax)^{3/2}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCos[a*x]^(3/2)/x,x]
```

```
[Out] Integrate[ArcCos[a*x]^(3/2)/x, x]
```

Maple [A] time = 0.095, size = 0, normalized size = 0.

$$\int \frac{1}{x} (\arccos(ax))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^(3/2)/x,x)
```

```
[Out] int(arccos(a*x)^(3/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acos}^{\frac{3}{2}}(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**(3/2)/x,x)
```

```
[Out] Integral(acos(a*x)**(3/2)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos{(ax)}^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^(3/2)/x, x)
```

3.86 $\int x^4 \cos^{-1}(ax)^{5/2} \, dx$

Optimal. Leaf size=298

$$\frac{15\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{32a^5}+\frac{\sqrt{\frac{3\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{320a^5}+\frac{\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{60a^5}+\frac{3\sqrt{\frac{\pi}{10}}\text{Fresn}}{}$$

```
[Out] (-2*x*Sqrt[ArcCos[a*x]])/(5*a^4) - (x^3*Sqrt[ArcCos[a*x]])/(15*a^2) - (3*x^
5*Sqrt[ArcCos[a*x]])/100 - (4*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(3/2))/(15*a^5)
 - (2*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(3/2))/(15*a^3) - (x^4*Sqrt[1 - a^2
*x^2]*ArcCos[a*x]^(3/2))/(10*a) + (x^5*ArcCos[a*x]^(5/2))/5 + (15*Sqrt[Pi/2
]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(32*a^5) + (Sqrt[Pi/6]*FresnelC[S
qrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(60*a^5) + (Sqrt[(3*Pi)/2]*FresnelC[Sqrt[6/Pi
]*Sqrt[ArcCos[a*x]]])/(320*a^5) + (3*Sqrt[Pi/10]*FresnelC[Sqrt[10/Pi]*Sqrt[
ArcCos[a*x]]])/(1600*a^5)
```

Rubi [A] time = 0.789701, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4630, 4708, 4678, 4620, 4724, 3304, 3352, 3312}

$$\frac{15\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{32a^5}+\frac{\sqrt{\frac{3\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{320a^5}+\frac{\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{60a^5}+\frac{3\sqrt{\frac{\pi}{10}}\text{Fresn}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCos[a*x]^(5/2),x]
```


```
[Out] (-2*x*Sqrt[ArcCos[a*x]])/(5*a^4) - (x^3*Sqrt[ArcCos[a*x]])/(15*a^2) - (3*x^
5*Sqrt[ArcCos[a*x]])/100 - (4*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(3/2))/(15*a^5)
 - (2*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(3/2))/(15*a^3) - (x^4*Sqrt[1 - a^2
*x^2]*ArcCos[a*x]^(3/2))/(10*a) + (x^5*ArcCos[a*x]^(5/2))/5 + (15*Sqrt[Pi/2
]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(32*a^5) + (Sqrt[Pi/6]*FresnelC[S
qrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(60*a^5) + (Sqrt[(3*Pi)/2]*FresnelC[Sqrt[6/Pi
]*Sqrt[ArcCos[a*x]]])/(320*a^5) + (3*Sqrt[Pi/10]*FresnelC[Sqrt[10/Pi]*Sqrt[
ArcCos[a*x]]])/(1600*a^5)
```


Rule 4630

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
x^(m + 1)*(a + b*ArcCos[c*x])^n)/(m + 1), x] + Dist[(b*c*n)/(m + 1), Int[(x
^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
```

```
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int x^4 \cos^{-1}(ax)^{5/2}\,dx = \frac{1}{5}x^5\cos^{-1}(ax)^{5/2} + \frac{1}{2}a\int \frac{x^5\cos^{-1}(ax)^{3/2}}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{x^4\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{10a} + \frac{1}{5}x^5\cos^{-1}(ax)^{5/2} - \frac{3}{20}\int x^4\sqrt{\cos^{-1}(ax)}\,dx + \frac{2\int \frac{x^3\cos^{-1}(ax)^{3/2}}{\sqrt{1-a^2x^2}}\,dx}{5a}$$

$$= -\frac{3}{100}x^5\sqrt{\cos^{-1}(ax)} - \frac{2x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{15a^3} - \frac{x^4\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{10a} + \frac{1}{5}x^5\cos^{-1}(ax)^{5/2}$$

$$= -\frac{x^3\sqrt{\cos^{-1}(ax)}}{15a^2} - \frac{3}{100}x^5\sqrt{\cos^{-1}(ax)} - \frac{4\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{15a^5} - \frac{2x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{15a^3} -$$

$$= -\frac{2x\sqrt{\cos^{-1}(ax)}}{5a^4} - \frac{x^3\sqrt{\cos^{-1}(ax)}}{15a^2} - \frac{3}{100}x^5\sqrt{\cos^{-1}(ax)} - \frac{4\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{15a^5} - \frac{2x^2\sqrt{1-a^2x^2}}{15}$$

$$= -\frac{2x\sqrt{\cos^{-1}(ax)}}{5a^4} - \frac{x^3\sqrt{\cos^{-1}(ax)}}{15a^2} - \frac{3}{100}x^5\sqrt{\cos^{-1}(ax)} - \frac{4\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{15a^5} - \frac{2x^2\sqrt{1-a^2x^2}}{15}$$

$$= -\frac{2x\sqrt{\cos^{-1}(ax)}}{5a^4} - \frac{x^3\sqrt{\cos^{-1}(ax)}}{15a^2} - \frac{3}{100}x^5\sqrt{\cos^{-1}(ax)} - \frac{4\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{15a^5} - \frac{2x^2\sqrt{1-a^2x^2}}{15}$$

$$= -\frac{2x\sqrt{\cos^{-1}(ax)}}{5a^4} - \frac{x^3\sqrt{\cos^{-1}(ax)}}{15a^2} - \frac{3}{100}x^5\sqrt{\cos^{-1}(ax)} - \frac{4\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{15a^5} - \frac{2x^2\sqrt{1-a^2x^2}}{15}$$

$$= -\frac{2x\sqrt{\cos^{-1}(ax)}}{5a^4} - \frac{x^3\sqrt{\cos^{-1}(ax)}}{15a^2} - \frac{3}{100}x^5\sqrt{\cos^{-1}(ax)} - \frac{4\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{15a^5} - \frac{2x^2\sqrt{1-a^2x^2}}{15}$$

Mathematica [C] time = 0.191514, size = 212, normalized size = 0.71

$$-\frac{-625\sqrt{3}\left(-i\cos^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{7}{2},-3i\cos^{-1}(ax)\right) - 27\sqrt{5}\left(-i\cos^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{7}{2},-5i\cos^{-1}(ax)\right) + 33750\sqrt{c}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*ArcCos[a*x]^(5/2),x]
```

```
[Out] -(33750*Sqrt[I*ArcCos[a*x]]*Sqrt[ArcCos[a*x]^2]*Gamma[7/2, (-I)*ArcCos[a*x]
] + 33750*Sqrt[(-I)*ArcCos[a*x]]*Sqrt[ArcCos[a*x]^2]*Gamma[7/2, I*ArcCos[a*
x]] - 625*Sqrt[3]*((-I)*ArcCos[a*x])^(3/2)*Gamma[7/2, (-3*I)*ArcCos[a*x]] -
 625*Sqrt[3]*(I*ArcCos[a*x])^(3/2)*Gamma[7/2, (3*I)*ArcCos[a*x]] - 27*Sqrt[
5]*((-I)*ArcCos[a*x])^(3/2)*Gamma[7/2, (-5*I)*ArcCos[a*x]] - 27*Sqrt[5]*(I*
ArcCos[a*x])^(3/2)*Gamma[7/2, (5*I)*ArcCos[a*x]])/(540000*a^5*ArcCos[a*x]^(
3/2))
```

Maple [A] time = 0.109, size = 233, normalized size = 0.8

$$\frac{1}{144000\,a^5}\left(18000\,ax\,(\arccos(ax))^3+9000\,(\arccos(ax))^3\cos(3\,\arccos(ax))+1800\,(\arccos(ax))^3\cos(5\,\arccos(ax)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccos(a*x)^(5/2),x)
```

```
[Out] 1/144000/a^5*(18000*a*x*arccos(a*x)^3+9000*arccos(a*x)^3*cos(3*arccos(a*x))
+1800*arccos(a*x)^3*cos(5*arccos(a*x))+27*5^(1/2)*2^(1/2)*arccos(a*x)^(1/2)
*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)*arccos(a*x)^(1/2))+625*3^(1/2)*
2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*arccos
(a*x)^(1/2))-45000*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)-7500*arccos(a*x)^2*sin(
3*arccos(a*x))-900*arccos(a*x)^2*sin(5*arccos(a*x))+33750*2^(1/2)*arccos(a*
x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))-67500*a*x*ar
ccos(a*x)-3750*arccos(a*x)*cos(3*arccos(a*x))-270*arccos(a*x)*cos(5*arccos(
a*x)))/arccos(a*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccos(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccos(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acos(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.45306, size = 738, normalized size = 2.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccos(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/320*i*arccos(a*x)^(3/2)*e^(5*i*arccos(a*x))/a^5 + 1/160*arccos(a*x)^(5/2)
*e^(5*i*arccos(a*x))/a^5 + 5/192*i*arccos(a*x)^(3/2)*e^(3*i*arccos(a*x))/a^
5 + 1/32*arccos(a*x)^(5/2)*e^(3*i*arccos(a*x))/a^5 + 5/32*i*arccos(a*x)^(3/
2)*e^(i*arccos(a*x))/a^5 + 1/16*arccos(a*x)^(5/2)*e^(i*arccos(a*x))/a^5 - 5
/32*i*arccos(a*x)^(3/2)*e^(-i*arccos(a*x))/a^5 + 1/16*arccos(a*x)^(5/2)*e^(
-i*arccos(a*x))/a^5 - 5/192*i*arccos(a*x)^(3/2)*e^(-3*i*arccos(a*x))/a^5 +
1/32*arccos(a*x)^(5/2)*e^(-3*i*arccos(a*x))/a^5 - 1/320*i*arccos(a*x)^(3/2)
*e^(-5*i*arccos(a*x))/a^5 + 1/160*arccos(a*x)^(5/2)*e^(-5*i*arccos(a*x))/a^
5 - 3/32000*sqrt(10)*sqrt(pi)*i*erf(sqrt(10)*sqrt(arccos(a*x))/(i - 1))/(a^
```

```
5*(i - 1)) - 5/2304*sqrt(6)*sqrt(pi)*i*erf(sqrt(6)*sqrt(arccos(a*x))/(i - 1
))/(a^5*(i - 1)) - 15/128*sqrt(2)*sqrt(pi)*i*erf(sqrt(2)*sqrt(arccos(a*x))/
(i - 1))/(a^5*(i - 1)) - 3/3200*sqrt(arccos(a*x))*e^(5*i*arccos(a*x))/a^5 -
 5/384*sqrt(arccos(a*x))*e^(3*i*arccos(a*x))/a^5 - 15/64*sqrt(arccos(a*x))*
e^(i*arccos(a*x))/a^5 - 15/64*sqrt(arccos(a*x))*e^(-i*arccos(a*x))/a^5 - 5/
384*sqrt(arccos(a*x))*e^(-3*i*arccos(a*x))/a^5 - 3/3200*sqrt(arccos(a*x))*e
^(-5*i*arccos(a*x))/a^5 + 3/32000*sqrt(10)*sqrt(pi)*erf(-sqrt(10)*i*sqrt(ar
ccos(a*x))/(i - 1))/(a^5*(i - 1)) + 5/2304*sqrt(6)*sqrt(pi)*erf(-sqrt(6)*i*
sqrt(arccos(a*x))/(i - 1))/(a^5*(i - 1)) + 15/128*sqrt(2)*sqrt(pi)*erf(-sqr
t(2)*i*sqrt(arccos(a*x))/(i - 1))/(a^5*(i - 1))
```

3.87 $\int x^3 \cos^{-1}(ax)^{5/2} \, dx$

Optimal. Leaf size=205

$$\frac{15\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{4096a^4}+\frac{15\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{256a^4}-\frac{5x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{32a}-\frac{45x^2\sqrt{\cos^{-1}(ax)}}{256a^2}-1$$

```
[Out] (225*Sqrt[ArcCos[a*x]])/(2048*a^4) - (45*x^2*Sqrt[ArcCos[a*x]])/(256*a^2) -
 (15*x^4*Sqrt[ArcCos[a*x]])/256 - (15*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(3/2)
)/(64*a^3) - (5*x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(3/2))/(32*a) - (3*ArcCos
[a*x]^(5/2))/(32*a^4) + (x^4*ArcCos[a*x]^(5/2))/4 + (15*Sqrt[Pi/2]*FresnelC
[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(4096*a^4) + (15*Sqrt[Pi]*FresnelC[(2*Sqr
t[ArcCos[a*x]])/Sqrt[Pi]])/(256*a^4)
```

Rubi [A] time = 0.58931, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {4630, 4708, 4642, 4724, 3312, 3304, 3352}

$$\frac{15\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{4096a^4}+\frac{15\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{256a^4}-\frac{5x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{32a}-\frac{45x^2\sqrt{\cos^{-1}(ax)}}{256a^2}-1$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCos[a*x]^(5/2),x]
```


```
[Out] (225*Sqrt[ArcCos[a*x]])/(2048*a^4) - (45*x^2*Sqrt[ArcCos[a*x]])/(256*a^2) -
 (15*x^4*Sqrt[ArcCos[a*x]])/256 - (15*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(3/2)
)/(64*a^3) - (5*x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(3/2))/(32*a) - (3*ArcCos
[a*x]^(5/2))/(32*a^4) + (x^4*ArcCos[a*x]^(5/2))/4 + (15*Sqrt[Pi/2]*FresnelC
[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(4096*a^4) + (15*Sqrt[Pi]*FresnelC[(2*Sqr
t[ArcCos[a*x]])/Sqrt[Pi]])/(256*a^4)
```


Rule 4630

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
x^(m + 1)*(a + b*ArcCos[c*x])^n)/(m + 1), x] + Dist[(b*c*n)/(m + 1), Int[(x
^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int x^3 \cos^{-1}(ax)^{5/2}\,dx = \frac{1}{4}x^4\cos^{-1}(ax)^{5/2} + \frac{1}{8}(5a)\int \frac{x^4\cos^{-1}(ax)^{3/2}}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{5x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{32a} + \frac{1}{4}x^4\cos^{-1}(ax)^{5/2} - \frac{15}{64}\int x^3\sqrt{\cos^{-1}(ax)}\,dx + \frac{15\int \frac{x^2\cos^{-1}(ax)^{3/2}}{\sqrt{1-a^2x^2}}\,dx}{32a}$$

$$= -\frac{15}{256}x^4\sqrt{\cos^{-1}(ax)} - \frac{15x\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{32a} + \frac{1}{4}x^4\cos^{-1}(ax)^{5/2}$$

$$= -\frac{45x^2\sqrt{\cos^{-1}(ax)}}{256a^2} - \frac{15}{256}x^4\sqrt{\cos^{-1}(ax)} - \frac{15x\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{32a}$$

$$= -\frac{45x^2\sqrt{\cos^{-1}(ax)}}{256a^2} - \frac{15}{256}x^4\sqrt{\cos^{-1}(ax)} - \frac{15x\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{32a}$$

$$= \frac{45\sqrt{\cos^{-1}(ax)}}{2048a^4} - \frac{45x^2\sqrt{\cos^{-1}(ax)}}{256a^2} - \frac{15}{256}x^4\sqrt{\cos^{-1}(ax)} - \frac{15x\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{64a^3} - 5x^3\sqrt{1-a}$$

$$= \frac{225\sqrt{\cos^{-1}(ax)}}{2048a^4} - \frac{45x^2\sqrt{\cos^{-1}(ax)}}{256a^2} - \frac{15}{256}x^4\sqrt{\cos^{-1}(ax)} - \frac{15x\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{64a^3} - 5x^3\sqrt{1-}$$

$$= \frac{225\sqrt{\cos^{-1}(ax)}}{2048a^4} - \frac{45x^2\sqrt{\cos^{-1}(ax)}}{256a^2} - \frac{15}{256}x^4\sqrt{\cos^{-1}(ax)} - \frac{15x\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{64a^3} - 5x^3\sqrt{1-}$$

$$= \frac{225\sqrt{\cos^{-1}(ax)}}{2048a^4} - \frac{45x^2\sqrt{\cos^{-1}(ax)}}{256a^2} - \frac{15}{256}x^4\sqrt{\cos^{-1}(ax)} - \frac{15x\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{64a^3} - 5x^3\sqrt{1-}$$

Mathematica [C] time = 0.126707, size = 140, normalized size = 0.68

$$-\frac{-16\sqrt{2}\left(-i\cos^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{7}{2},-2i\cos^{-1}(ax)\right) - 16\sqrt{2}\left(i\cos^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{7}{2},2i\cos^{-1}(ax)\right) + \sqrt{\cos^{-1}(ax)^2}\,(}{2048a^4\cos^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*ArcCos[a*x]^(5/2),x]
```

```
[Out] -(-16*Sqrt[2]*((-I)*ArcCos[a*x])^(3/2)*Gamma[7/2, (-2*I)*ArcCos[a*x]] - 16*
Sqrt[2]*(I*ArcCos[a*x])^(3/2)*Gamma[7/2, (2*I)*ArcCos[a*x]] + Sqrt[ArcCos[a
*x]^2]*(Sqrt[I*ArcCos[a*x]]*Gamma[7/2, (-4*I)*ArcCos[a*x]] + Sqrt[(-I)*ArcC
os[a*x]]*Gamma[7/2, (4*I)*ArcCos[a*x]]))/(2048*a^4*ArcCos[a*x]^(3/2))
```

Maple [A] time = 0.094, size = 154, normalized size = 0.8

$$\frac{1}{8192\,a^4\sqrt{\pi}}\left(1024\,(\arccos(ax))^{5/2}\sqrt{\pi}\cos(2\arccos(ax))+256\,(\arccos(ax))^{5/2}\sqrt{\pi}\cos(4\arccos(ax))-1280\,(\text{arcco}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccos(a*x)^(5/2),x)
```

```
[Out] 1/8192/a^4/Pi^(1/2)*(1024*arccos(a*x)^(5/2)*Pi^(1/2)*cos(2*arccos(a*x))+256
*arccos(a*x)^(5/2)*Pi^(1/2)*cos(4*arccos(a*x))-1280*arccos(a*x)^(3/2)*Pi^(1
/2)*sin(2*arccos(a*x))-160*arccos(a*x)^(3/2)*Pi^(1/2)*sin(4*arccos(a*x))+15
*Pi*2^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))-960*Pi^(1/2)*arc
cos(a*x)^(1/2)*cos(2*arccos(a*x))-60*arccos(a*x)^(1/2)*Pi^(1/2)*cos(4*arcco
s(a*x))+480*Pi*FresnelC(2*arccos(a*x)^(1/2)/Pi^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acos(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32306, size = 466, normalized size = 2.27

$$\frac{5\,i\arccos\left(ax\right)^{\frac{3}{2}}e^{\left(4\,i\arccos\left(ax\right)\right)}}{512\,a^{4}}+\frac{\arccos\left(ax\right)^{\frac{5}{2}}e^{\left(4\,i\arccos\left(ax\right)\right)}}{64\,a^{4}}+\frac{5\,i\arccos\left(ax\right)^{\frac{3}{2}}e^{\left(2\,i\arccos\left(ax\right)\right)}}{64\,a^{4}}+\frac{\arccos\left(ax\right)^{\frac{5}{2}}e^{\left(2\,i\arccos\left(ax\right)\right)}}{16\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] 5/512*i*arccos(a*x)^(3/2)*e^(4*i*arccos(a*x))/a^4 + 1/64*arccos(a*x)^(5/2)*
e^(4*i*arccos(a*x))/a^4 + 5/64*i*arccos(a*x)^(3/2)*e^(2*i*arccos(a*x))/a^4
+ 1/16*arccos(a*x)^(5/2)*e^(2*i*arccos(a*x))/a^4 - 5/64*i*arccos(a*x)^(3/2)
*e^(-2*i*arccos(a*x))/a^4 + 1/16*arccos(a*x)^(5/2)*e^(-2*i*arccos(a*x))/a^4
 - 5/512*i*arccos(a*x)^(3/2)*e^(-4*i*arccos(a*x))/a^4 + 1/64*arccos(a*x)^(5
/2)*e^(-4*i*arccos(a*x))/a^4 - 15/16384*sqrt(2)*sqrt(pi)*i*erf(-sqrt(2)*(i
+ 1)*sqrt(arccos(a*x)))/(a^4*(i - 1)) - 15/512*sqrt(pi)*i*erf(-(i + 1)*sqrt
(arccos(a*x)))/(a^4*(i - 1)) - 15/4096*sqrt(arccos(a*x))*e^(4*i*arccos(a*x)
)/a^4 - 15/256*sqrt(arccos(a*x))*e^(2*i*arccos(a*x))/a^4 - 15/256*sqrt(arcc
os(a*x))*e^(-2*i*arccos(a*x))/a^4 - 15/4096*sqrt(arccos(a*x))*e^(-4*i*arcco
s(a*x))/a^4 + 15/16384*sqrt(2)*sqrt(pi)*erf(sqrt(2)*(i - 1)*sqrt(arccos(a*x
)))/(a^4*(i - 1)) + 15/512*sqrt(pi)*erf((i - 1)*sqrt(arccos(a*x)))/(a^4*(i
- 1))
```

3.88 $\int x^2 \cos^{-1}(ax)^{5/2} dx$

Optimal. Leaf size=178

$$\frac{15\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{16a^3}+\frac{5\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{144a^3}-\frac{5x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{18a}-\frac{5\sqrt{1-a^2x^2}\cos^{-1}}{9a^3}$$

```
[Out] (-5*x*Sqrt[ArcCos[a*x]])/(6*a^2) - (5*x^3*Sqrt[ArcCos[a*x]])/36 - (5*Sqrt[1
 - a^2*x^2]*ArcCos[a*x]^(3/2))/(9*a^3) - (5*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*
x]^(3/2))/(18*a) + (x^3*ArcCos[a*x]^(5/2))/3 + (15*Sqrt[Pi/2]*FresnelC[Sqrt
[2/Pi]*Sqrt[ArcCos[a*x]]])/(16*a^3) + (5*Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqr
t[ArcCos[a*x]]])/(144*a^3)
```

Rubi [A] time = 0.456678, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4630, 4708, 4678, 4620, 4724, 3304, 3352, 3312}

$$\frac{15\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{16a^3}+\frac{5\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{144a^3}-\frac{5x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{18a}-\frac{5\sqrt{1-a^2x^2}\cos^{-1}}{9a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCos[a*x]^(5/2),x]
```


```
[Out] (-5*x*Sqrt[ArcCos[a*x]])/(6*a^2) - (5*x^3*Sqrt[ArcCos[a*x]])/36 - (5*Sqrt[1
 - a^2*x^2]*ArcCos[a*x]^(3/2))/(9*a^3) - (5*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*
x]^(3/2))/(18*a) + (x^3*ArcCos[a*x]^(5/2))/3 + (15*Sqrt[Pi/2]*FresnelC[Sqrt
[2/Pi]*Sqrt[ArcCos[a*x]]])/(16*a^3) + (5*Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqr
t[ArcCos[a*x]]])/(144*a^3)
```


Rule 4630

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
x^(m + 1)*(a + b*ArcCos[c*x])^n)/(m + 1), x] + Dist[(b*c*n)/(m + 1), Int[(x
^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
```

```
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int x^2\cos^{-1}(ax)^{5/2}\,dx = \frac{1}{3}x^3\cos^{-1}(ax)^{5/2} + \frac{1}{6}(5a)\int\frac{x^3\cos^{-1}(ax)^{3/2}}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{5x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{18a} + \frac{1}{3}x^3\cos^{-1}(ax)^{5/2} - \frac{5}{12}\int x^2\sqrt{\cos^{-1}(ax)}\,dx + \frac{5\int\frac{x\cos^{-1}(ax)^{3/2}}{\sqrt{1-a^2x^2}}\,dx}{9a}$$

$$= -\frac{5}{36}x^3\sqrt{\cos^{-1}(ax)} - \frac{5\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{18a} + \frac{1}{3}x^3\cos^{-1}(ax)^{5/2} -$$

$$= -\frac{5x\sqrt{\cos^{-1}(ax)}}{6a^2} - \frac{5}{36}x^3\sqrt{\cos^{-1}(ax)} - \frac{5\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{18a} +$$

$$= -\frac{5x\sqrt{\cos^{-1}(ax)}}{6a^2} - \frac{5}{36}x^3\sqrt{\cos^{-1}(ax)} - \frac{5\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{18a} +$$

$$= -\frac{5x\sqrt{\cos^{-1}(ax)}}{6a^2} - \frac{5}{36}x^3\sqrt{\cos^{-1}(ax)} - \frac{5\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{18a} +$$

$$= -\frac{5x\sqrt{\cos^{-1}(ax)}}{6a^2} - \frac{5}{36}x^3\sqrt{\cos^{-1}(ax)} - \frac{5\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{18a} +$$

$$= -\frac{5x\sqrt{\cos^{-1}(ax)}}{6a^2} - \frac{5}{36}x^3\sqrt{\cos^{-1}(ax)} - \frac{5\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{18a} +$$

Mathematica [C] time = 0.118874, size = 122, normalized size = 0.69

$$-\frac{81i\sqrt{\cos^{-1}(ax)^2}\text{Gamma}\left(\frac{7}{2},-i\cos^{-1}(ax)\right) + 81\cos^{-1}(ax)\text{Gamma}\left(\frac{7}{2},i\cos^{-1}(ax)\right) + \sqrt{3}\left(i\sqrt{\cos^{-1}(ax)^2}\text{Gamma}\left(\frac{7}{2},-\right.\right.}{648a^3\sqrt{i\cos^{-1}(ax)}\sqrt{\cos^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*ArcCos[a*x]^(5/2),x]
```

```
[Out] -((81*I)*Sqrt[ArcCos[a*x]^2]*Gamma[7/2, (-I)*ArcCos[a*x]] + 81*ArcCos[a*x]*
Gamma[7/2, I*ArcCos[a*x]] + Sqrt[3]*(I*Sqrt[ArcCos[a*x]^2]*Gamma[7/2, (-3*I
)*ArcCos[a*x]] + ArcCos[a*x]*Gamma[7/2, (3*I)*ArcCos[a*x]]))/(648*a^3*Sqrt[
I*ArcCos[a*x]]*Sqrt[ArcCos[a*x]])
```

Maple [A] time = 0.09, size = 156, normalized size = 0.9

$$\frac{1}{864\,a^3}\left(216\,ax\,(\arccos(ax))^3+72\,(\arccos(ax))^3\cos(3\,\arccos(ax))+5\,\sqrt{3}\sqrt{2}\sqrt{\arccos(ax)}\sqrt{\pi}\mathrm{FresnelC}\left(\frac{\sqrt{3}\sqrt{2}\sqrt{\mathrm{arcc}}}{\sqrt{\pi}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccos(a*x)^(5/2),x)
```

```
[Out] 1/864/a^3*(216*a*x*arccos(a*x)^3+72*arccos(a*x)^3*cos(3*arccos(a*x))+5*3^(1
/2)*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*ar
ccos(a*x)^(1/2))-540*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)-60*arccos(a*x)^2*sin(
3*arccos(a*x))+405*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(
1/2)*arccos(a*x)^(1/2))-810*a*x*arccos(a*x)-30*arccos(a*x)*cos(3*arccos(a*x
)))/arccos(a*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acos(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31501, size = 491, normalized size = 2.76

$$\frac{5\,i\arccos\left(ax\right)^{\frac{3}{2}}e^{\left(3\,i\arccos\left(ax\right)\right)}}{144\,a^{3}}+\frac{\arccos\left(ax\right)^{\frac{5}{2}}e^{\left(3\,i\arccos\left(ax\right)\right)}}{24\,a^{3}}+\frac{5\,i\arccos\left(ax\right)^{\frac{3}{2}}e^{\left(i\arccos\left(ax\right)\right)}}{16\,a^{3}}+\frac{\arccos\left(ax\right)^{\frac{5}{2}}e^{\left(i\arccos\left(ax\right)\right)}}{8\,a^{3}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] 5/144*i*arccos(a*x)^(3/2)*e^(3*i*arccos(a*x))/a^3 + 1/24*arccos(a*x)^(5/2)*
e^(3*i*arccos(a*x))/a^3 + 5/16*i*arccos(a*x)^(3/2)*e^(i*arccos(a*x))/a^3 +
1/8*arccos(a*x)^(5/2)*e^(i*arccos(a*x))/a^3 - 5/16*i*arccos(a*x)^(3/2)*e^(-
i*arccos(a*x))/a^3 + 1/8*arccos(a*x)^(5/2)*e^(-i*arccos(a*x))/a^3 - 5/144*i
*arccos(a*x)^(3/2)*e^(-3*i*arccos(a*x))/a^3 + 1/24*arccos(a*x)^(5/2)*e^(-3*
i*arccos(a*x))/a^3 - 5/1728*sqrt(6)*sqrt(pi)*i*erf(sqrt(6)*sqrt(arccos(a*x)
)/(i - 1))/(a^3*(i - 1)) - 15/64*sqrt(2)*sqrt(pi)*i*erf(sqrt(2)*sqrt(arccos
(a*x))/(i - 1))/(a^3*(i - 1)) - 5/288*sqrt(arccos(a*x))*e^(3*i*arccos(a*x))
/a^3 - 15/32*sqrt(arccos(a*x))*e^(i*arccos(a*x))/a^3 - 15/32*sqrt(arccos(a*
x))*e^(-i*arccos(a*x))/a^3 - 5/288*sqrt(arccos(a*x))*e^(-3*i*arccos(a*x))/a
^3 + 5/1728*sqrt(6)*sqrt(pi)*erf(-sqrt(6)*i*sqrt(arccos(a*x))/(i - 1))/(a^3
*(i - 1)) + 15/64*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*i*sqrt(arccos(a*x))/(i - 1)
)/(a^3*(i - 1))
```

3.89 $\int x \cos^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=119

$$\frac{15\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{128a^2}-\frac{5x\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{8a}-\frac{\cos^{-1}(ax)^{5/2}}{4a^2}+\frac{15\sqrt{\cos^{-1}(ax)}}{64a^2}+\frac{1}{2}x^2\cos^{-1}(ax)^{5/2}-\frac{15}{32}x^2\sqrt{\text{co}}$$

```
[Out] (15*Sqrt[ArcCos[a*x]])/(64*a^2) - (15*x^2*Sqrt[ArcCos[a*x]])/32 - (5*x*Sqrt
[1 - a^2*x^2]*ArcCos[a*x]^(3/2))/(8*a) - ArcCos[a*x]^(5/2)/(4*a^2) + (x^2*A
rcCos[a*x]^(5/2))/2 + (15*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]]
)/(128*a^2)
```

Rubi [A] time = 0.287317, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {4630, 4708, 4642, 4724, 3312, 3304, 3352}

$$\frac{15\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{128a^2}-\frac{5x\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{8a}-\frac{\cos^{-1}(ax)^{5/2}}{4a^2}+\frac{15\sqrt{\cos^{-1}(ax)}}{64a^2}+\frac{1}{2}x^2\cos^{-1}(ax)^{5/2}-\frac{15}{32}x^2\sqrt{\text{co}}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCos[a*x]^(5/2),x]
```

```
[Out] (15*Sqrt[ArcCos[a*x]])/(64*a^2) - (15*x^2*Sqrt[ArcCos[a*x]])/32 - (5*x*Sqrt
[1 - a^2*x^2]*ArcCos[a*x]^(3/2))/(8*a) - ArcCos[a*x]^(5/2)/(4*a^2) + (x^2*A
rcCos[a*x]^(5/2))/2 + (15*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]]
)/(128*a^2)
```

Rule 4630

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
x^(m + 1)*(a + b*ArcCos[c*x])^n)/(m + 1), x] + Dist[(b*c*n)/(m + 1), Int[(x
^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
```

```
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int x\cos^{-1}(ax)^{5/2}\,dx = \frac{1}{2}x^2\cos^{-1}(ax)^{5/2}+\frac{1}{4}(5a)\int\frac{x^2\cos^{-1}(ax)^{3/2}}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{5x\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{8a}+\frac{1}{2}x^2\cos^{-1}(ax)^{5/2}-\frac{15}{16}\int x\sqrt{\cos^{-1}(ax)}\,dx+\frac{5\int\frac{\cos^{-1}(ax)^{3/2}}{\sqrt{1-a^2x^2}}\,dx}{8a}$$

$$= -\frac{15}{32}x^2\sqrt{\cos^{-1}(ax)}-\frac{5x\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{8a}-\frac{\cos^{-1}(ax)^{5/2}}{4a^2}+\frac{1}{2}x^2\cos^{-1}(ax)^{5/2}-\frac{1}{64}(15a)\int$$

$$= -\frac{15}{32}x^2\sqrt{\cos^{-1}(ax)}-\frac{5x\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{8a}-\frac{\cos^{-1}(ax)^{5/2}}{4a^2}+\frac{1}{2}x^2\cos^{-1}(ax)^{5/2}+\frac{15\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{15}{32}x^2\sqrt{\cos^{-1}(ax)}-\frac{5x\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{8a}-\frac{\cos^{-1}(ax)^{5/2}}{4a^2}+\frac{1}{2}x^2\cos^{-1}(ax)^{5/2}+\frac{15\,\text{Subst}\left(\int\left(\right.\right.}{}$$

$$= \frac{15\sqrt{\cos^{-1}(ax)}}{64a^2}-\frac{15}{32}x^2\sqrt{\cos^{-1}(ax)}-\frac{5x\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{8a}-\frac{\cos^{-1}(ax)^{5/2}}{4a^2}+\frac{1}{2}x^2\cos^{-1}(ax)^{5/2}$$

$$= \frac{15\sqrt{\cos^{-1}(ax)}}{64a^2}-\frac{15}{32}x^2\sqrt{\cos^{-1}(ax)}-\frac{5x\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{8a}-\frac{\cos^{-1}(ax)^{5/2}}{4a^2}+\frac{1}{2}x^2\cos^{-1}(ax)^{5/2}$$

$$= \frac{15\sqrt{\cos^{-1}(ax)}}{64a^2}-\frac{15}{32}x^2\sqrt{\cos^{-1}(ax)}-\frac{5x\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{8a}-\frac{\cos^{-1}(ax)^{5/2}}{4a^2}+\frac{1}{2}x^2\cos^{-1}(ax)^{5/2}$$

Mathematica [A] time = 0.0892726, size = 73, normalized size = 0.61

$$\frac{15\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)-2\sqrt{\cos^{-1}(ax)}\left(\left(15-16\cos^{-1}(ax)^2\right)\cos\left(2\cos^{-1}(ax)\right)+20\cos^{-1}(ax)\sin\left(2\cos^{-1}(ax)\right)\right)}{128a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCos[a*x]^(5/2),x]
```

```
[Out] (15*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]] - 2*Sqrt[ArcCos[a*x]]
*((15 - 16*ArcCos[a*x]^2)*Cos[2*ArcCos[a*x]] + 20*ArcCos[a*x]*Sin[2*ArcCos[
a*x]]))/(128*a^2)
```

Maple [A] time = 0.078, size = 79, normalized size = 0.7

$$\frac{1}{128\,a^2\sqrt{\pi}}\left(32\,\left(\arccos(ax)\right)^{5/2}\sqrt{\pi}\cos(2\,\arccos(ax))-40\,\left(\arccos(ax)\right)^{3/2}\sqrt{\pi}\sin(2\,\arccos(ax))+15\,\pi\,\text{FresnelC}\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccos(a*x)^(5/2),x)
```

```
[Out] 1/128/a^2/Pi^(1/2)*(32*arccos(a*x)^(5/2)*Pi^(1/2)*cos(2*arccos(a*x))-40*arc
cos(a*x)^(3/2)*Pi^(1/2)*sin(2*arccos(a*x))+15*Pi*FresnelC(2*arccos(a*x)^(1/
2)/Pi^(1/2))-30*Pi^(1/2)*arccos(a*x)^(1/2)*cos(2*arccos(a*x)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acos(a*x)**(5/2),x)
```

[Out] Timed out

Giac [A] time = 1.23437, size = 225, normalized size = 1.89

$$\frac{5\,i\arccos\left(ax\right)^{\frac{3}{2}}e^{\left(2\,i\arccos\left(ax\right)\right)}}{32\,a^{2}}+\frac{\arccos\left(ax\right)^{\frac{5}{2}}e^{\left(2\,i\arccos\left(ax\right)\right)}}{8\,a^{2}}-\frac{5\,i\arccos\left(ax\right)^{\frac{3}{2}}e^{\left(-2\,i\arccos\left(ax\right)\right)}}{32\,a^{2}}+\frac{\arccos\left(ax\right)^{\frac{5}{2}}e^{\left(-2\,i\arccos\left(ax\right)\right)}}{8\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] 5/32*i*arccos(a*x)^(3/2)*e^(2*i*arccos(a*x))/a^2 + 1/8*arccos(a*x)^(5/2)*e^
(2*i*arccos(a*x))/a^2 - 5/32*i*arccos(a*x)^(3/2)*e^(-2*i*arccos(a*x))/a^2 +
 1/8*arccos(a*x)^(5/2)*e^(-2*i*arccos(a*x))/a^2 - 15/256*sqrt(pi)*i*erf(-(i
 + 1)*sqrt(arccos(a*x)))/(a^2*(i - 1)) - 15/128*sqrt(arccos(a*x))*e^(2*i*ar
ccos(a*x))/a^2 - 15/128*sqrt(arccos(a*x))*e^(-2*i*arccos(a*x))/a^2 + 15/256
*sqrt(pi)*erf((i - 1)*sqrt(arccos(a*x)))/(a^2*(i - 1))
```

3.90 $\int \cos^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=88

$$-\frac{5\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{2a}+\frac{15\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{4a}+x\cos^{-1}(ax)^{5/2}-\frac{15}{4}x\sqrt{\cos^{-1}(ax)}$$

```
[Out] (-15*x*Sqrt[ArcCos[a*x]])/4 - (5*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(3/2))/(2*a)
 + x*ArcCos[a*x]^(5/2) + (15*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x
]]])/(4*a)
```

Rubi [A] time = 0.160196, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4620, 4678, 4724, 3304, 3352}

$$-\frac{5\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{2a}+\frac{15\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{4a}+x\cos^{-1}(ax)^{5/2}-\frac{15}{4}x\sqrt{\cos^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^(5/2),x]
```

```
[Out] (-15*x*Sqrt[ArcCos[a*x]])/4 - (5*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(3/2))/(2*a)
 + x*ArcCos[a*x]^(5/2) + (15*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x
]]])/(4*a)
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^{-1}(ax)^{5/2}\,dx &= x\cos^{-1}(ax)^{5/2}+\frac{1}{2}(5a)\int \frac{x\cos^{-1}(ax)^{3/2}}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{5\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{2a}+x\cos^{-1}(ax)^{5/2}-\frac{15}{4}\int \sqrt{\cos^{-1}(ax)}\,dx\\
&= -\frac{15}{4}x\sqrt{\cos^{-1}(ax)}-\frac{5\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{2a}+x\cos^{-1}(ax)^{5/2}-\frac{1}{8}(15a)\int \frac{x}{\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}\,dx\\
&= -\frac{15}{4}x\sqrt{\cos^{-1}(ax)}-\frac{5\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{2a}+x\cos^{-1}(ax)^{5/2}+\frac{15\,\mathrm{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx,x,\cos^{-1}(ax)\right)}{8a}\\
&= -\frac{15}{4}x\sqrt{\cos^{-1}(ax)}-\frac{5\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{2a}+x\cos^{-1}(ax)^{5/2}+\frac{15\,\mathrm{Subst}\left(\int \cos\left(x^2\right)\,dx,x,\sqrt{\cos^{-1}(\right.}{4a}\\
&= -\frac{15}{4}x\sqrt{\cos^{-1}(ax)}-\frac{5\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}{2a}+x\cos^{-1}(ax)^{5/2}+\frac{15\sqrt{\frac{\pi}{2}}C\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{4a}
\end{aligned}$$

Mathematica [C] time = 0.0374966, size = 76, normalized size = 0.86

$$-\frac{\sqrt{\cos^{-1}(ax)}\left(\sqrt{i\cos^{-1}(ax)}\,\mathrm{Gamma}\left(\frac{7}{2},-i\cos^{-1}(ax)\right)+\sqrt{-i\cos^{-1}(ax)}\,\mathrm{Gamma}\left(\frac{7}{2},i\cos^{-1}(ax)\right)\right)}{2a\sqrt{\cos^{-1}(ax)^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCos[a*x]^(5/2),x]
```

```
[Out] -(Sqrt[ArcCos[a*x]]*(Sqrt[I*ArcCos[a*x]]*Gamma[7/2, (-I)*ArcCos[a*x]] + Sqr
t[(-I)*ArcCos[a*x]]*Gamma[7/2, I*ArcCos[a*x]]))/(2*a*Sqrt[ArcCos[a*x]^2])
```

Maple [A] time = 0.073, size = 88, normalized size = 1.

$$\frac{\sqrt{2}}{8\,a\sqrt{\pi}}\left(4\,(\arccos(ax))^{5/2}\sqrt{2}\sqrt{\pi}xa-10\,(\arccos(ax))^{3/2}\sqrt{2}\sqrt{\pi}\sqrt{-a^2x^2+1}-15\,\sqrt{2}\sqrt{\pi}\sqrt{\arccos(ax)}xa+15\,\pi\,\mathrm{Fresnel}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^(5/2),x)
```

```
[Out] 1/8/a*2^(1/2)/Pi^(1/2)*(4*arccos(a*x)^(5/2)*2^(1/2)*Pi^(1/2)*x*a-10*arccos(
a*x)^(3/2)*2^(1/2)*Pi^(1/2)*(-a^2*x^2+1)^(1/2)-15*2^(1/2)*Pi^(1/2)*arccos(a
*x)^(1/2)*x*a+15*Pi*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32006, size = 244, normalized size = 2.77

$$\frac{5\,i\arccos\left(ax\right)^{\frac{3}{2}}e^{\left(i\arccos\left(ax\right)\right)}}{4\,a}+\frac{\arccos\left(ax\right)^{\frac{5}{2}}e^{\left(i\arccos\left(ax\right)\right)}}{2\,a}-\frac{5\,i\arccos\left(ax\right)^{\frac{3}{2}}e^{\left(-i\arccos\left(ax\right)\right)}}{4\,a}+\frac{\arccos\left(ax\right)^{\frac{5}{2}}e^{\left(-i\arccos\left(ax\right)\right)}}{2\,a}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] 5/4*i*arccos(a*x)^(3/2)*e^(i*arccos(a*x))/a + 1/2*arccos(a*x)^(5/2)*e^(i*ar
ccos(a*x))/a - 5/4*i*arccos(a*x)^(3/2)*e^(-i*arccos(a*x))/a + 1/2*arccos(a*
x)^(5/2)*e^(-i*arccos(a*x))/a - 15/16*sqrt(2)*sqrt(pi)*i*erf(sqrt(2)*sqrt(a
rccos(a*x))/(i - 1))/(a*(i - 1)) - 15/8*sqrt(arccos(a*x))*e^(i*arccos(a*x))
/a - 15/8*sqrt(arccos(a*x))*e^(-i*arccos(a*x))/a + 15/16*sqrt(2)*sqrt(pi)*e
rf(-sqrt(2)*i*sqrt(arccos(a*x))/(i - 1))/(a*(i - 1))
```

3.91 $\int \frac{\cos^{-1}(ax)^{5/2}}{x} \, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{\cos^{-1}(ax)^{5/2}}{x},x\right)$$

```
[Out] Unintegrable[ArcCos[a*x]^(5/2)/x, x]
```

Rubi [A] time = 0.0123123, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos^{-1}(ax)^{5/2}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCos[a*x]^(5/2)/x,x]
```

```
[Out] Defer[Int][ArcCos[a*x]^(5/2)/x, x]
```

Rubi steps

$$\int \frac{\cos^{-1}(ax)^{5/2}}{x} \, dx = \int \frac{\cos^{-1}(ax)^{5/2}}{x} \, dx$$

Mathematica [A] time = 0.391111, size = 0, normalized size = 0.

$$\int \frac{\cos^{-1}(ax)^{5/2}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCos[a*x]^(5/2)/x,x]
```

```
[Out] Integrate[ArcCos[a*x]^(5/2)/x, x]
```

Maple [A] time = 0.097, size = 0, normalized size = 0.

$$\int \frac{1}{x} (\arccos(ax))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^(5/2)/x,x)
```

```
[Out] int(arccos(a*x)^(5/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**(5/2)/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(ax)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^(5/2)/x, x)
```

3.92 $\int \frac{x^4}{\sqrt{\cos^{-1}(ax)}} dx$

Optimal. Leaf size=106

$$-\frac{\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{4a^5}-\frac{\sqrt{\frac{3\pi}{2}}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{8a^5}-\frac{\sqrt{\frac{\pi}{10}}S\left(\sqrt{\frac{10}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{8a^5}$$

```
[Out] -(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(4*a^5) - (Sqrt[(3*Pi)
/2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(8*a^5) - (Sqrt[Pi/10]*FresnelS
[Sqrt[10/Pi]*Sqrt[ArcCos[a*x]]])/(8*a^5)
```

Rubi [A] time = 0.104841, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4636, 4406, 3305, 3351}

$$-\frac{\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{4a^5}-\frac{\sqrt{\frac{3\pi}{2}}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{8a^5}-\frac{\sqrt{\frac{\pi}{10}}S\left(\sqrt{\frac{10}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[ArcCos[a*x]],x]
```

```
[Out] -(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(4*a^5) - (Sqrt[(3*Pi)
/2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(8*a^5) - (Sqrt[Pi/10]*FresnelS
[Sqrt[10/Pi]*Sqrt[ArcCos[a*x]]])/(8*a^5)
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{\cos^{-1}(ax)}}\,dx = -\frac{\text{Subst}\left(\int \frac{\cos^4(x)\sin(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{a^5}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{\sin(x)}{8\sqrt{x}} + \frac{3\sin(3x)}{16\sqrt{x}} + \frac{\sin(5x)}{16\sqrt{x}}\right) dx, x, \cos^{-1}(ax)\right)}{a^5}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sin(5x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{16a^5} - \frac{\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{8a^5} - \frac{3\,\text{Subst}\left(\int \frac{\sin(3x)}{\sqrt{x}}\,dx, x, \text{c}\right.}{16a^5}$$

$$= -\frac{\text{Subst}\left(\int \sin\left(5x^2\right) dx, x, \sqrt{\cos^{-1}(ax)}\right)}{8a^5} - \frac{\text{Subst}\left(\int \sin\left(x^2\right) dx, x, \sqrt{\cos^{-1}(ax)}\right)}{4a^5} - \frac{3\,\text{Subst}\left(\int \text{sin}\right.}{}$$

$$= -\frac{\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{4a^5} - \frac{\sqrt{\frac{3\pi}{2}}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{8a^5} - \frac{\sqrt{\frac{\pi}{10}}S\left(\sqrt{\frac{10}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{8a^5}$$

Mathematica [C] time = 0.0999738, size = 192, normalized size = 1.81

$$-\frac{-10\sqrt{-i\cos^{-1}(ax)}\text{Gamma}\left(\frac{1}{2}, -i\cos^{-1}(ax)\right) - 10\sqrt{i\cos^{-1}(ax)}\text{Gamma}\left(\frac{1}{2}, i\cos^{-1}(ax)\right) - 5\sqrt{3}\sqrt{-i\cos^{-1}(ax)}\text{Gamma}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/Sqrt[ArcCos[a*x]],x]
```

```
[Out] -(-10*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-I)*ArcCos[a*x]] - 10*Sqrt[I*ArcCo
s[a*x]]*Gamma[1/2, I*ArcCos[a*x]] - 5*Sqrt[3]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[
1/2, (-3*I)*ArcCos[a*x]] - 5*Sqrt[3]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (3*I)*A
rcCos[a*x]] - Sqrt[5]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-5*I)*ArcCos[a*x]]
```

```
 - Sqrt[5]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (5*I)*ArcCos[a*x]])/(160*a^5*Sqrt
[ArcCos[a*x]])
```

Maple [A] time = 0.078, size = 72, normalized size = 0.7

$$-\frac{\sqrt{2}\sqrt{\pi}}{80\,a^{5}}\left(\sqrt{5}\mathrm{FresnelS}\left(\frac{\sqrt{2}\sqrt{5}}{\sqrt{\pi}}\sqrt{\arccos(ax)}\right)+5\sqrt{3}\mathrm{FresnelS}\left(\frac{\sqrt{2}\sqrt{3}\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+10\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arccos(a*x)^(1/2),x)
```

```
[Out] -1/80/a^5*2^(1/2)*Pi^(1/2)*(5^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)*arcco
s(a*x)^(1/2))+5*3^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*arccos(a*x)^(1/2)
)+10*FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccos(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccos(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{\operatorname{acos}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/acos(a*x)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(acos(a*x)), x)
```

Giac [B] time = 1.41067, size = 273, normalized size = 2.58

$$\frac{\sqrt{10}\sqrt{\pi}i\operatorname{erf}\left(-\frac{\sqrt{10}i\sqrt{\arccos(ax)}}{i-1}\right)}{160\,a^5(i-1)}+\frac{\sqrt{6}\sqrt{\pi}i\operatorname{erf}\left(-\frac{\sqrt{6}i\sqrt{\arccos(ax)}}{i-1}\right)}{32\,a^5(i-1)}+\frac{\sqrt{2}\sqrt{\pi}i\operatorname{erf}\left(-\frac{\sqrt{2}i\sqrt{\arccos(ax)}}{i-1}\right)}{16\,a^5(i-1)}-\frac{\sqrt{10}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{10}\sqrt{\arccos(}}{i-1}\right.}{160\,a^5(i-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccos(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/160*sqrt(10)*sqrt(pi)*i*erf(-sqrt(10)*i*sqrt(arccos(a*x))/(i - 1))/(a^5*(
i - 1)) + 1/32*sqrt(6)*sqrt(pi)*i*erf(-sqrt(6)*i*sqrt(arccos(a*x))/(i - 1))
/(a^5*(i - 1)) + 1/16*sqrt(2)*sqrt(pi)*i*erf(-sqrt(2)*i*sqrt(arccos(a*x))/(
i - 1))/(a^5*(i - 1)) - 1/160*sqrt(10)*sqrt(pi)*erf(sqrt(10)*sqrt(arccos(a*
x))/(i - 1))/(a^5*(i - 1)) - 1/32*sqrt(6)*sqrt(pi)*erf(sqrt(6)*sqrt(arccos(
a*x))/(i - 1))/(a^5*(i - 1)) - 1/16*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(arcco
s(a*x))/(i - 1))/(a^5*(i - 1))
```

3.93 $\int \frac{x^3}{\sqrt{\cos^{-1}(ax)}}\,dx$

Optimal. Leaf size=65

$$-\frac{\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{8a^4}-\frac{\sqrt{\pi}\,S\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{4a^4}$$

```
[Out] -(Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(8*a^4) - (Sqrt[Pi]*
FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/(4*a^4)
```

Rubi [A] time = 0.0777967, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4636, 4406, 3305, 3351}

$$-\frac{\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{8a^4}-\frac{\sqrt{\pi}\,S\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[ArcCos[a*x]],x]
```

```
[Out] -(Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(8*a^4) - (Sqrt[Pi]*
FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/(4*a^4)
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{\cos^{-1}(ax)}}\,dx &= -\frac{\text{Subst}\left(\int \frac{\cos^3(x)\sin(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{a^4}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{\sin(2x)}{4\sqrt{x}}+\frac{\sin(4x)}{8\sqrt{x}}\right)dx, x, \cos^{-1}(ax)\right)}{a^4}\\
&= -\frac{\text{Subst}\left(\int \frac{\sin(4x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{8a^4}-\frac{\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{4a^4}\\
&= -\frac{\text{Subst}\left(\int \sin\left(4x^2\right)dx, x, \sqrt{\cos^{-1}(ax)}\right)}{4a^4}-\frac{\text{Subst}\left(\int \sin\left(2x^2\right)dx, x, \sqrt{\cos^{-1}(ax)}\right)}{2a^4}\\
&= -\frac{\sqrt{\frac{\pi}{2}}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{8a^4}-\frac{\sqrt{\pi}S\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{4a^4}
\end{aligned}$$

Mathematica [C] time = 0.0678617, size = 130, normalized size = 2.

$$-\frac{-2\sqrt{2}\sqrt{-i\cos^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-2i\cos^{-1}(ax)\right)-2\sqrt{2}\sqrt{i\cos^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},2i\cos^{-1}(ax)\right)-\sqrt{-i\cos^{-1}(ax)}\text{Gamm}}{32a^4\sqrt{\cos^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/Sqrt[ArcCos[a*x]],x]
```

```
[Out] -(-2*Sqrt[2]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-2*I)*ArcCos[a*x]] - 2*Sqrt
[2]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (2*I)*ArcCos[a*x]] - Sqrt[(-I)*ArcCos[a*
x]]*Gamma[1/2, (-4*I)*ArcCos[a*x]] - Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (4*I)*A
rcCos[a*x]])/(32*a^4*Sqrt[ArcCos[a*x]])
```

Maple [A] time = 0.07, size = 43, normalized size = 0.7

$$-\frac{\sqrt{\pi}}{16\,a^4}\left(\sqrt{2}\mathrm{FresnelS}\left(2\,\frac{\sqrt{2}\sqrt{\arccos\left(ax\right)}}{\sqrt{\pi}}\right)+4\,\mathrm{FresnelS}\left(2\,\frac{\sqrt{\arccos\left(ax\right)}}{\sqrt{\pi}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccos(a*x)^(1/2),x)
```

```
[Out] -1/16/a^4*Pi^(1/2)*(2^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))+
4*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccos(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccos(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{\sqrt{\operatorname{acos}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acos(a*x)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(acos(a*x)), x)
```

Giac [B] time = 1.3581, size = 153, normalized size = 2.35

$$\frac{\sqrt{2}\sqrt{\pi}i\operatorname{erf}\left(\sqrt{2}(i-1)\sqrt{\arccos(ax)}\right)}{32a^4(i-1)}+\frac{\sqrt{\pi}i\operatorname{erf}\left((i-1)\sqrt{\arccos(ax)}\right)}{8a^4(i-1)}-\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\sqrt{2}(i+1)\sqrt{\arccos(ax)}\right)}{32a^4(i-1)}-\frac{\sqrt{\pi}\operatorname{erf}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccos(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/32*sqrt(2)*sqrt(pi)*i*erf(sqrt(2)*(i - 1)*sqrt(arccos(a*x)))/(a^4*(i - 1)
) + 1/8*sqrt(pi)*i*erf((i - 1)*sqrt(arccos(a*x)))/(a^4*(i - 1)) - 1/32*sqrt
(2)*sqrt(pi)*erf(-sqrt(2)*(i + 1)*sqrt(arccos(a*x)))/(a^4*(i - 1)) - 1/8*sq
rt(pi)*erf(-(i + 1)*sqrt(arccos(a*x)))/(a^4*(i - 1))
```

3.94 $\int \frac{x^2}{\sqrt{\cos^{-1}(ax)}}\,dx$

Optimal. Leaf size=71

$$-\frac{\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{2a^3}-\frac{\sqrt{\frac{\pi}{6}}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{2a^3}$$

```
[Out] -(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(2*a^3) - (Sqrt[Pi/6]*
FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(2*a^3)
```

Rubi [A] time = 0.079498, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4636, 4406, 3305, 3351}

$$-\frac{\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{2a^3}-\frac{\sqrt{\frac{\pi}{6}}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[ArcCos[a*x]],x]
```

```
[Out] -(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(2*a^3) - (Sqrt[Pi/6]*
FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(2*a^3)
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{\cos^{-1}(ax)}}\,dx &= -\frac{\text{Subst}\left(\int \frac{\cos^2(x)\sin(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{a^3}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{\sin(x)}{4\sqrt{x}} + \frac{\sin(3x)}{4\sqrt{x}}\right) dx, x, \cos^{-1}(ax)\right)}{a^3}\\
&= -\frac{\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{4a^3} - \frac{\text{Subst}\left(\int \frac{\sin(3x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{4a^3}\\
&= -\frac{\text{Subst}\left(\int \sin\left(x^2\right) dx, x, \sqrt{\cos^{-1}(ax)}\right)}{2a^3} - \frac{\text{Subst}\left(\int \sin\left(3x^2\right) dx, x, \sqrt{\cos^{-1}(ax)}\right)}{2a^3}\\
&= -\frac{\sqrt{\frac{\pi}{2}}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{2a^3} - \frac{\sqrt{\frac{\pi}{6}}\,S\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{2a^3}
\end{aligned}$$

Mathematica [C] time = 0.0819761, size = 126, normalized size = 1.77

$$-\frac{-3\sqrt{-i\cos^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2}, -i\cos^{-1}(ax)\right) - 3\sqrt{i\cos^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2}, i\cos^{-1}(ax)\right) - \sqrt{3}\left(\sqrt{-i\cos^{-1}(ax)}\,\text{Gamma}\left(\right.\right.}{24a^3\sqrt{\cos^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/Sqrt[ArcCos[a*x]],x]
```

```
[Out] -(-3*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-I)*ArcCos[a*x]] - 3*Sqrt[I*ArcCos[
a*x]]*Gamma[1/2, I*ArcCos[a*x]] - Sqrt[3]*(Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2
, (-3*I)*ArcCos[a*x]] + Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (3*I)*ArcCos[a*x]]))
/(24*a^3*Sqrt[ArcCos[a*x]])
```

Maple [A] time = 0.069, size = 50, normalized size = 0.7

$$-\frac{\sqrt{2}\,\sqrt{\pi}}{12\,a^{3}}\left(\sqrt{3}\mathrm{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{3}}{\sqrt{\pi}}\sqrt{\arccos\left(ax\right)}\right)+3\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{\arccos\left(ax\right)}}{\sqrt{\pi}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccos(a*x)^(1/2),x)
```

```
[Out] -1/12/a^3*2^(1/2)*Pi^(1/2)*(3^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*arcco
s(a*x)^(1/2))+3*FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccos(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccos(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{2}}{\sqrt{\operatorname{acos}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acos(a*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(acos(a*x)), x)
```

Giac [B] time = 1.35129, size = 182, normalized size = 2.56

$$\frac{\sqrt{6}\sqrt{\pi}i\,\mathrm{erf}\left(-\frac{\sqrt{6}i\sqrt{\arccos(ax)}}{i-1}\right)}{24\,a^3(i-1)}+\frac{\sqrt{2}\sqrt{\pi}i\,\mathrm{erf}\left(-\frac{\sqrt{2}i\sqrt{\arccos(ax)}}{i-1}\right)}{8\,a^3(i-1)}-\frac{\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{6}\sqrt{\arccos(ax)}}{i-1}\right)}{24\,a^3(i-1)}-\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{\arccos(ax)}}{i-1}\right)}{8\,a^3(i-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccos(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(6)*sqrt(pi)*i*erf(-sqrt(6)*i*sqrt(arccos(a*x))/(i - 1))/(a^3*(i -
 1)) + 1/8*sqrt(2)*sqrt(pi)*i*erf(-sqrt(2)*i*sqrt(arccos(a*x))/(i - 1))/(a^
3*(i - 1)) - 1/24*sqrt(6)*sqrt(pi)*erf(sqrt(6)*sqrt(arccos(a*x))/(i - 1))/(
a^3*(i - 1)) - 1/8*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(arccos(a*x))/(i - 1))/
(a^3*(i - 1))
```

3.95 $\int \frac{x}{\sqrt{\cos^{-1}(ax)}} dx$

Optimal. Leaf size=28

$$-\frac{\sqrt{\pi} S\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{2a^2}$$

```
[Out] -(Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/(2*a^2)
```

Rubi [A] time = 0.0402895, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4636, 4406, 12, 3305, 3351}

$$-\frac{\sqrt{\pi} S\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[ArcCos[a*x]],x]
```

```
[Out] -(Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/(2*a^2)
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{\cos^{-1}(ax)}}\,dx &= -\frac{\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{a^2}\\ &= -\frac{\text{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{a^2}\\ &= -\frac{\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{2a^2}\\ &= -\frac{\text{Subst}\left(\int \sin\left(2x^2\right)dx, x, \sqrt{\cos^{-1}(ax)}\right)}{a^2}\\ &= -\frac{\sqrt{\pi}S\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.0230338, size = 28, normalized size = 1.

$$-\frac{\sqrt{\pi}S\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[ArcCos[a*x]],x]
```

```
[Out] -(Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/(2*a^2)
```

Maple [A] time = 0.054, size = 21, normalized size = 0.8

$$-\frac{\sqrt{\pi}}{2\,a^{2}}\mathrm{FresnelS}\left(2\,\frac{\sqrt{\arccos\left(ax\right)}}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccos(a*x)^(1/2),x)
```

```
[Out] -1/2*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{\operatorname{acos}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acos(a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(acos(a*x)), x)
```

Giac [B] time = 1.31227, size = 69, normalized size = 2.46

$$\frac{\sqrt{\pi}i\,\mathrm{erf}\left((i-1)\sqrt{\arccos{(ax)}}\right)}{4\,a^2(i-1)}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-(i+1)\sqrt{\arccos{(ax)}}\right)}{4\,a^2(i-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*i*erf((i - 1)*sqrt(arccos(a*x)))/(a^2*(i - 1)) - 1/4*sqrt(pi)*
erf(-(i + 1)*sqrt(arccos(a*x)))/(a^2*(i - 1))
```

3.96 $\int \frac{1}{\sqrt{\cos^{-1}(ax)}}\,dx$

Optimal. Leaf size=31

$$-\frac{\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a}$$

```
[Out] -((Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/a)
```

Rubi [A] time = 0.022666, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4624, 3305, 3351}

$$-\frac{\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[ArcCos[a*x]],x]
```

```
[Out] -((Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/a)
```

Rule 4624

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Sub
st[Int[x^n*Sin[a/b - x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c,
 n}, x]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos^{-1}(ax)}}\,dx = -\frac{\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{a}$$
$$= -\frac{2\,\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{\cos^{-1}(ax)}\right)}{a}$$
$$= -\frac{\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a}$$

Mathematica [C] time = 0.0242184, size = 68, normalized size = 2.19

$$-\frac{-\sqrt{-i\cos^{-1}(ax)}\text{Gamma}\left(\frac{1}{2}, -i\cos^{-1}(ax)\right) - \sqrt{i\cos^{-1}(ax)}\text{Gamma}\left(\frac{1}{2}, i\cos^{-1}(ax)\right)}{2a\sqrt{\cos^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[ArcCos[a*x]],x]
```

```
[Out] -(-(Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-I)*ArcCos[a*x]]) - Sqrt[I*ArcCos[a*
x]]*Gamma[1/2, I*ArcCos[a*x]])/(2*a*Sqrt[ArcCos[a*x]])
```

Maple [A] time = 0.049, size = 26, normalized size = 0.8

$$-\frac{\sqrt{2}\sqrt{\pi}}{a}\text{FresnelS}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\sqrt{\arccos(ax)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos(a*x)^(1/2),x)
```

```
[Out] -FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\operatorname{acos}\left(ax\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acos(a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(acos(a*x)), x)
```

Giac [B] time = 1.24571, size = 92, normalized size = 2.97

$$\frac{\sqrt{2}\sqrt{\pi} i \operatorname{erf}\left(-\frac{\sqrt{2} i \sqrt{\arccos(ax)}}{i-1}\right)}{2\, a(i-1)} - \frac{\sqrt{2}\sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\arccos(ax)}}{i-1}\right)}{2\, a(i-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*sqrt(pi)*i*erf(-sqrt(2)*i*sqrt(arccos(a*x))/(i - 1))/(a*(i - 1)
) - 1/2*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(arccos(a*x))/(i - 1))/(a*(i - 1))
```

3.97 $\int \frac{1}{x\sqrt{\cos^{-1}(ax)}}\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{1}{x\sqrt{\cos^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[1/(x*Sqrt[ArcCos[a*x]]), x]
```

Rubi [A] time = 0.0126049, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\sqrt{\cos^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[ArcCos[a*x]]),x]
```

```
[Out] Defer[Int][1/(x*Sqrt[ArcCos[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{\cos^{-1}(ax)}}\,dx=\int \frac{1}{x\sqrt{\cos^{-1}(ax)}}\,dx$$

Mathematica [A] time = 0.309415, size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\cos^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[ArcCos[a*x]]),x]
```

```
[Out] Integrate[1/(x*Sqrt[ArcCos[a*x]]), x]
```

Maple [A] time = 0.08, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt{\arccos\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccos(a*x)^(1/2),x)
```

```
[Out] int(1/x/arccos(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccos(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccos(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\operatorname{acos}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acos(a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(acos(a*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\arccos(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccos(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sqrt(arccos(a*x))), x)
```

3.98 $\int \frac{1}{x^2\sqrt{\cos^{-1}(ax)}}\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{1}{x^2\sqrt{\cos^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[1/(x^2*Sqrt[ArcCos[a*x]]), x]
```

Rubi [A] time = 0.0130397, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\sqrt{\cos^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[ArcCos[a*x]]),x]
```

```
[Out] Defer[Int][1/(x^2*Sqrt[ArcCos[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{\cos^{-1}(ax)}}\,dx=\int \frac{1}{x^2\sqrt{\cos^{-1}(ax)}}\,dx$$

Mathematica [A] time = 3.46933, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{\cos^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[ArcCos[a*x]]),x]
```

```
[Out] Integrate[1/(x^2*Sqrt[ArcCos[a*x]]), x]
```

Maple [A] time = 0.154, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \frac{1}{\sqrt{\arccos{(ax)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arccos(a*x)^(1/2),x)
```

```
[Out] int(1/x^2/arccos(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccos(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccos(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{\operatorname{acos}{(ax)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/acos(a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(acos(a*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt{\arccos(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccos(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*sqrt(arccos(a*x))), x)
```

3.99 $\int \frac{x^6}{\cos^{-1}(ax)^{3/2}} dx$

Optimal. Leaf size=171

$$-\frac{5\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{16a^7}-\frac{9\sqrt{\frac{3\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{16a^7}-\frac{5\sqrt{\frac{5\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{16a^7}-\sqrt{\frac{7\pi}{2}}\text{Fre}$$

```
[Out] (2*x^6*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (5*Sqrt[Pi/2]*FresnelC[Sq
rt[2/Pi]*Sqrt[ArcCos[a*x]]])/(16*a^7) - (9*Sqrt[(3*Pi)/2]*FresnelC[Sqrt[6/P
i]*Sqrt[ArcCos[a*x]]])/(16*a^7) - (5*Sqrt[(5*Pi)/2]*FresnelC[Sqrt[10/Pi]*Sq
rt[ArcCos[a*x]]])/(16*a^7) - (Sqrt[(7*Pi)/2]*FresnelC[Sqrt[14/Pi]*Sqrt[ArcC
os[a*x]]])/(16*a^7)
```

Rubi [A] time = 0.145915, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4632, 3304, 3352}

$$-\frac{5\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{16a^7}-\frac{9\sqrt{\frac{3\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{16a^7}-\frac{5\sqrt{\frac{5\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{16a^7}-\sqrt{\frac{7\pi}{2}}\text{Fre}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/ArcCos[a*x]^(3/2),x]
```

```
[Out] (2*x^6*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (5*Sqrt[Pi/2]*FresnelC[Sq
rt[2/Pi]*Sqrt[ArcCos[a*x]]])/(16*a^7) - (9*Sqrt[(3*Pi)/2]*FresnelC[Sqrt[6/P
i]*Sqrt[ArcCos[a*x]]])/(16*a^7) - (5*Sqrt[(5*Pi)/2]*FresnelC[Sqrt[10/Pi]*Sq
rt[ArcCos[a*x]]])/(16*a^7) - (Sqrt[(7*Pi)/2]*FresnelC[Sqrt[14/Pi]*Sqrt[ArcC
os[a*x]]])/(16*a^7)
```

Rule 4632

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dis
t[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), Co
s[x]^(m - 1)*(m - (m + 1)*Cos[x]^2), x], x], x, ArcCos[c*x]], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^6}{\cos^{-1}(ax)^{3/2}}\,dx = \frac{2x^6\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} + \frac{2\,\text{Subst}\left(\int\left(-\frac{5\cos(x)}{64\sqrt{x}} - \frac{27\cos(3x)}{64\sqrt{x}} - \frac{25\cos(5x)}{64\sqrt{x}} - \frac{7\cos(7x)}{64\sqrt{x}}\right)dx, x, \cos^{-1}(ax)\right)}{a^7}$$

$$= \frac{2x^6\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} - \frac{5\,\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{32a^7} - \frac{7\,\text{Subst}\left(\int \frac{\cos(7x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{32a^7} - \frac{25\,\text{Su}}{}$$

$$= \frac{2x^6\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} - \frac{5\,\text{Subst}\left(\int \cos\left(x^2\right)dx, x, \sqrt{\cos^{-1}(ax)}\right)}{16a^7} - \frac{7\,\text{Subst}\left(\int \cos\left(7x^2\right)dx, x, \sqrt{\cos^{-1}(ax}\right.}{16a^7}$$

$$= \frac{2x^6\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} - \frac{5\sqrt{\frac{\pi}{2}}C\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{16a^7} - \frac{9\sqrt{\frac{3\pi}{2}}C\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{16a^7} - \frac{5\sqrt{\frac{5\pi}{2}}C\left(\sqrt{\frac{10}{\pi}}\sqrt{\cos^{-1}(a}\right.}{16a^7}$$

Mathematica [C] time = 0.286505, size = 306, normalized size = 1.79

$$\frac{i\left(5\sqrt{-i\cos^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2}, -i\cos^{-1}(ax)\right) - 5\sqrt{i\cos^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2}, i\cos^{-1}(ax)\right) + 9\sqrt{3}\sqrt{-i\cos^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/ArcCos[a*x]^(3/2),x]
```

```
[Out] ((I/64)*((-10*I)*Sqrt[1 - a^2*x^2] + 5*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-
I)*ArcCos[a*x]] - 5*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, I*ArcCos[a*x]] + 9*Sqrt[
3]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-3*I)*ArcCos[a*x]] - 9*Sqrt[3]*Sqrt[I
*ArcCos[a*x]]*Gamma[1/2, (3*I)*ArcCos[a*x]] + 5*Sqrt[5]*Sqrt[(-I)*ArcCos[a*
x]]*Gamma[1/2, (-5*I)*ArcCos[a*x]] - 5*Sqrt[5]*Sqrt[I*ArcCos[a*x]]*Gamma[1/
2, (5*I)*ArcCos[a*x]] + Sqrt[7]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-7*I)*Ar
cCos[a*x]] - Sqrt[7]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (7*I)*ArcCos[a*x]] - (1
```

```
8*I)*Sin[3*ArcCos[a*x]] - (10*I)*Sin[5*ArcCos[a*x]] - (2*I)*Sin[7*ArcCos[a*
x]]))/(a^7*Sqrt[ArcCos[a*x]])
```

Maple [A] time = 0.109, size = 182, normalized size = 1.1

$$\frac{1}{32\,a^7}\left(-5\,\sqrt{5}\sqrt{2}\sqrt{\arccos(ax)}\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{5}\sqrt{2}\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-\sqrt{2}\sqrt{\pi}\sqrt{7}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\sqrt{7}}{\sqrt{\pi}}\sqrt{\arccos(ax)}\right)\sqrt{\arccos}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/arccos(a*x)^(3/2),x)
```

```
[Out] 1/32/a^7*(-5*5^(1/2)*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi
^(1/2)*5^(1/2)*arccos(a*x)^(1/2))-2^(1/2)*Pi^(1/2)*7^(1/2)*FresnelC(2^(1/2)
/Pi^(1/2)*7^(1/2)*arccos(a*x)^(1/2))*arccos(a*x)^(1/2)-9*3^(1/2)*2^(1/2)*ar
ccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*arccos(a*x)^(1/2
))-5*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*
x)^(1/2))+5*(-a^2*x^2+1)^(1/2)+9*sin(3*arccos(a*x))+5*sin(5*arccos(a*x))+si
n(7*arccos(a*x)))/arccos(a*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/arccos(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/arccos(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\operatorname{acos}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/acos(a*x)**(3/2),x)
```

```
[Out] Integral(x**6/acos(a*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\arccos{(ax)}^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/arccos(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/arccos(a*x)^(3/2), x)
```

3.100 $\int \frac{x^5}{\cos^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a^6}-\frac{\sqrt{3\pi}\text{FresnelC}\left(2\sqrt{\frac{3}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{8a^6}-\frac{5\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{8a^6}+\frac{2x^5\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}}$$

```
[Out] (2*x^5*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (Sqrt[Pi/2]*FresnelC[2*Sq
rt[2/Pi]*Sqrt[ArcCos[a*x]]])/a^6 - (Sqrt[3*Pi]*FresnelC[2*Sqrt[3/Pi]*Sqrt[A
rcCos[a*x]]])/(8*a^6) - (5*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]
])/(8*a^6)
```

Rubi [A] time = 0.103803, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4632, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a^6}-\frac{\sqrt{3\pi}\text{FresnelC}\left(2\sqrt{\frac{3}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{8a^6}-\frac{5\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{8a^6}+\frac{2x^5\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/ArcCos[a*x]^(3/2),x]
```

```
[Out] (2*x^5*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (Sqrt[Pi/2]*FresnelC[2*Sq
rt[2/Pi]*Sqrt[ArcCos[a*x]]])/a^6 - (Sqrt[3*Pi]*FresnelC[2*Sqrt[3/Pi]*Sqrt[A
rcCos[a*x]]])/(8*a^6) - (5*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]
])/(8*a^6)
```

Rule 4632

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dis
t[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), Co
s[x]^(m - 1)*(m - (m + 1)*Cos[x]^2), x], x], x, ArcCos[c*x]], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^5}{\cos^{-1}(ax)^{3/2}}\,dx = \frac{2x^5\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} + \frac{2\,\text{Subst}\left(\int\left(-\frac{5\cos(2x)}{16\sqrt{x}}-\frac{\cos(4x)}{2\sqrt{x}}-\frac{3\cos(6x)}{16\sqrt{x}}\right)dx, x, \cos^{-1}(ax)\right)}{a^6}$$

$$= \frac{2x^5\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} - \frac{3\,\text{Subst}\left(\int\frac{\cos(6x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{8a^6} - \frac{5\,\text{Subst}\left(\int\frac{\cos(2x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{8a^6} - \text{Sub}$$

$$= \frac{2x^5\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} - \frac{3\,\text{Subst}\left(\int\cos\left(6x^2\right)dx, x, \sqrt{\cos^{-1}(ax)}\right)}{4a^6} - \frac{5\,\text{Subst}\left(\int\cos\left(2x^2\right)dx, x, \sqrt{\cos^{-1}(a}\right.}{4a^6}$$

$$= \frac{2x^5\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} - \frac{\sqrt{\frac{\pi}{2}}C\left(2\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a^6} - \frac{\sqrt{3\pi}C\left(2\sqrt{\frac{3}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{8a^6} - \frac{5\sqrt{\pi}C\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{8a^6}$$

Mathematica [C] time = 0.482975, size = 226, normalized size = 1.78

$$\frac{i\left(5\sqrt{2}\sqrt{-i\cos^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-2i\cos^{-1}(ax)\right)-5\sqrt{2}\sqrt{i\cos^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},2i\cos^{-1}(ax)\right)+8\sqrt{-i\cos^{-1}(ax)}\text{Gamm}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5/ArcCos[a*x]^(3/2),x]
```

```
[Out] ((I/32)*(5*Sqrt[2]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-2*I)*ArcCos[a*x]] -
5*Sqrt[2]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (2*I)*ArcCos[a*x]] + 8*Sqrt[(-I)*A
rcCos[a*x]]*Gamma[1/2, (-4*I)*ArcCos[a*x]] - 8*Sqrt[I*ArcCos[a*x]]*Gamma[1/
2, (4*I)*ArcCos[a*x]] + Sqrt[6]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-6*I)*Ar
cCos[a*x]] - Sqrt[6]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (6*I)*ArcCos[a*x]] - (1
0*I)*Sin[2*ArcCos[a*x]] - (8*I)*Sin[4*ArcCos[a*x]] - (2*I)*Sin[6*ArcCos[a*x
]]))/(a^6*Sqrt[ArcCos[a*x]])
```

Maple [A] time = 0.092, size = 121, normalized size = 1.

$$\frac{1}{16\,a^{6}}\left(-2\,\sqrt{\pi}\sqrt{3}\mathrm{FresnelC}\left(\frac{\sqrt{2}\sqrt{6}\sqrt{\arccos\left(ax\right)}}{\sqrt{\pi}}\right)\sqrt{\arccos\left(ax\right)}-8\,\sqrt{2}\sqrt{\pi}\sqrt{\arccos\left(ax\right)}\mathrm{FresnelC}\left(2\,\frac{\sqrt{2}\sqrt{\arccos\left(ax\right)}}{\sqrt{\pi}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/arccos(a*x)^(3/2),x)
```

```
[Out] 1/16/a^6/arccos(a*x)^(1/2)*(-2*Pi^(1/2)*3^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*6
^(1/2)*arccos(a*x)^(1/2))*arccos(a*x)^(1/2)-8*2^(1/2)*Pi^(1/2)*arccos(a*x)^
(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))-10*Pi^(1/2)*arccos(a*x
)^(1/2)*FresnelC(2*arccos(a*x)^(1/2)/Pi^(1/2))+5*sin(2*arccos(a*x))+4*sin(4
*arccos(a*x))+sin(6*arccos(a*x)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arccos(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arccos(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\operatorname{acos}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/acos(a*x)**(3/2),x)
```

```
[Out] Integral(x**5/acos(a*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\arccos(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arccos(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/arccos(a*x)^(3/2), x)
```

3.101 $\int \frac{x^4}{\cos^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=136

$$-\frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{2a^5}-\frac{3\sqrt{\frac{3\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{4a^5}-\frac{\sqrt{\frac{5\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{4a^5}+\frac{2x^4\sqrt{1-a}}{a\sqrt{\cos^{-1}(}}$$

```
[Out] (2*x^4*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (Sqrt[Pi/2]*FresnelC[Sqrt
[2/Pi]*Sqrt[ArcCos[a*x]]])/(2*a^5) - (3*Sqrt[(3*Pi)/2]*FresnelC[Sqrt[6/Pi]*
Sqrt[ArcCos[a*x]]])/(4*a^5) - (Sqrt[(5*Pi)/2]*FresnelC[Sqrt[10/Pi]*Sqrt[Arc
Cos[a*x]]])/(4*a^5)
```

Rubi [A] time = 0.0946939, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4632, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{2a^5}-\frac{3\sqrt{\frac{3\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{4a^5}-\frac{\sqrt{\frac{5\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{4a^5}+\frac{2x^4\sqrt{1-a}}{a\sqrt{\cos^{-1}(}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcCos[a*x]^(3/2),x]
```


```
[Out] (2*x^4*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (Sqrt[Pi/2]*FresnelC[Sqrt
[2/Pi]*Sqrt[ArcCos[a*x]]])/(2*a^5) - (3*Sqrt[(3*Pi)/2]*FresnelC[Sqrt[6/Pi]*
Sqrt[ArcCos[a*x]]])/(4*a^5) - (Sqrt[(5*Pi)/2]*FresnelC[Sqrt[10/Pi]*Sqrt[Arc
Cos[a*x]]])/(4*a^5)
```


Rule 4632

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dis
t[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), Co
s[x]^(m - 1)*(m - (m + 1)*Cos[x]^2), x], x], x, ArcCos[c*x]], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^4}{\cos^{-1}(ax)^{3/2}}\,dx = \frac{2x^4\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} + \frac{2\,\mathrm{Subst}\left(\int\left(-\frac{\cos(x)}{8\sqrt{x}}-\frac{9\cos(3x)}{16\sqrt{x}}-\frac{5\cos(5x)}{16\sqrt{x}}\right)dx, x, \cos^{-1}(ax)\right)}{a^5}$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} - \frac{\mathrm{Subst}\left(\int\frac{\cos(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{4a^5} - \frac{5\,\mathrm{Subst}\left(\int\frac{\cos(5x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{8a^5} - \frac{9\,\mathrm{Subs}}{}$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} - \frac{\mathrm{Subst}\left(\int\cos\left(x^2\right)dx, x, \sqrt{\cos^{-1}(ax)}\right)}{2a^5} - \frac{5\,\mathrm{Subst}\left(\int\cos\left(5x^2\right)dx, x, \sqrt{\cos^{-1}(ax)}\right)}{4a^5}$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} - \frac{\sqrt{\frac{\pi}{2}}C\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{2a^5} - \frac{3\sqrt{\frac{3\pi}{2}}C\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{4a^5} - \frac{\sqrt{\frac{5\pi}{2}}C\left(\sqrt{\frac{10}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{4a^5}$$

Mathematica [C] time = 0.199778, size = 233, normalized size = 1.71

$$\frac{i\left(2\sqrt{-i\cos^{-1}(ax)}\mathrm{Gamma}\left(\frac{1}{2},-i\cos^{-1}(ax)\right)-2\sqrt{i\cos^{-1}(ax)}\mathrm{Gamma}\left(\frac{1}{2},i\cos^{-1}(ax)\right)+3\sqrt{3}\sqrt{-i\cos^{-1}(ax)}\mathrm{Gamma}\left(\frac{1}{2}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/ArcCos[a*x]^(3/2),x]
```

```
[Out] ((I/16)*((-4*I)*Sqrt[1 - a^2*x^2] + 2*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-I
)*ArcCos[a*x]] - 2*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, I*ArcCos[a*x]] + 3*Sqrt[3
]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-3*I)*ArcCos[a*x]] - 3*Sqrt[3]*Sqrt[I*
ArcCos[a*x]]*Gamma[1/2, (3*I)*ArcCos[a*x]] + Sqrt[5]*Sqrt[(-I)*ArcCos[a*x]]
*Gamma[1/2, (-5*I)*ArcCos[a*x]] - Sqrt[5]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (5
*I)*ArcCos[a*x]] - (6*I)*Sin[3*ArcCos[a*x]] - (2*I)*Sin[5*ArcCos[a*x]]))/(a
^5*Sqrt[ArcCos[a*x]])
```

Maple [A] time = 0.089, size = 140, normalized size = 1.

$$-\frac{1}{8\,a^5}\left(\sqrt{5}\sqrt{2}\sqrt{\arccos\left(ax\right)}\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{5}\sqrt{2}}{\sqrt{\pi}}\sqrt{\arccos\left(ax\right)}\right)+3\sqrt{3}\sqrt{2}\sqrt{\arccos\left(ax\right)}\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{3}\sqrt{2}\sqrt{\arccos\left(a\right.}}{\sqrt{\pi}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arccos(a*x)^(3/2),x)
```

```
[Out] -1/8/a^5*(5^(1/2)*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1
/2)*5^(1/2)*arccos(a*x)^(1/2))+3*3^(1/2)*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)
*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*arccos(a*x)^(1/2))+2*2^(1/2)*arccos(a*x)
^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))-3*sin(3*arccos
(a*x))-sin(5*arccos(a*x))-2*(-a^2*x^2+1)^(1/2))/arccos(a*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccos(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccos(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

<u>Sympy</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{acos}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/acos(a*x)**(3/2),x)
```

```
[Out] Integral(x**4/acos(a*x)**(3/2), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\arccos(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccos(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/arccos(a*x)^(3/2), x)
```

3.102 $\int \frac{x^3}{\cos^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a^4}-\frac{\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^4}+\frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}}$$

```
[Out] (2*x^3*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (Sqrt[Pi/2]*FresnelC[2*Sq
rt[2/Pi]*Sqrt[ArcCos[a*x]]])/a^4 - (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])
/Sqrt[Pi]])/a^4
```

Rubi [A] time = 0.0651208, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4632, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a^4}-\frac{\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^4}+\frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcCos[a*x]^(3/2),x]
```

```
[Out] (2*x^3*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (Sqrt[Pi/2]*FresnelC[2*Sq
rt[2/Pi]*Sqrt[ArcCos[a*x]]])/a^4 - (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])
/Sqrt[Pi]])/a^4
```

Rule 4632

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dis
t[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), Co
s[x]^(m - 1)*(m - (m + 1)*Cos[x]^2), x], x], x, ArcCos[c*x]], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^3}{\cos^{-1}(ax)^{3/2}}\,dx = \frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} + \frac{2\,\mathrm{Subst}\left(\int\left(-\frac{\cos(2x)}{2\sqrt{x}}-\frac{\cos(4x)}{2\sqrt{x}}\right)dx, x, \cos^{-1}(ax)\right)}{a^4}$$

$$= \frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} - \frac{\mathrm{Subst}\left(\int\frac{\cos(2x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{a^4} - \frac{\mathrm{Subst}\left(\int\frac{\cos(4x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{a^4}$$

$$= \frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} - \frac{2\,\mathrm{Subst}\left(\int\cos\left(2x^2\right)dx, x, \sqrt{\cos^{-1}(ax)}\right)}{a^4} - \frac{2\,\mathrm{Subst}\left(\int\cos\left(4x^2\right)dx, x, \sqrt{\cos^{-1}(a}\right.}{a^4}$$

$$= \frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} - \frac{\sqrt{\frac{\pi}{2}}C\left(2\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a^4} - \frac{\sqrt{\pi}C\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^4}$$

Mathematica [C] time = 0.39551, size = 154, normalized size = 1.69

$$\frac{i\sqrt{2}\sqrt{-i\cos^{-1}(ax)}\mathrm{Gamma}\left(\frac{1}{2},-2i\cos^{-1}(ax)\right) - i\sqrt{2}\sqrt{i\cos^{-1}(ax)}\mathrm{Gamma}\left(\frac{1}{2},2i\cos^{-1}(ax)\right) + i\sqrt{-i\cos^{-1}(ax)}\mathrm{Gamma}\left(\right.}{4a^4\sqrt{\cos^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/ArcCos[a*x]^(3/2),x]
```

```
[Out] (I*Sqrt[2]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-2*I)*ArcCos[a*x]] - I*Sqrt[2
]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (2*I)*ArcCos[a*x]] + I*Sqrt[(-I)*ArcCos[a*
x]]*Gamma[1/2, (-4*I)*ArcCos[a*x]] - I*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (4*I)
*ArcCos[a*x]] + 2*Sin[2*ArcCos[a*x]] + Sin[4*ArcCos[a*x]])/(4*a^4*Sqrt[ArcC
os[a*x]])
```

Maple [A] time = 0.074, size = 81, normalized size = 0.9

$$\frac{1}{4\,a^4}\left(-2\,\sqrt{2}\sqrt{\pi}\sqrt{\arccos(ax)}\mathrm{FresnelC}\left(2\,\frac{\sqrt{2}\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right) - 4\,\sqrt{\pi}\sqrt{\arccos(ax)}\mathrm{FresnelC}\left(2\,\frac{\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right) + 2\,\sin\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccos(a*x)^(3/2),x)
```

```
[Out] 1/4/a^4/arccos(a*x)^(1/2)*(-2*2^(1/2)*Pi^(1/2)*arccos(a*x)^(1/2)*FresnelC(2
*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))-4*Pi^(1/2)*arccos(a*x)^(1/2)*FresnelC(
2*arccos(a*x)^(1/2)/Pi^(1/2))+2*sin(2*arccos(a*x))+sin(4*arccos(a*x)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccos(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccos(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{acos}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acos(a*x)**(3/2),x)
```

```
[Out] Integral(x**3/acos(a*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\arccos(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccos(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/arccos(a*x)^(3/2), x)
```

3.103 $\int \frac{x^2}{\cos^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a^3}-\frac{\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a^3}+\frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}}$$

```
[Out] (2*x^2*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (Sqrt[Pi/2]*FresnelC[Sqrt
[2/Pi]*Sqrt[ArcCos[a*x]]])/a^3 - (Sqrt[(3*Pi)/2]*FresnelC[Sqrt[6/Pi]*Sqrt[A
rcCos[a*x]]])/a^3
```

Rubi [A] time = 0.0687269, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4632, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a^3}-\frac{\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a^3}+\frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcCos[a*x]^(3/2),x]
```

```
[Out] (2*x^2*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (Sqrt[Pi/2]*FresnelC[Sqrt
[2/Pi]*Sqrt[ArcCos[a*x]]])/a^3 - (Sqrt[(3*Pi)/2]*FresnelC[Sqrt[6/Pi]*Sqrt[A
rcCos[a*x]]])/a^3
```

Rule 4632

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dis
t[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), Co
s[x]^(m - 1)*(m - (m + 1)*Cos[x]^2), x], x], x, ArcCos[c*x]], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\cos^{-1}(ax)^{3/2}}\,dx &= \frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} + \frac{2\,\mathrm{Subst}\left(\int\left(-\frac{\cos(x)}{4\sqrt{x}}-\frac{3\cos(3x)}{4\sqrt{x}}\right)dx,x,\cos^{-1}(ax)\right)}{a^3}\\
&= \frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} - \frac{\mathrm{Subst}\left(\int\frac{\cos(x)}{\sqrt{x}}\,dx,x,\cos^{-1}(ax)\right)}{2a^3} - \frac{3\,\mathrm{Subst}\left(\int\frac{\cos(3x)}{\sqrt{x}}\,dx,x,\cos^{-1}(ax)\right)}{2a^3}\\
&= \frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} - \frac{\mathrm{Subst}\left(\int\cos\left(x^2\right)dx,x,\sqrt{\cos^{-1}(ax)}\right)}{a^3} - \frac{3\,\mathrm{Subst}\left(\int\cos\left(3x^2\right)dx,x,\sqrt{\cos^{-1}(ax)}\right)}{a^3}\\
&= \frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} - \frac{\sqrt{\frac{\pi}{2}}C\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a^3} - \frac{\sqrt{\frac{3\pi}{2}}C\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a^3}
\end{aligned}$$

Mathematica [C] time = 0.10879, size = 159, normalized size = 1.64

$$\frac{i\left(\sqrt{-i\cos^{-1}(ax)}\mathrm{Gamma}\left(\frac{1}{2},-i\cos^{-1}(ax)\right)-\sqrt{i\cos^{-1}(ax)}\mathrm{Gamma}\left(\frac{1}{2},i\cos^{-1}(ax)\right)+\sqrt{3}\sqrt{-i\cos^{-1}(ax)}\mathrm{Gamma}\left(\frac{1}{2},-3\right.\right.}{4a^3\sqrt{\cos^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/ArcCos[a*x]^(3/2),x]
```

```
[Out] ((I/4)*((-2*I)*Sqrt[1 - a^2*x^2] + Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-I)*A
rcCos[a*x]] - Sqrt[I*ArcCos[a*x]]*Gamma[1/2, I*ArcCos[a*x]] + Sqrt[3]*Sqrt[
(-I)*ArcCos[a*x]]*Gamma[1/2, (-3*I)*ArcCos[a*x]] - Sqrt[3]*Sqrt[I*ArcCos[a*
x]]*Gamma[1/2, (3*I)*ArcCos[a*x]] - (2*I)*Sin[3*ArcCos[a*x]]))/(a^3*Sqrt[Ar
cCos[a*x]])
```

Maple [A] time = 0.079, size = 96, normalized size = 1.

$$-\frac{1}{2a^3}\left(\sqrt{3}\sqrt{2}\sqrt{\arccos(ax)}\sqrt{\pi}\mathrm{FresnelC}\left(\frac{\sqrt{3}\sqrt{2}}{\sqrt{\pi}}\sqrt{\arccos(ax)}\right)+\sqrt{2}\sqrt{\arccos(ax)}\sqrt{\pi}\mathrm{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\sqrt{\arccos(ax)}\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccos(a*x)^(3/2),x)
```

```
[Out] -1/2/a^3*(3^(1/2)*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1
/2)*3^(1/2)*arccos(a*x)^(1/2))+2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(
2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))-sin(3*arccos(a*x))-(-a^2*x^2+1)^(1/2))/
arccos(a*x)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccos(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccos(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{acos}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acos(a*x)**(3/2),x)
```

```
[Out] Integral(x**2/acos(a*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\arccos(ax)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccos(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/arccos(a*x)^(3/2), x)
```

3.104 $\int \frac{x}{\cos^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}}-\frac{2\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^2}$$

```
[Out] (2*x*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (2*Sqrt[Pi]*FresnelC[(2*Sqr
t[ArcCos[a*x]])/Sqrt[Pi]])/a^2
```

Rubi [A] time = 0.0310101, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4632, 3304, 3352}

$$\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}}-\frac{2\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcCos[a*x]^(3/2),x]
```

```
[Out] (2*x*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (2*Sqrt[Pi]*FresnelC[(2*Sqr
t[ArcCos[a*x]])/Sqrt[Pi]])/a^2
```

Rule 4632

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dis
t[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), Co
s[x]^(m - 1)*(m - (m + 1)*Cos[x]^2), x], x], x, ArcCos[c*x]], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\cos^{-1}(ax)^{3/2}}\,dx &= \frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} - \frac{2\,\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{a^2}\\ &= \frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} - \frac{4\,\text{Subst}\left(\int \cos\left(2x^2\right)dx, x, \sqrt{\cos^{-1}(ax)}\right)}{a^2}\\ &= \frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} - \frac{2\sqrt{\pi}C\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.0482697, size = 44, normalized size = 0.8

$$\frac{\frac{\sin\left(2\cos^{-1}(ax)\right)}{\sqrt{\cos^{-1}(ax)}} - 2\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcCos[a*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]] + Sin[2*ArcCos[a*x]]/
Sqrt[ArcCos[a*x]])/a^2
```

Maple [A] time = 0.066, size = 42, normalized size = 0.8

$$\frac{1}{a^2}\left(-2\sqrt{\pi}\sqrt{\arccos(ax)}\,\text{FresnelC}\left(2\,\frac{\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right) + \sin(2\arccos(ax))\right)\frac{1}{\sqrt{\arccos(ax)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccos(a*x)^(3/2),x)
```

```
[Out] 1/a^2/arccos(a*x)^(1/2)*(-2*Pi^(1/2)*arccos(a*x)^(1/2)*FresnelC(2*arccos(a*
x)^(1/2)/Pi^(1/2))+sin(2*arccos(a*x)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{acos}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acos(a*x)**(3/2),x)
```

```
[Out] Integral(x/acos(a*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\arccos(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/arccos(a*x)^(3/2), x)
```

3.105 $\int \frac{1}{\cos^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=59

$$\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}}-\frac{2\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a}$$

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (2*Sqrt[2*Pi]*FresnelC[Sqrt[2
/Pi]*Sqrt[ArcCos[a*x]]])/a
```

Rubi [A] time = 0.0867448, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4622, 4724, 3304, 3352}

$$\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}}-\frac{2\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^(-3/2),x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (2*Sqrt[2*Pi]*FresnelC[Sqrt[2
/Pi]*Sqrt[ArcCos[a*x]]])/a
```

Rule 4622

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(Sqrt[1 - c
^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1))
, Int[(x*(a + b*ArcCos[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && LtQ[n, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\cos^{-1}(ax)^{3/2}}\,dx &= \frac{2\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} + (2a)\int \frac{x}{\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}\,dx \\
&= \frac{2\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} - \frac{2\,\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{a} \\
&= \frac{2\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} - \frac{4\,\text{Subst}\left(\int \cos\left(x^2\right)\,dx, x, \sqrt{\cos^{-1}(ax)}\right)}{a} \\
&= \frac{2\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}} - \frac{2\sqrt{2\pi}C\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a}
\end{aligned}$$

Mathematica [C] time = 0.0346069, size = 86, normalized size = 1.46

$$-\frac{-i\sqrt{-i\cos^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-i\cos^{-1}(ax)\right)+i\sqrt{i\cos^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},i\cos^{-1}(ax)\right)-2\sqrt{1-a^2x^2}}{a\sqrt{\cos^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCos[a*x]^(-3/2),x]
```

```
[Out] -((-2*Sqrt[1 - a^2*x^2] - I*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-I)*ArcCos[a
*x]] + I*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, I*ArcCos[a*x]])/(a*Sqrt[ArcCos[a*x]
]))
```

Maple [A] time = 0.07, size = 66, normalized size = 1.1

$$-\frac{\sqrt{2}}{a\sqrt{\pi}\arccos(ax)}\left(2\arccos(ax)\,\pi\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-\sqrt{2}\sqrt{\arccos(ax)}\sqrt{\pi}\sqrt{-a^2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos(a*x)^(3/2),x)
```

```
[Out] -1/a*2^(1/2)/Pi^(1/2)/arccos(a*x)*(2*arccos(a*x)*Pi*FresnelC(2^(1/2)/Pi^(1/
2)*arccos(a*x)^(1/2))-2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*(-a^2*x^2+1)^(1/2)
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\operatorname{acos}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acos(a*x)**(3/2),x)
```

```
[Out] Integral(acos(a*x)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\arccos{(ax)}^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^(-3/2), x)
```

3.106 $\int \frac{1}{x \cos^{-1}(ax)^{3/2}} \, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{1}{x \cos^{-1}(ax)^{3/2}}, x\right)$$

```
[Out] Unintegrable[1/(x*ArcCos[a*x]^(3/2)), x]
```

Rubi [A] time = 0.0123989, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x \cos^{-1}(ax)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcCos[a*x]^(3/2)),x]
```

```
[Out] Defer[Int][1/(x*ArcCos[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{x \cos^{-1}(ax)^{3/2}} \, dx = \int \frac{1}{x \cos^{-1}(ax)^{3/2}} \, dx$$

Mathematica [A] time = 0.44617, size = 0, normalized size = 0.

$$\int \frac{1}{x \cos^{-1}(ax)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcCos[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x*ArcCos[a*x]^(3/2)), x]
```

Maple [A] time = 0.095, size = 0, normalized size = 0.

$$\int \frac{1}{x} (\arccos{(ax)})^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccos(a*x)^(3/2),x)
```

```
[Out] int(1/x/arccos(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccos(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccos(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{acos}^{\frac{3}{2}}{(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acos(a*x)**(3/2),x)
```

```
[Out] Integral(1/(x*acos(a*x)**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \arccos{(ax)}^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccos(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*arccos(a*x)^(3/2)), x)
```

3.107 $\int \frac{x^4}{\cos^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=235

$$-\frac{4\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a^5}+\frac{25\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{3a^5}-\frac{4\sqrt{\frac{2\pi}{3}}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a^5}+\frac{25\sqrt{\frac{\pi}{6}}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{2a^5}+\frac{5\cdots}{\cdots}$$

```
[Out] (2*x^4*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^(3/2)) - (16*x^3)/(3*a^2*Sqrt[Ar
cCos[a*x]]) + (20*x^5)/(3*Sqrt[ArcCos[a*x]]) + (25*Sqrt[Pi/2]*FresnelS[Sqrt
[2/Pi]*Sqrt[ArcCos[a*x]]])/(3*a^5) - (4*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt
[ArcCos[a*x]]])/a^5 + (25*Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]]
)/(2*a^5) - (4*Sqrt[(2*Pi)/3]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/a^5 +
 (5*Sqrt[(5*Pi)/2]*FresnelS[Sqrt[10/Pi]*Sqrt[ArcCos[a*x]]])/(6*a^5)
```

Rubi [A] time = 0.4096, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4634, 4720, 4636, 4406, 3305, 3351}

$$-\frac{4\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a^5}+\frac{25\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{3a^5}-\frac{4\sqrt{\frac{2\pi}{3}}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a^5}+\frac{25\sqrt{\frac{\pi}{6}}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{2a^5}+\frac{5\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcCos[a*x]^(5/2),x]
```

```
[Out] (2*x^4*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^(3/2)) - (16*x^3)/(3*a^2*Sqrt[Ar
cCos[a*x]]) + (20*x^5)/(3*Sqrt[ArcCos[a*x]]) + (25*Sqrt[Pi/2]*FresnelS[Sqrt
[2/Pi]*Sqrt[ArcCos[a*x]]])/(3*a^5) - (4*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt
[ArcCos[a*x]]])/a^5 + (25*Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]]
)/(2*a^5) - (4*Sqrt[(2*Pi)/3]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/a^5 +
 (5*Sqrt[(5*Pi)/2]*FresnelS[Sqrt[10/Pi]*Sqrt[ArcCos[a*x]]])/(6*a^5)
```

Rule 4634

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-D
ist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCos[c*x])^(n + 1))/Sq
rt[1 - c^2*x^2], x], x] + Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*ArcCo
s[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[
```

```
m, 0] && LtQ[n, -2]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^4}{\cos^{-1}(ax)^{5/2}}\,dx = \frac{2x^4\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{8\int \frac{x^3}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}\,dx}{3a} + \frac{1}{3}(10a)\int \frac{x^5}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}\,dx$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\cos^{-1}(ax)}} + \frac{20x^5}{3\sqrt{\cos^{-1}(ax)}} - \frac{100}{3}\int \frac{x^4}{\sqrt{\cos^{-1}(ax)}}\,dx + \frac{16\int \frac{x^2}{\sqrt{\cos^{-1}(ax)}}\,dx}{a^2}$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\cos^{-1}(ax)}} + \frac{20x^5}{3\sqrt{\cos^{-1}(ax)}} - \frac{16\,\text{Subst}\left(\int \frac{\cos^2(x)\sin(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{a^5} + \cdots$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\cos^{-1}(ax)}} + \frac{20x^5}{3\sqrt{\cos^{-1}(ax)}} - \frac{16\,\text{Subst}\left(\int \left(\frac{\sin(x)}{4\sqrt{x}} + \frac{\sin(3x)}{4\sqrt{x}}\right) dx, x, \cos^{-1}(ax\cdots\right.}{a^5}$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\cos^{-1}(ax)}} + \frac{20x^5}{3\sqrt{\cos^{-1}(ax)}} + \frac{25\,\text{Subst}\left(\int \frac{\sin(5x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{12a^5} - \frac{4\,\text{Sub}\cdots}{}$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\cos^{-1}(ax)}} + \frac{20x^5}{3\sqrt{\cos^{-1}(ax)}} + \frac{25\,\text{Subst}\left(\int \sin\left(5x^2\right) dx, x, \sqrt{\cos^{-1}(ax)}\right)}{6a^5} - \cdots$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\cos^{-1}(ax)}} + \frac{20x^5}{3\sqrt{\cos^{-1}(ax)}} + \frac{25\sqrt{\frac{\pi}{2}}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{3a^5} - \frac{4\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{c\cdots}\right.}{a^5}$$

Mathematica [C] time = 1.63439, size = 322, normalized size = 1.37

$$-\frac{2\left(\sqrt{-i\cos^{-1}(ax)}\cos^{-1}(ax)\text{Gamma}\left(\frac{1}{2}, -i\cos^{-1}(ax)\right) + \sqrt{i\cos^{-1}(ax)}\cos^{-1}(ax)\text{Gamma}\left(\frac{1}{2}, i\cos^{-1}(ax)\right) - \sqrt{1-a^2x^2}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/ArcCos[a*x]^(5/2),x]
```

```
[Out] -(2*(-Sqrt[1 - a^2*x^2] - ArcCos[a*x]/E^(I*ArcCos[a*x]) - E^(I*ArcCos[a*x])
*ArcCos[a*x] + Sqrt[(-I)*ArcCos[a*x]]*ArcCos[a*x]*Gamma[1/2, (-I)*ArcCos[a*
x]] + Sqrt[I*ArcCos[a*x]]*ArcCos[a*x]*Gamma[1/2, I*ArcCos[a*x]]) - 5*ArcCos
[a*x]*(E^((-5*I)*ArcCos[a*x]) + E^((5*I)*ArcCos[a*x]) - Sqrt[5]*Sqrt[(-I)*A
rcCos[a*x]]*Gamma[1/2, (-5*I)*ArcCos[a*x]] - Sqrt[5]*Sqrt[I*ArcCos[a*x]]*Ga
mma[1/2, (5*I)*ArcCos[a*x]]) - 3*(3*ArcCos[a*x]*(E^((-3*I)*ArcCos[a*x]) + E
^((3*I)*ArcCos[a*x]) - Sqrt[3]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-3*I)*Arc
Cos[a*x]] - Sqrt[3]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (3*I)*ArcCos[a*x]]) + Si
n[3*ArcCos[a*x]]) - Sin[5*ArcCos[a*x]])/(24*a^5*ArcCos[a*x]^(3/2))
```

Maple [A] time = 0.102, size = 173, normalized size = 0.7

$$\frac{1}{24\,a^{5}}\left(18\,\sqrt{2}\sqrt{\pi}\sqrt{3}\mathrm{FresnelS}\left(\frac{\sqrt{2}\sqrt{3}\sqrt{\arccos\left(ax\right)}}{\sqrt{\pi}}\right)\left(\arccos\left(ax\right)\right)^{3/2}+10\,\sqrt{2}\sqrt{\pi}\sqrt{5}\mathrm{FresnelS}\left(\frac{\sqrt{2}\sqrt{5}\sqrt{\arccos\left(ax\right)}}{\sqrt{\pi}}\right)(\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arccos(a*x)^(5/2),x)
```

```
[Out] 1/24/a^5*(18*2^(1/2)*Pi^(1/2)*3^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*arc
cos(a*x)^(1/2))*arccos(a*x)^(3/2)+10*2^(1/2)*Pi^(1/2)*5^(1/2)*FresnelS(2^(1
/2)/Pi^(1/2)*5^(1/2)*arccos(a*x)^(1/2))*arccos(a*x)^(3/2)+4*2^(1/2)*Pi^(1/2
)*FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*arccos(a*x)^(3/2)+4*a*x*arcc
os(a*x)+18*arccos(a*x)*cos(3*arccos(a*x))+10*arccos(a*x)*cos(5*arccos(a*x))
+2*(-a^2*x^2+1)^(1/2)+3*sin(3*arccos(a*x))+sin(5*arccos(a*x)))/arccos(a*x)^
(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccos(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccos(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{acos}^{\frac{5}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/acos(a*x)**(5/2),x)
```

```
[Out] Integral(x**4/acos(a*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\arccos(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccos(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/arccos(a*x)^(5/2), x)
```

3.108 $\int \frac{x^3}{\cos^{-1}(ax)^{5/2}} dx$

Optimal. Leaf size=126

$$\frac{4\sqrt{2\pi}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{3a^4}+\frac{4\sqrt{\pi}S\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{3a^4}+\frac{2x^3\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}}-\frac{4x^2}{a^2\sqrt{\cos^{-1}(ax)}}+\frac{16x^4}{3\sqrt{\cos^{-1}(ax)}}$$

```
[Out] (2*x^3*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^(3/2)) - (4*x^2)/(a^2*Sqrt[ArcCo
s[a*x]]) + (16*x^4)/(3*Sqrt[ArcCos[a*x]]) + (4*Sqrt[2*Pi]*FresnelS[2*Sqrt[2
/Pi]*Sqrt[ArcCos[a*x]]])/(3*a^4) + (4*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]
])/Sqrt[Pi]])/(3*a^4)
```

Rubi [A] time = 0.325012, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {4634, 4720, 4636, 4406, 3305, 3351, 12}

$$\frac{4\sqrt{2\pi}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{3a^4}+\frac{4\sqrt{\pi}S\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{3a^4}+\frac{2x^3\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}}-\frac{4x^2}{a^2\sqrt{\cos^{-1}(ax)}}+\frac{16x^4}{3\sqrt{\cos^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcCos[a*x]^(5/2),x]
```

```
[Out] (2*x^3*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^(3/2)) - (4*x^2)/(a^2*Sqrt[ArcCo
s[a*x]]) + (16*x^4)/(3*Sqrt[ArcCos[a*x]]) + (4*Sqrt[2*Pi]*FresnelS[2*Sqrt[2
/Pi]*Sqrt[ArcCos[a*x]]])/(3*a^4) + (4*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]
])/Sqrt[Pi]])/(3*a^4)
```

Rule 4634

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-D
ist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCos[c*x])^(n + 1))/Sq
rt[1 - c^2*x^2], x], x] + Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*ArcCo
s[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[
m, 0] && LtQ[n, -2]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{x^3}{\cos^{-1}(ax)^{5/2}}\,dx = \frac{2x^3\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{2\int \frac{x^2}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}\,dx}{a} + \frac{1}{3}(8a)\int \frac{x^4}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}\,dx$$

$$= \frac{2x^3\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\cos^{-1}(ax)}} + \frac{16x^4}{3\sqrt{\cos^{-1}(ax)}} - \frac{64}{3}\int \frac{x^3}{\sqrt{\cos^{-1}(ax)}}\,dx + \frac{8\int \frac{x}{\sqrt{\cos^{-1}(ax)}}\,dx}{a^2}$$

$$= \frac{2x^3\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\cos^{-1}(ax)}} + \frac{16x^4}{3\sqrt{\cos^{-1}(ax)}} - \frac{8\,\mathrm{Subst}\left(\int \frac{\cos(x)\sin(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{a^4} + \frac{64\,\mathrm{Su}}{}$$

$$= \frac{2x^3\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\cos^{-1}(ax)}} + \frac{16x^4}{3\sqrt{\cos^{-1}(ax)}} - \frac{8\,\mathrm{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{a^4} + \frac{64\,\mathrm{Subst}}{}$$

$$= \frac{2x^3\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\cos^{-1}(ax)}} + \frac{16x^4}{3\sqrt{\cos^{-1}(ax)}} + \frac{8\,\mathrm{Subst}\left(\int \frac{\sin(4x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{3a^4} - \frac{4\,\mathrm{Subst}\left(\right.}{}$$

$$= \frac{2x^3\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\cos^{-1}(ax)}} + \frac{16x^4}{3\sqrt{\cos^{-1}(ax)}} + \frac{16\,\mathrm{Subst}\left(\int \sin\left(4x^2\right)dx, x, \sqrt{\cos^{-1}(ax)}\right)}{3a^4} - \frac{8\,\mathrm{S}}{}$$

$$= \frac{2x^3\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\cos^{-1}(ax)}} + \frac{16x^4}{3\sqrt{\cos^{-1}(ax)}} + \frac{4\sqrt{2\pi}S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{3a^4} + \frac{4\sqrt{\pi}S\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right.}{3a^4}$$

Mathematica [C] time = 0.890657, size = 203, normalized size = 1.61

$$-\frac{-4\cos^{-1}(ax)\left(-2\sqrt{-i\cos^{-1}(ax)}\mathrm{Gamma}\left(\frac{1}{2}, -4i\cos^{-1}(ax)\right) - 2\sqrt{i\cos^{-1}(ax)}\mathrm{Gamma}\left(\frac{1}{2}, 4i\cos^{-1}(ax)\right) + e^{-4i\cos^{-1}(ax)} + e\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/ArcCos[a*x]^(5/2),x]
```

```
[Out] -(-4*ArcCos[a*x]*(E^((-4*I)*ArcCos[a*x]) + E^((4*I)*ArcCos[a*x]) - 2*Sqrt[(
-I)*ArcCos[a*x]]*Gamma[1/2, (-4*I)*ArcCos[a*x]] - 2*Sqrt[I*ArcCos[a*x]]*Gam
ma[1/2, (4*I)*ArcCos[a*x]]) - 2*(2*ArcCos[a*x]*(E^((-2*I)*ArcCos[a*x]) + E^
((2*I)*ArcCos[a*x]) - Sqrt[2]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-2*I)*ArcC
os[a*x]] - Sqrt[2]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (2*I)*ArcCos[a*x]]) + Sin
[2*ArcCos[a*x]]) - Sin[4*ArcCos[a*x]])/(12*a^4*ArcCos[a*x]^(3/2))
```

Maple [A] time = 0.085, size = 107, normalized size = 0.9

$$\frac{1}{12\,a^4}\left(16\,\sqrt{2}\sqrt{\pi}\mathrm{FresnelS}\left(2\,\frac{\sqrt{2}\sqrt{\arccos\left(ax\right)}}{\sqrt{\pi}}\right)\left(\arccos\left(ax\right)\right)^{3/2}+16\,\sqrt{\pi}\mathrm{FresnelS}\left(2\,\frac{\sqrt{\arccos\left(ax\right)}}{\sqrt{\pi}}\right)\left(\arccos\left(ax\right)\right)^{3/2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccos(a*x)^(5/2),x)
```

```
[Out] 1/12/a^4*(16*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2)
)*arccos(a*x)^(3/2)+16*Pi^(1/2)*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2))*arcc
os(a*x)^(3/2)+8*arccos(a*x)*cos(2*arccos(a*x))+8*arccos(a*x)*cos(4*arccos(a
*x))+2*sin(2*arccos(a*x))+sin(4*arccos(a*x)))/arccos(a*x)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccos(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccos(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{acos}^{\frac{5}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acos(a*x)**(5/2),x)
```

```
[Out] Integral(x**3/acos(a*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\arccos(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccos(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/arccos(a*x)^(5/2), x)
```

3.109 $\int \frac{x^2}{\cos^{-1}(ax)^{5/2}} dx$

Optimal. Leaf size=125

$$\frac{\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{3a^3}+\frac{\sqrt{6\pi}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a^3}+\frac{2x^2\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}}-\frac{8x}{3a^2\sqrt{\cos^{-1}(ax)}}+\frac{4x^3}{\sqrt{\cos^{-1}(ax)}}$$

```
[Out] (2*x^2*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^(3/2)) - (8*x)/(3*a^2*Sqrt[ArcCo
s[a*x]]) + (4*x^3)/Sqrt[ArcCos[a*x]] + (Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt
[ArcCos[a*x]]])/(3*a^3) + (Sqrt[6*Pi]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]
])/a^3
```

Rubi [A] time = 0.286432, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {4634, 4720, 4636, 4406, 3305, 3351, 4624}

$$\frac{\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{3a^3}+\frac{\sqrt{6\pi}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a^3}+\frac{2x^2\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}}-\frac{8x}{3a^2\sqrt{\cos^{-1}(ax)}}+\frac{4x^3}{\sqrt{\cos^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcCos[a*x]^(5/2),x]
```

```
[Out] (2*x^2*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^(3/2)) - (8*x)/(3*a^2*Sqrt[ArcCo
s[a*x]]) + (4*x^3)/Sqrt[ArcCos[a*x]] + (Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt
[ArcCos[a*x]]])/(3*a^3) + (Sqrt[6*Pi]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]
])/a^3
```

Rule 4634

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-D
ist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCos[c*x])^(n + 1))/Sq
rt[1 - c^2*x^2], x], x] + Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*ArcCo
s[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[
m, 0] && LtQ[n, -2]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4624

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Sub
st[Int[x^n*Sin[a/b - x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c,
 n}, x]
```

Rubi steps

$$\int \frac{x^2}{\cos^{-1}(ax)^{5/2}}\,dx = \frac{2x^2\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{4\int \frac{x}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}\,dx}{3a} + (2a)\int \frac{x^3}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}\,dx$$

$$= \frac{2x^2\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\cos^{-1}(ax)}} + \frac{4x^3}{\sqrt{\cos^{-1}(ax)}} - 12\int \frac{x^2}{\sqrt{\cos^{-1}(ax)}}\,dx + \frac{8\int \frac{1}{\sqrt{\cos^{-1}(ax)}}\,dx}{3a^2}$$

$$= \frac{2x^2\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\cos^{-1}(ax)}} + \frac{4x^3}{\sqrt{\cos^{-1}(ax)}} - \frac{8\,\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{3a^3} + \frac{12\,\text{Subst}}{}$$

$$= \frac{2x^2\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\cos^{-1}(ax)}} + \frac{4x^3}{\sqrt{\cos^{-1}(ax)}} - \frac{16\,\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{\cos^{-1}(ax)}\right)}{3a^3} + \frac{12}{}$$

$$= \frac{2x^2\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\cos^{-1}(ax)}} + \frac{4x^3}{\sqrt{\cos^{-1}(ax)}} - \frac{8\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{3a^3} + \frac{3\,\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\right.}{a}$$

$$= \frac{2x^2\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\cos^{-1}(ax)}} + \frac{4x^3}{\sqrt{\cos^{-1}(ax)}} - \frac{8\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{3a^3} + \frac{6\,\text{Subst}\left(\int \sin\left(x\right.\right.}{}$$

$$= \frac{2x^2\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\cos^{-1}(ax)}} + \frac{4x^3}{\sqrt{\cos^{-1}(ax)}} + \frac{\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{3a^3} + \frac{\sqrt{6\pi}S\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-}}\right.}{a^3}$$

Mathematica [C] time = 0.821265, size = 220, normalized size = 1.76

$$-\frac{\sqrt{-i\cos^{-1}(ax)}\cos^{-1}(ax)\text{Gamma}\left(\frac{1}{2}, -i\cos^{-1}(ax)\right) + \sqrt{i\cos^{-1}(ax)}\cos^{-1}(ax)\text{Gamma}\left(\frac{1}{2}, i\cos^{-1}(ax)\right) - 3\cos^{-1}(ax)\left(-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/ArcCos[a*x]^(5/2),x]
```

```
[Out] -(-Sqrt[1 - a^2*x^2] - ArcCos[a*x]/E^(I*ArcCos[a*x]) - E^(I*ArcCos[a*x])*Ar
cCos[a*x] + Sqrt[(-I)*ArcCos[a*x]]*ArcCos[a*x]*Gamma[1/2, (-I)*ArcCos[a*x]]
 + Sqrt[I*ArcCos[a*x]]*ArcCos[a*x]*Gamma[1/2, I*ArcCos[a*x]] - 3*ArcCos[a*x
]*(E^((-3*I)*ArcCos[a*x]) + E^((3*I)*ArcCos[a*x]) - Sqrt[3]*Sqrt[(-I)*ArcCo
s[a*x]]*Gamma[1/2, (-3*I)*ArcCos[a*x]] - Sqrt[3]*Sqrt[I*ArcCos[a*x]]*Gamma[
1/2, (3*I)*ArcCos[a*x]]) - Sin[3*ArcCos[a*x]])/(6*a^3*ArcCos[a*x]^(3/2))
```

Maple [A] time = 0.085, size = 115, normalized size = 0.9

$$\frac{1}{6\,a^{3}}\left(6\,\sqrt{2}\sqrt{\pi}\sqrt{3}\mathrm{FresnelS}\left(\frac{\sqrt{2}\sqrt{3}\sqrt{\arccos\left(ax\right)}}{\sqrt{\pi}}\right)\left(\arccos\left(ax\right)\right)^{3/2}+2\,\sqrt{2}\sqrt{\pi}\mathrm{FresnelS}\left(\frac{\sqrt{2}\sqrt{\arccos\left(ax\right)}}{\sqrt{\pi}}\right)\left(\arccos\left(ax\right)\right)^{3/}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccos(a*x)^(5/2),x)
```

```
[Out] 1/6/a^3*(6*2^(1/2)*Pi^(1/2)*3^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*arcco
s(a*x)^(1/2))*arccos(a*x)^(3/2)+2*2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2
)*arccos(a*x)^(1/2))*arccos(a*x)^(3/2)+2*a*x*arccos(a*x)+6*arccos(a*x)*cos(
3*arccos(a*x))+sin(3*arccos(a*x))+(-a^2*x^2+1)^(1/2))/arccos(a*x)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccos(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccos(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\operatorname{acos}^{\frac{5}{2}}{\left(ax \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acos(a*x)**(5/2),x)
```

```
[Out] Integral(x**2/acos(a*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\arccos(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccos(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/arccos(a*x)^(5/2), x)
```

3.110 $\int \frac{x}{\cos^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=89

$$\frac{8\sqrt{\pi}S\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{3a^2}+\frac{2x\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\cos^{-1}(ax)}}+\frac{8x^2}{3\sqrt{\cos^{-1}(ax)}}$$

```
[Out] (2*x*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^(3/2)) - 4/(3*a^2*Sqrt[ArcCos[a*x]
]) + (8*x^2)/(3*Sqrt[ArcCos[a*x]]) + (8*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*
x]])/Sqrt[Pi]])/(3*a^2)
```

Rubi [A] time = 0.172752, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {4634, 4720, 4636, 4406, 12, 3305, 3351, 4642}

$$\frac{8\sqrt{\pi}S\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{3a^2}+\frac{2x\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\cos^{-1}(ax)}}+\frac{8x^2}{3\sqrt{\cos^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcCos[a*x]^(5/2),x]
```

```
[Out] (2*x*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^(3/2)) - 4/(3*a^2*Sqrt[ArcCos[a*x]
]) + (8*x^2)/(3*Sqrt[ArcCos[a*x]]) + (8*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*
x]])/Sqrt[Pi]])/(3*a^2)
```

Rule 4634

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-D
ist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCos[c*x])^(n + 1))/Sq
rt[1 - c^2*x^2], x], x] + Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*ArcCo
s[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[
m, 0] && LtQ[n, -2]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
```

```
1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{x}{\cos^{-1}(ax)^{5/2}}\,dx = \frac{2x\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{2\int \frac{1}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}\,dx}{3a} + \frac{1}{3}(4a)\int \frac{x^2}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}\,dx$$
$$= \frac{2x\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\cos^{-1}(ax)}} + \frac{8x^2}{3\sqrt{\cos^{-1}(ax)}} - \frac{16}{3}\int \frac{x}{\sqrt{\cos^{-1}(ax)}}\,dx$$
$$= \frac{2x\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\cos^{-1}(ax)}} + \frac{8x^2}{3\sqrt{\cos^{-1}(ax)}} + \frac{16\,\mathrm{Subst}\left(\int \frac{\cos(x)\sin(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{3a^2}$$
$$= \frac{2x\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\cos^{-1}(ax)}} + \frac{8x^2}{3\sqrt{\cos^{-1}(ax)}} + \frac{16\,\mathrm{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{3a^2}$$
$$= \frac{2x\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\cos^{-1}(ax)}} + \frac{8x^2}{3\sqrt{\cos^{-1}(ax)}} + \frac{8\,\mathrm{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{3a^2}$$
$$= \frac{2x\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\cos^{-1}(ax)}} + \frac{8x^2}{3\sqrt{\cos^{-1}(ax)}} + \frac{16\,\mathrm{Subst}\left(\int \sin\left(2x^2\right)\,dx, x, \sqrt{\cos^{-1}(ax)}\right)}{3a^2}$$
$$= \frac{2x\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\cos^{-1}(ax)}} + \frac{8x^2}{3\sqrt{\cos^{-1}(ax)}} + \frac{8\sqrt{\pi}S\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{3a^2}$$

Mathematica [A] time = 0.0832623, size = 61, normalized size = 0.69

$$\frac{8\sqrt{\pi}S\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right) + \frac{4\cos^{-1}(ax)\cos\left(2\cos^{-1}(ax)\right)+\sin\left(2\cos^{-1}(ax)\right)}{\cos^{-1}(ax)^{3/2}}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcCos[a*x]^(5/2),x]
```

```
[Out] (8*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]] + (4*ArcCos[a*x]*Cos[2
*ArcCos[a*x]] + Sin[2*ArcCos[a*x]])/ArcCos[a*x]^(3/2))/(3*a^2)
```

Maple [A] time = 0.069, size = 56, normalized size = 0.6

$$\frac{1}{3\,a^2}\left(8\,\sqrt{\pi}\mathrm{FresnelS}\left(2\,\frac{\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)(\arccos(ax))^{3/2} + 4\,\arccos(ax)\cos(2\,\arccos(ax)) + \sin(2\,\arccos(ax))\right)(\mathrm{arcco}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccos(a*x)^(5/2),x)
```

```
[Out] 1/3/a^2*(8*Pi^(1/2)*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2))*arccos(a*x)^(3/2
)+4*arccos(a*x)*cos(2*arccos(a*x))+sin(2*arccos(a*x)))/arccos(a*x)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{acos}^{\frac{5}{2}}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acos(a*x)**(5/2),x)
```

```
[Out] Integral(x/acos(a*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\arccos(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/arccos(a*x)^(5/2), x)
```

3.111 $\int \frac{1}{\cos^{-1}(ax)^{5/2}} dx$

Optimal. Leaf size=76

$$\frac{2\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}}+\frac{4\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{3a}+\frac{4x}{3\sqrt{\cos^{-1}(ax)}}$$

```
[Out] (2*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^(3/2)) + (4*x)/(3*Sqrt[ArcCos[a*x]])
 + (4*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(3*a)
```

Rubi [A] time = 0.0929396, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4622, 4720, 4624, 3305, 3351}

$$\frac{2\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}}+\frac{4\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{3a}+\frac{4x}{3\sqrt{\cos^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^(-5/2),x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^(3/2)) + (4*x)/(3*Sqrt[ArcCos[a*x]])
 + (4*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(3*a)
```

Rule 4622

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(Sqrt[1 - c
^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1))
, Int[(x*(a + b*ArcCos[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && LtQ[n, -1]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4624

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Sub
st[Int[x^n*Sin[a/b - x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c,
 n}, x]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\cos^{-1}(ax)^{5/2}}\,dx &= \frac{2\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}}+\frac{1}{3}(2a)\int \frac{x}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^{3/2}}\,dx\\
&= \frac{2\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}}+\frac{4x}{3\sqrt{\cos^{-1}(ax)}}-\frac{4}{3}\int \frac{1}{\sqrt{\cos^{-1}(ax)}}\,dx\\
&= \frac{2\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}}+\frac{4x}{3\sqrt{\cos^{-1}(ax)}}+\frac{4\,\mathrm{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx,x,\cos^{-1}(ax)\right)}{3a}\\
&= \frac{2\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}}+\frac{4x}{3\sqrt{\cos^{-1}(ax)}}+\frac{8\,\mathrm{Subst}\left(\int \sin\left(x^2\right)\,dx,x,\sqrt{\cos^{-1}(ax)}\right)}{3a}\\
&= \frac{2\sqrt{1-a^2x^2}}{3a\cos^{-1}(ax)^{3/2}}+\frac{4x}{3\sqrt{\cos^{-1}(ax)}}+\frac{4\sqrt{2\pi}S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{3a}
\end{aligned}$$

Mathematica [C] time = 0.23861, size = 122, normalized size = 1.61

$$-\frac{2\left(\sqrt{-i\cos^{-1}(ax)}\cos^{-1}(ax)\mathrm{Gamma}\left(\frac{1}{2},-i\cos^{-1}(ax)\right)+\sqrt{i\cos^{-1}(ax)}\cos^{-1}(ax)\mathrm{Gamma}\left(\frac{1}{2},i\cos^{-1}(ax)\right)-\sqrt{1-a^2x^2}-\right.}{3a\cos^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCos[a*x]^(-5/2),x]
```

```
[Out] (-2*(-Sqrt[1 - a^2*x^2] - ArcCos[a*x]/E^(I*ArcCos[a*x]) - E^(I*ArcCos[a*x])
*ArcCos[a*x] + Sqrt[(-I)*ArcCos[a*x]]*ArcCos[a*x]*Gamma[1/2, (-I)*ArcCos[a*
x]] + Sqrt[I*ArcCos[a*x]]*ArcCos[a*x]*Gamma[1/2, I*ArcCos[a*x]]))/(3*a*ArcC
os[a*x]^(3/2))
```

Maple [A] time = 0.072, size = 83, normalized size = 1.1

$$\frac{\sqrt{2}}{3\,a\sqrt{\pi}\,(\arccos(ax))^2}\left(4\,\pi\,(\arccos(ax))^2\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+2\,(\arccos(ax))^{3/2}\sqrt{2}\sqrt{\pi}xa+\sqrt{2}\sqrt{\arccos(ax)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos(a*x)^(5/2),x)
```

```
[Out] 1/3/a*2^(1/2)/Pi^(1/2)*(4*Pi*arccos(a*x)^2*FresnelS(2^(1/2)/Pi^(1/2)*arccos
(a*x)^(1/2))+2*arccos(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*x*a+2^(1/2)*arccos(a*x)^(
1/2)*Pi^(1/2)*(-a^2*x^2+1)^(1/2))/arccos(a*x)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{acos}^{\frac{5}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acos(a*x)**(5/2),x)
```

```
[Out] Integral(acos(a*x)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\arccos(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^(-5/2), x)
```

3.112 $\int \frac{1}{x\cos^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{1}{x\cos^{-1}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable[1/(x*ArcCos[a*x]^(5/2)), x]
```

Rubi [A] time = 0.0125784, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\cos^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcCos[a*x]^(5/2)),x]
```

```
[Out] Defer[Int][1/(x*ArcCos[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{1}{x\cos^{-1}(ax)^{5/2}}\,dx=\int \frac{1}{x\cos^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A] time = 0.4656, size = 0, normalized size = 0.

$$\int \frac{1}{x\cos^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcCos[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x*ArcCos[a*x]^(5/2)), x]
```

Maple [A] time = 0.098, size = 0, normalized size = 0.

$$\int \frac{1}{x} (\arccos (ax))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccos(a*x)^(5/2),x)

[Out] int(1/x/arccos(a*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccos(a*x)^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccos(a*x)^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{acos}^{\frac{5}{2}} (ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acos(a*x)**(5/2),x)
```

```
[Out] Integral(1/(x*acos(a*x)**(5/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \arccos(ax)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccos(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*arccos(a*x)^(5/2)), x)
```

3.113 $\int \frac{x^4}{\cos^{-1}(ax)^{7/2}}\,dx$

Optimal. Leaf size=264

$$\frac{\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{15a^5}-\frac{8\sqrt{6\pi}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{5a^5}+\frac{5\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a^5}+\frac{5\sqrt{\frac{5\pi}{2}}\,\text{Fres}}{}$$

```
[Out] (2*x^4*Sqrt[1 - a^2*x^2])/(5*a*ArcCos[a*x]^(5/2)) - (16*x^3)/(15*a^2*ArcCos
[a*x]^(3/2)) + (4*x^5)/(3*ArcCos[a*x]^(3/2)) + (32*x^2*Sqrt[1 - a^2*x^2])/(
5*a^3*Sqrt[ArcCos[a*x]]) - (40*x^4*Sqrt[1 - a^2*x^2])/(3*a*Sqrt[ArcCos[a*x]
]) + (Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(15*a^5) + (5*Sqrt
[(3*Pi)/2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/a^5 - (8*Sqrt[6*Pi]*Fres
nelC[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(5*a^5) + (5*Sqrt[(5*Pi)/2]*FresnelC[Sq
rt[10/Pi]*Sqrt[ArcCos[a*x]]])/(3*a^5)
```

Rubi [A] time = 0.38294, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4634, 4720, 4632, 3304, 3352}

$$\frac{\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{15a^5}-\frac{8\sqrt{6\pi}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{5a^5}+\frac{5\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{a^5}+\frac{5\sqrt{\frac{5\pi}{2}}\,\text{Fres}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcCos[a*x]^(7/2),x]
```


```
[Out] (2*x^4*Sqrt[1 - a^2*x^2])/(5*a*ArcCos[a*x]^(5/2)) - (16*x^3)/(15*a^2*ArcCos
[a*x]^(3/2)) + (4*x^5)/(3*ArcCos[a*x]^(3/2)) + (32*x^2*Sqrt[1 - a^2*x^2])/(
5*a^3*Sqrt[ArcCos[a*x]]) - (40*x^4*Sqrt[1 - a^2*x^2])/(3*a*Sqrt[ArcCos[a*x]
]) + (Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(15*a^5) + (5*Sqrt
[(3*Pi)/2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/a^5 - (8*Sqrt[6*Pi]*Fres
nelC[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(5*a^5) + (5*Sqrt[(5*Pi)/2]*FresnelC[Sq
rt[10/Pi]*Sqrt[ArcCos[a*x]]])/(3*a^5)
```


Rule 4634

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-D
ist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCos[c*x])^(n + 1))/Sq
```

```
rt[1 - c^2*x^2], x], x] + Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*ArcCo
s[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[
m, 0] && LtQ[n, -2]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4632

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dis
t[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), Co
s[x]^(m - 1)*(m - (m + 1)*Cos[x]^2), x], x], x, ArcCos[c*x]], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^4}{\cos^{-1}(ax)^{7/2}}\,dx = \frac{2x^4\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} - \frac{8\int \frac{x^3}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^{5/2}}\,dx}{5a} + (2a)\int \frac{x^5}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^{5/2}}\,dx$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} - \frac{16x^3}{15a^2\cos^{-1}(ax)^{3/2}} + \frac{4x^5}{3\cos^{-1}(ax)^{3/2}} - \frac{20}{3}\int \frac{x^4}{\cos^{-1}(ax)^{3/2}}\,dx + \frac{16\int \frac{x^2}{\cos^{-1}(ax)^{3/2}}\,dx}{5a^2}$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} - \frac{16x^3}{15a^2\cos^{-1}(ax)^{3/2}} + \frac{4x^5}{3\cos^{-1}(ax)^{3/2}} + \frac{32x^2\sqrt{1-a^2x^2}}{5a^3\sqrt{\cos^{-1}(ax)}} - \frac{40x^4\sqrt{1-a^2x^2}}{3a\sqrt{\cos^{-1}(ax)}} + \frac{32\,\text{Sub}}{}$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} - \frac{16x^3}{15a^2\cos^{-1}(ax)^{3/2}} + \frac{4x^5}{3\cos^{-1}(ax)^{3/2}} + \frac{32x^2\sqrt{1-a^2x^2}}{5a^3\sqrt{\cos^{-1}(ax)}} - \frac{40x^4\sqrt{1-a^2x^2}}{3a\sqrt{\cos^{-1}(ax)}} - \frac{8\,\text{Subs}}{}$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} - \frac{16x^3}{15a^2\cos^{-1}(ax)^{3/2}} + \frac{4x^5}{3\cos^{-1}(ax)^{3/2}} + \frac{32x^2\sqrt{1-a^2x^2}}{5a^3\sqrt{\cos^{-1}(ax)}} - \frac{40x^4\sqrt{1-a^2x^2}}{3a\sqrt{\cos^{-1}(ax)}} - \frac{16\,\text{Sub}}{}$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} - \frac{16x^3}{15a^2\cos^{-1}(ax)^{3/2}} + \frac{4x^5}{3\cos^{-1}(ax)^{3/2}} + \frac{32x^2\sqrt{1-a^2x^2}}{5a^3\sqrt{\cos^{-1}(ax)}} - \frac{40x^4\sqrt{1-a^2x^2}}{3a\sqrt{\cos^{-1}(ax)}} + \frac{\sqrt{2\pi}C}{}$$

Mathematica [C] time = 7.73285, size = 418, normalized size = 1.58

$$-\frac{2\left(-4\cos^{-1}(ax)\left(-i\cos^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-i\cos^{-1}(ax)\right)+e^{-i\cos^{-1}(ax)}\cos^{-1}(ax)\left(-4e^{i\cos^{-1}(ax)}\left(i\cos^{-1}(ax)\right)^{3/2}\text{Gamm}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/ArcCos[a*x]^(7/2),x]
```

```
[Out] -(2*(-6*Sqrt[1 - a^2*x^2] - (2*I)*E^(I*ArcCos[a*x])*ArcCos[a*x]*(-I + 2*Arc
Cos[a*x]) - 4*((-I)*ArcCos[a*x])^(3/2)*ArcCos[a*x]*Gamma[1/2, (-I)*ArcCos[a
*x]] + (ArcCos[a*x]*(-2 + (4*I)*ArcCos[a*x] - 4*E^(I*ArcCos[a*x])*(I*ArcCos
[a*x])^(3/2)*Gamma[1/2, I*ArcCos[a*x]]))/E^(I*ArcCos[a*x])) - 5*ArcCos[a*x]
*(2*E^((5*I)*ArcCos[a*x])*(1 + (10*I)*ArcCos[a*x]) + 20*Sqrt[5]*((-I)*ArcCo
s[a*x])^(3/2)*Gamma[1/2, (-5*I)*ArcCos[a*x]] + (2 - (20*I)*ArcCos[a*x] + 20
*Sqrt[5]*E^((5*I)*ArcCos[a*x])*(I*ArcCos[a*x])^(3/2)*Gamma[1/2, (5*I)*ArcCo
s[a*x]])/E^((5*I)*ArcCos[a*x])) + 9*(-2*ArcCos[a*x]*(E^((3*I)*ArcCos[a*x])*
(1 + (6*I)*ArcCos[a*x]) + 6*Sqrt[3]*((-I)*ArcCos[a*x])^(3/2)*Gamma[1/2, (-3
*I)*ArcCos[a*x]] + (1 - (6*I)*ArcCos[a*x] + 6*Sqrt[3]*E^((3*I)*ArcCos[a*x])
*(I*ArcCos[a*x])^(3/2)*Gamma[1/2, (3*I)*ArcCos[a*x]])/E^((3*I)*ArcCos[a*x])
) - 2*Sin[3*ArcCos[a*x]]) - 6*Sin[5*ArcCos[a*x]])/(240*a^5*ArcCos[a*x]^(5/2
```

```
))
```

Maple [A] time = 0.11, size = 225, normalized size = 0.9

$$-\frac{1}{120\,a^5}\left(-100\,\sqrt{2}\sqrt{\pi}\sqrt{5}\mathrm{FresnelC}\left(\frac{\sqrt{2}\sqrt{5}\sqrt{\arccos{(ax)}}}{\sqrt{\pi}}\right)(\arccos{(ax)})^{5/2}-108\,\sqrt{2}\sqrt{\pi}\sqrt{3}\mathrm{FresnelC}\left(\frac{\sqrt{2}\sqrt{3}\sqrt{\arccos{(a}}}{\sqrt{\pi}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arccos(a*x)^(7/2),x)
```

```
[Out] -1/120/a^5*(-100*2^(1/2)*Pi^(1/2)*5^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)
*arccos(a*x)^(1/2))*arccos(a*x)^(5/2)-108*2^(1/2)*Pi^(1/2)*3^(1/2)*FresnelC
(2^(1/2)/Pi^(1/2)*3^(1/2)*arccos(a*x)^(1/2))*arccos(a*x)^(5/2)-8*2^(1/2)*Pi
^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*arccos(a*x)^(5/2)+8*arc
cos(a*x)^2*(-a^2*x^2+1)^(1/2)+108*arccos(a*x)^2*sin(3*arccos(a*x))+100*arcc
os(a*x)^2*sin(5*arccos(a*x))-4*a*x*arccos(a*x)-18*arccos(a*x)*cos(3*arccos(
a*x))-10*arccos(a*x)*cos(5*arccos(a*x))-6*(-a^2*x^2+1)^(1/2)-9*sin(3*arccos
(a*x))-3*sin(5*arccos(a*x)))/arccos(a*x)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccos(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccos(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/acos(a*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\arccos{(ax)}^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccos(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/arccos(a*x)^(7/2), x)
```

3.114 $\int \frac{x^3}{\cos^{-1}(ax)^{7/2}}\,dx$

Optimal. Leaf size=190

$$\frac{32\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{15a^4}+\frac{16\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{15a^4}-\frac{128x^3\sqrt{1-a^2x^2}}{15a\sqrt{\cos^{-1}(ax)}}+\frac{2x^3\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}}-\frac{4x^2}{5a^2\cos^{-1}(\ldots}$$

```
[Out] (2*x^3*Sqrt[1 - a^2*x^2])/(5*a*ArcCos[a*x]^(5/2)) - (4*x^2)/(5*a^2*ArcCos[a
*x]^(3/2)) + (16*x^4)/(15*ArcCos[a*x]^(3/2)) + (16*x*Sqrt[1 - a^2*x^2])/(5*
a^3*Sqrt[ArcCos[a*x]]) - (128*x^3*Sqrt[1 - a^2*x^2])/(15*a*Sqrt[ArcCos[a*x]
]) + (32*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(15*a^4) + (1
6*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/(15*a^4)
```

Rubi [A] time = 0.323089, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4634, 4720, 4632, 3304, 3352}

$$\frac{32\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{15a^4}+\frac{16\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{15a^4}-\frac{128x^3\sqrt{1-a^2x^2}}{15a\sqrt{\cos^{-1}(ax)}}+\frac{2x^3\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}}-\frac{4x^2}{5a^2\cos^{-1}(\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcCos[a*x]^(7/2),x]
```

```
[Out] (2*x^3*Sqrt[1 - a^2*x^2])/(5*a*ArcCos[a*x]^(5/2)) - (4*x^2)/(5*a^2*ArcCos[a
*x]^(3/2)) + (16*x^4)/(15*ArcCos[a*x]^(3/2)) + (16*x*Sqrt[1 - a^2*x^2])/(5*
a^3*Sqrt[ArcCos[a*x]]) - (128*x^3*Sqrt[1 - a^2*x^2])/(15*a*Sqrt[ArcCos[a*x]
]) + (32*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(15*a^4) + (1
6*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/(15*a^4)
```

Rule 4634

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-D
ist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCos[c*x])^(n + 1))/Sq
rt[1 - c^2*x^2], x], x] + Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*ArcCo
s[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[
m, 0] && LtQ[n, -2]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4632

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dis
t[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), Co
s[x]^(m - 1)*(m - (m + 1)*Cos[x]^2), x], x], x, ArcCos[c*x]], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^3}{\cos^{-1}(ax)^{7/2}}\,dx = \frac{2x^3\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} - \frac{6\int \frac{x^2}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^{5/2}}\,dx}{5a} + \frac{1}{5}(8a)\int \frac{x^4}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^{5/2}}\,dx$$

$$= \frac{2x^3\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} - \frac{4x^2}{5a^2\cos^{-1}(ax)^{3/2}} + \frac{16x^4}{15\cos^{-1}(ax)^{3/2}} - \frac{64}{15}\int \frac{x^3}{\cos^{-1}(ax)^{3/2}}\,dx + \frac{8\int \frac{x}{\cos^{-1}(ax)^{3/2}}\,dx}{5a^2}$$

$$= \frac{2x^3\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} - \frac{4x^2}{5a^2\cos^{-1}(ax)^{3/2}} + \frac{16x^4}{15\cos^{-1}(ax)^{3/2}} + \frac{16x\sqrt{1-a^2x^2}}{5a^3\sqrt{\cos^{-1}(ax)}} - \frac{128x^3\sqrt{1-a^2x^2}}{15a\sqrt{\cos^{-1}(ax)}} - \frac{16\,\mathrm{S}}{}$$

$$= \frac{2x^3\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} - \frac{4x^2}{5a^2\cos^{-1}(ax)^{3/2}} + \frac{16x^4}{15\cos^{-1}(ax)^{3/2}} + \frac{16x\sqrt{1-a^2x^2}}{5a^3\sqrt{\cos^{-1}(ax)}} - \frac{128x^3\sqrt{1-a^2x^2}}{15a\sqrt{\cos^{-1}(ax)}} + \frac{64\,\mathrm{S}}{}$$

$$= \frac{2x^3\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} - \frac{4x^2}{5a^2\cos^{-1}(ax)^{3/2}} + \frac{16x^4}{15\cos^{-1}(ax)^{3/2}} + \frac{16x\sqrt{1-a^2x^2}}{5a^3\sqrt{\cos^{-1}(ax)}} - \frac{128x^3\sqrt{1-a^2x^2}}{15a\sqrt{\cos^{-1}(ax)}} - \frac{16\sqrt{}}{}$$

$$= \frac{2x^3\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} - \frac{4x^2}{5a^2\cos^{-1}(ax)^{3/2}} + \frac{16x^4}{15\cos^{-1}(ax)^{3/2}} + \frac{16x\sqrt{1-a^2x^2}}{5a^3\sqrt{\cos^{-1}(ax)}} - \frac{128x^3\sqrt{1-a^2x^2}}{15a\sqrt{\cos^{-1}(ax)}} + \frac{32\sqrt{}}{}$$

Mathematica [C] time = 4.23985, size = 264, normalized size = 1.39

$$-\frac{\frac{16\sqrt{2}\cos^{-1}(ax)^3\text{Gamma}\left(\frac{1}{2},-2i\cos^{-1}(ax)\right)}{\sqrt{-i\cos^{-1}(ax)}} - 2\cos^{-1}(ax)\left(32\left(-i\cos^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-4i\cos^{-1}(ax)\right) + 2e^{-4i\cos^{-1}(ax)}\left(16e^{4}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/ArcCos[a*x]^(7/2),x]
```

```
[Out] -((-4*(1 + E^((4*I)*ArcCos[a*x])*(1 + (4*I)*ArcCos[a*x]) - (4*I)*ArcCos[a*x
])*ArcCos[a*x])/E^((2*I)*ArcCos[a*x]) + (16*Sqrt[2]*ArcCos[a*x]^3*Gamma[1/2
, (-2*I)*ArcCos[a*x]])/Sqrt[(-I)*ArcCos[a*x]] + (16*I)*Sqrt[2]*(I*ArcCos[a*
x])^(5/2)*Gamma[1/2, (2*I)*ArcCos[a*x]] - 2*ArcCos[a*x]*(2*E^((4*I)*ArcCos[
a*x])*(1 + (8*I)*ArcCos[a*x]) + 32*((-I)*ArcCos[a*x])^(3/2)*Gamma[1/2, (-4*
I)*ArcCos[a*x]] + (2*(1 - (8*I)*ArcCos[a*x] + 16*E^((4*I)*ArcCos[a*x])*(I*A
rcCos[a*x])^(3/2)*Gamma[1/2, (4*I)*ArcCos[a*x]]))/E^((4*I)*ArcCos[a*x])) -
6*Sin[2*ArcCos[a*x]] - 3*Sin[4*ArcCos[a*x]])/(60*a^4*ArcCos[a*x]^(5/2))
```

Maple [A] time = 0.091, size = 139, normalized size = 0.7

$$-\frac{1}{60\,a^{4}}\left(-128\,\sqrt{2}\sqrt{\pi}\mathrm{FresnelC}\left(2\,\frac{\sqrt{2}\sqrt{\arccos\left(ax\right)}}{\sqrt{\pi}}\right)\left(\arccos\left(ax\right)\right)^{5/2}-64\,\sqrt{\pi}\mathrm{FresnelC}\left(2\,\frac{\sqrt{\arccos\left(ax\right)}}{\sqrt{\pi}}\right)\left(\arccos\left(ax\right)\right)^{5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccos(a*x)^(7/2),x)
```

```
[Out] -1/60/a^4*(-128*2^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1
/2))*arccos(a*x)^(5/2)-64*Pi^(1/2)*FresnelC(2*arccos(a*x)^(1/2)/Pi^(1/2))*a
rccos(a*x)^(5/2)+32*sin(2*arccos(a*x))*arccos(a*x)^2+64*sin(4*arccos(a*x))*
arccos(a*x)^2-8*arccos(a*x)*cos(2*arccos(a*x))-8*arccos(a*x)*cos(4*arccos(a
*x))-6*sin(2*arccos(a*x))-3*sin(4*arccos(a*x)))/arccos(a*x)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccos(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccos(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acos(a*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\arccos(ax)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccos(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/arccos(a*x)^(7/2), x)
```

3.115 $\int \frac{x^2}{\cos^{-1}(ax)^{7/2}}\,dx$

Optimal. Leaf size=191

$$\frac{2\sqrt{2\pi}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{15a^3}+\frac{6\sqrt{6\pi}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{5a^3}-\frac{24x^2\sqrt{1-a^2x^2}}{5a\sqrt{\cos^{-1}(ax)}}+\frac{2x^2\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}}+\frac{16\sqrt{1-}}{15a^3\sqrt{\cos}}$$

```
[Out] (2*x^2*Sqrt[1 - a^2*x^2])/(5*a*ArcCos[a*x]^(5/2)) - (8*x)/(15*a^2*ArcCos[a*
x]^(3/2)) + (4*x^3)/(5*ArcCos[a*x]^(3/2)) + (16*Sqrt[1 - a^2*x^2])/(15*a^3*
Sqrt[ArcCos[a*x]]) - (24*x^2*Sqrt[1 - a^2*x^2])/(5*a*Sqrt[ArcCos[a*x]]) + (
2*Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(15*a^3) + (6*Sqrt[6*P
i]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(5*a^3)
```

Rubi [A] time = 0.340763, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {4634, 4720, 4632, 3304, 3352, 4622, 4724}

$$\frac{2\sqrt{2\pi}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{15a^3}+\frac{6\sqrt{6\pi}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{5a^3}-\frac{24x^2\sqrt{1-a^2x^2}}{5a\sqrt{\cos^{-1}(ax)}}+\frac{2x^2\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}}+\frac{16\sqrt{1-}}{15a^3\sqrt{\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcCos[a*x]^(7/2),x]
```

```
[Out] (2*x^2*Sqrt[1 - a^2*x^2])/(5*a*ArcCos[a*x]^(5/2)) - (8*x)/(15*a^2*ArcCos[a*
x]^(3/2)) + (4*x^3)/(5*ArcCos[a*x]^(3/2)) + (16*Sqrt[1 - a^2*x^2])/(15*a^3*
Sqrt[ArcCos[a*x]]) - (24*x^2*Sqrt[1 - a^2*x^2])/(5*a*Sqrt[ArcCos[a*x]]) + (
2*Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(15*a^3) + (6*Sqrt[6*P
i]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(5*a^3)
```

Rule 4634

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-D
ist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCos[c*x])^(n + 1))/Sq
rt[1 - c^2*x^2], x], x] + Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*ArcCo
s[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[
m, 0] && LtQ[n, -2]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4632

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dis
t[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), Co
s[x]^(m - 1)*(m - (m + 1)*Cos[x]^2), x], x], x, ArcCos[c*x]], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4622

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(Sqrt[1 - c
^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1))
, Int[(x*(a + b*ArcCos[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && LtQ[n, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{\cos^{-1}(ax)^{7/2}}\,dx = \frac{2x^2\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} - \frac{4\int \frac{x}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^{5/2}}\,dx}{5a} + \frac{1}{5}(6a)\int \frac{x^3}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^{5/2}}\,dx$$

$$= \frac{2x^2\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} - \frac{8x}{15a^2\cos^{-1}(ax)^{3/2}} + \frac{4x^3}{5\cos^{-1}(ax)^{3/2}} - \frac{12}{5}\int \frac{x^2}{\cos^{-1}(ax)^{3/2}}\,dx + \frac{8\int \frac{1}{\cos^{-1}(ax)^{3/2}}\,dx}{15a^2}$$

$$= \frac{2x^2\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} - \frac{8x}{15a^2\cos^{-1}(ax)^{3/2}} + \frac{4x^3}{5\cos^{-1}(ax)^{3/2}} + \frac{16\sqrt{1-a^2x^2}}{15a^3\sqrt{\cos^{-1}(ax)}} - \frac{24x^2\sqrt{1-a^2x^2}}{5a\sqrt{\cos^{-1}(ax)}} - \frac{24\,\mathrm{Su}}{}$$

$$= \frac{2x^2\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} - \frac{8x}{15a^2\cos^{-1}(ax)^{3/2}} + \frac{4x^3}{5\cos^{-1}(ax)^{3/2}} + \frac{16\sqrt{1-a^2x^2}}{15a^3\sqrt{\cos^{-1}(ax)}} - \frac{24x^2\sqrt{1-a^2x^2}}{5a\sqrt{\cos^{-1}(ax)}} - \frac{16\,\mathrm{Su}}{}$$

$$= \frac{2x^2\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} - \frac{8x}{15a^2\cos^{-1}(ax)^{3/2}} + \frac{4x^3}{5\cos^{-1}(ax)^{3/2}} + \frac{16\sqrt{1-a^2x^2}}{15a^3\sqrt{\cos^{-1}(ax)}} - \frac{24x^2\sqrt{1-a^2x^2}}{5a\sqrt{\cos^{-1}(ax)}} - \frac{32\,\mathrm{Su}}{}$$

$$= \frac{2x^2\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} - \frac{8x}{15a^2\cos^{-1}(ax)^{3/2}} + \frac{4x^3}{5\cos^{-1}(ax)^{3/2}} + \frac{16\sqrt{1-a^2x^2}}{15a^3\sqrt{\cos^{-1}(ax)}} - \frac{24x^2\sqrt{1-a^2x^2}}{5a\sqrt{\cos^{-1}(ax)}} + \frac{2\sqrt{2\pi}}{}$$

Mathematica [C] time = 2.72618, size = 281, normalized size = 1.47

$$-\frac{-4\cos^{-1}(ax)\left(-i\cos^{-1}(ax)\right)^{3/2}\mathrm{Gamma}\left(\frac{1}{2},-i\cos^{-1}(ax)\right)+e^{-i\cos^{-1}(ax)}\cos^{-1}(ax)\left(-4e^{i\cos^{-1}(ax)}\left(i\cos^{-1}(ax)\right)^{3/2}\mathrm{Gamma}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/ArcCos[a*x]^(7/2),x]
```

```
[Out] -(-6*Sqrt[1 - a^2*x^2] - (2*I)*E^(I*ArcCos[a*x])*ArcCos[a*x]*(-I + 2*ArcCos
[a*x]) - 4*((-I)*ArcCos[a*x])^(3/2)*ArcCos[a*x]*Gamma[1/2, (-I)*ArcCos[a*x]
] + (ArcCos[a*x]*(-2 + (4*I)*ArcCos[a*x] - 4*E^(I*ArcCos[a*x])*(I*ArcCos[a*
x])^(3/2)*Gamma[1/2, I*ArcCos[a*x]]))/E^(I*ArcCos[a*x]) - 6*ArcCos[a*x]*(E^
((3*I)*ArcCos[a*x])*(1 + (6*I)*ArcCos[a*x]) + 6*Sqrt[3]*((-I)*ArcCos[a*x])^
(3/2)*Gamma[1/2, (-3*I)*ArcCos[a*x]] + (1 - (6*I)*ArcCos[a*x] + 6*Sqrt[3]*E
^((3*I)*ArcCos[a*x])*(I*ArcCos[a*x])^(3/2)*Gamma[1/2, (3*I)*ArcCos[a*x]])/E
^((3*I)*ArcCos[a*x])) - 6*Sin[3*ArcCos[a*x]])/(60*a^3*ArcCos[a*x]^(5/2))
```

Maple [A] time = 0.091, size = 154, normalized size = 0.8

$$-\frac{1}{30a^3}\left(-36\sqrt{2}\sqrt{\pi}\sqrt{3}\mathrm{FresnelC}\left(\frac{\sqrt{2}\sqrt{3}\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)(\arccos(ax))^{5/2}-4\sqrt{2}\sqrt{\pi}\mathrm{FresnelC}\left(\frac{\sqrt{2}\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)(\arccos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccos(a*x)^(7/2),x)
```

```
[Out] -1/30/a^3*(-36*2^(1/2)*Pi^(1/2)*3^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*a
rccos(a*x)^(1/2))*arccos(a*x)^(5/2)-4*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^
(1/2)*arccos(a*x)^(1/2))*arccos(a*x)^(5/2)+36*arccos(a*x)^2*sin(3*arccos(a*
x))+4*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)-2*a*x*arccos(a*x)-6*arccos(a*x)*cos(
3*arccos(a*x))-3*sin(3*arccos(a*x))-3*(-a^2*x^2+1)^(1/2))/arccos(a*x)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccos(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccos(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acos(a*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\arccos(ax)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccos(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/arccos(a*x)^(7/2), x)
```

3.116 $\int \frac{x}{\cos^{-1}(ax)^{7/2}}\,dx$

Optimal. Leaf size=119

$$\frac{32\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{15a^2}-\frac{32x\sqrt{1-a^2x^2}}{15a\sqrt{\cos^{-1}(ax)}}+\frac{2x\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}}-\frac{4}{15a^2\cos^{-1}(ax)^{3/2}}+\frac{8x^2}{15\cos^{-1}(ax)^{3/2}}$$

```
[Out] (2*x*Sqrt[1 - a^2*x^2])/(5*a*ArcCos[a*x]^(5/2)) - 4/(15*a^2*ArcCos[a*x]^(3/
2)) + (8*x^2)/(15*ArcCos[a*x]^(3/2)) - (32*x*Sqrt[1 - a^2*x^2])/(15*a*Sqrt[
ArcCos[a*x]]) + (32*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/(15*
a^2)
```

Rubi [A] time = 0.172477, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4634, 4720, 4632, 3304, 3352, 4642}

$$\frac{32\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{15a^2}-\frac{32x\sqrt{1-a^2x^2}}{15a\sqrt{\cos^{-1}(ax)}}+\frac{2x\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}}-\frac{4}{15a^2\cos^{-1}(ax)^{3/2}}+\frac{8x^2}{15\cos^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcCos[a*x]^(7/2),x]
```

```
[Out] (2*x*Sqrt[1 - a^2*x^2])/(5*a*ArcCos[a*x]^(5/2)) - 4/(15*a^2*ArcCos[a*x]^(3/
2)) + (8*x^2)/(15*ArcCos[a*x]^(3/2)) - (32*x*Sqrt[1 - a^2*x^2])/(15*a*Sqrt[
ArcCos[a*x]]) + (32*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/(15*
a^2)
```

Rule 4634

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-D
ist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCos[c*x])^(n + 1))/Sq
rt[1 - c^2*x^2], x], x] + Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*ArcCo
s[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[
m, 0] && LtQ[n, -2]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4632

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dis
t[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), Co
s[x]^(m - 1)*(m - (m + 1)*Cos[x]^2), x], x], x, ArcCos[c*x]], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{x}{\cos^{-1}(ax)^{7/2}}\,dx = \frac{2x\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} - \frac{2\int \frac{1}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^{5/2}}\,dx}{5a} + \frac{1}{5}(4a)\int \frac{x^2}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^{5/2}}\,dx$$

$$= \frac{2x\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} - \frac{4}{15a^2\cos^{-1}(ax)^{3/2}} + \frac{8x^2}{15\cos^{-1}(ax)^{3/2}} - \frac{16}{15}\int \frac{x}{\cos^{-1}(ax)^{3/2}}\,dx$$

$$= \frac{2x\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} - \frac{4}{15a^2\cos^{-1}(ax)^{3/2}} + \frac{8x^2}{15\cos^{-1}(ax)^{3/2}} - \frac{32x\sqrt{1-a^2x^2}}{15a\sqrt{\cos^{-1}(ax)}} + \frac{32\,\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}}\,dx\right.}{15a^2}$$

$$= \frac{2x\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} - \frac{4}{15a^2\cos^{-1}(ax)^{3/2}} + \frac{8x^2}{15\cos^{-1}(ax)^{3/2}} - \frac{32x\sqrt{1-a^2x^2}}{15a\sqrt{\cos^{-1}(ax)}} + \frac{64\,\text{Subst}\left(\int \cos\left(2x^2\right)\right.}{15a}$$

$$= \frac{2x\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} - \frac{4}{15a^2\cos^{-1}(ax)^{3/2}} + \frac{8x^2}{15\cos^{-1}(ax)^{3/2}} - \frac{32x\sqrt{1-a^2x^2}}{15a\sqrt{\cos^{-1}(ax)}} + \frac{32\sqrt{\pi}C\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right)}{15a^2}$$

Mathematica [A] time = 0.0989414, size = 75, normalized size = 0.63

$$\frac{32\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\cos^{-1}(ax)}}{\sqrt{\pi}}\right) + \frac{4\cos\left(2\cos^{-1}(ax)\right)}{\cos^{-1}(ax)^{3/2}} - \frac{\left(16\cos^{-1}(ax)^2-3\right)\sin\left(2\cos^{-1}(ax)\right)}{\cos^{-1}(ax)^{5/2}}}{15a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcCos[a*x]^(7/2),x]
```

```
[Out] ((4*Cos[2*ArcCos[a*x]])/ArcCos[a*x]^(3/2) + 32*Sqrt[Pi]*FresnelC[(2*Sqrt[Ar
cCos[a*x]])/Sqrt[Pi]] - ((-3 + 16*ArcCos[a*x]^2)*Sin[2*ArcCos[a*x]])/ArcCos
[a*x]^(5/2))/(15*a^2)
```

Maple [A] time = 0.075, size = 73, normalized size = 0.6

$$-\frac{1}{15\,a^2}\left(-32\sqrt{\pi}\text{FresnelC}\left(2\,\frac{\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)(\arccos(ax))^{5/2} + 16\,\sin(2\,\arccos(ax))(\arccos(ax))^2 - 4\,\arccos(ax)\,\text{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccos(a*x)^(7/2),x)
```

```
[Out] -1/15/a^2*(-32*Pi^(1/2)*FresnelC(2*arccos(a*x)^(1/2)/Pi^(1/2))*arccos(a*x)^
(5/2)+16*sin(2*arccos(a*x))*arccos(a*x)^2-4*arccos(a*x)*cos(2*arccos(a*x))-
3*sin(2*arccos(a*x)))/arccos(a*x)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acos(a*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\arccos(ax)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x/arccos(a*x)^(7/2), x)
```

3.117 $\int \frac{1}{\cos^{-1}(ax)^{7/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{8\sqrt{1-a^2x^2}}{15a\sqrt{\cos^{-1}(ax)}}+\frac{2\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}}+\frac{8\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{15a}+\frac{4x}{15\cos^{-1}(ax)^{3/2}}$$

```
[Out] (2*Sqrt[1 - a^2*x^2])/(5*a*ArcCos[a*x]^(5/2)) + (4*x)/(15*ArcCos[a*x]^(3/2)
) - (8*Sqrt[1 - a^2*x^2])/(15*a*Sqrt[ArcCos[a*x]]) + (8*Sqrt[2*Pi]*FresnelC
[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(15*a)
```

Rubi [A] time = 0.165503, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4622, 4720, 4724, 3304, 3352}

$$-\frac{8\sqrt{1-a^2x^2}}{15a\sqrt{\cos^{-1}(ax)}}+\frac{2\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}}+\frac{8\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{15a}+\frac{4x}{15\cos^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^(-7/2),x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2])/(5*a*ArcCos[a*x]^(5/2)) + (4*x)/(15*ArcCos[a*x]^(3/2)
) - (8*Sqrt[1 - a^2*x^2])/(15*a*Sqrt[ArcCos[a*x]]) + (8*Sqrt[2*Pi]*FresnelC
[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(15*a)
```

Rule 4622

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(Sqrt[1 - c
^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1))
, Int[(x*(a + b*ArcCos[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && LtQ[n, -1]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
```

```
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\cos^{-1}(ax)^{7/2}}\,dx &= \frac{2\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} + \frac{1}{5}(2a)\int \frac{x}{\sqrt{1-a^2x^2}\cos^{-1}(ax)^{5/2}}\,dx \\
&= \frac{2\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} + \frac{4x}{15\cos^{-1}(ax)^{3/2}} - \frac{4}{15}\int \frac{1}{\cos^{-1}(ax)^{3/2}}\,dx \\
&= \frac{2\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} + \frac{4x}{15\cos^{-1}(ax)^{3/2}} - \frac{8\sqrt{1-a^2x^2}}{15a\sqrt{\cos^{-1}(ax)}} - \frac{1}{15}(8a)\int \frac{x}{\sqrt{1-a^2x^2}\sqrt{\cos^{-1}(ax)}}\,dx \\
&= \frac{2\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} + \frac{4x}{15\cos^{-1}(ax)^{3/2}} - \frac{8\sqrt{1-a^2x^2}}{15a\sqrt{\cos^{-1}(ax)}} + \frac{8\,\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(ax)\right)}{15a} \\
&= \frac{2\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} + \frac{4x}{15\cos^{-1}(ax)^{3/2}} - \frac{8\sqrt{1-a^2x^2}}{15a\sqrt{\cos^{-1}(ax)}} + \frac{16\,\text{Subst}\left(\int \cos\left(x^2\right)\,dx, x, \sqrt{\cos^{-1}(ax)}\right)}{15a} \\
&= \frac{2\sqrt{1-a^2x^2}}{5a\cos^{-1}(ax)^{5/2}} + \frac{4x}{15\cos^{-1}(ax)^{3/2}} - \frac{8\sqrt{1-a^2x^2}}{15a\sqrt{\cos^{-1}(ax)}} + \frac{8\sqrt{2\pi}C\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(ax)}\right)}{15a}
\end{aligned}
$$

Mathematica [C] time = 1.04488, size = 151, normalized size = 1.44

$$-\frac{-4\cos^{-1}(ax)\left(-i\cos^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-i\cos^{-1}(ax)\right)+e^{-i\cos^{-1}(ax)}\cos^{-1}(ax)\left(-4e^{i\cos^{-1}(ax)}\left(i\cos^{-1}(ax)\right)^{3/2}\text{Gamma}\right.}{15a\cos^{-1}(ax)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCos[a*x]^(-7/2),x]
```

```
[Out] -(-6*Sqrt[1 - a^2*x^2] - (2*I)*E^(I*ArcCos[a*x])*ArcCos[a*x]*(-I + 2*ArcCos
[a*x]) - 4*((-I)*ArcCos[a*x])^(3/2)*ArcCos[a*x]*Gamma[1/2, (-I)*ArcCos[a*x]
] + (ArcCos[a*x]*(-2 + (4*I)*ArcCos[a*x] - 4*E^(I*ArcCos[a*x])*(I*ArcCos[a*
x])^(3/2)*Gamma[1/2, I*ArcCos[a*x]]))/E^(I*ArcCos[a*x]))/(15*a*ArcCos[a*x]^
(5/2))
```

Maple [A] time = 0.075, size = 110, normalized size = 1.1

$$\frac{\sqrt{2}}{15\,a\sqrt{\pi}\,(\arccos(ax))^3}\left(8\,(\arccos(ax))^3\,\pi\,\text{FresnelC}\left(\frac{\sqrt{2}\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-4\,(\arccos(ax))^{5/2}\sqrt{2}\sqrt{\pi}\sqrt{-a^2x^2+1}+2\,(\text{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos(a*x)^(7/2),x)
```

```
[Out] 1/15/a*2^(1/2)/Pi^(1/2)*(8*arccos(a*x)^3*Pi*FresnelC(2^(1/2)/Pi^(1/2)*arcco
s(a*x)^(1/2))-4*arccos(a*x)^(5/2)*2^(1/2)*Pi^(1/2)*(-a^2*x^2+1)^(1/2)+2*arc
cos(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*x*a+3*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*(-
a^2*x^2+1)^(1/2))/arccos(a*x)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acos(a*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\arccos{(ax)}^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^(-7/2), x)
```

3.118 $\int \frac{1}{x \cos^{-1}(ax)^{7/2}} \, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{1}{x \cos^{-1}(ax)^{7/2}}, x\right)$$

```
[Out] Unintegrable[1/(x*ArcCos[a*x]^(7/2)), x]
```

Rubi [A] time = 0.0128925, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{x \cos^{-1}(ax)^{7/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcCos[a*x]^(7/2)),x]
```

```
[Out] Defer[Int][1/(x*ArcCos[a*x]^(7/2)), x]
```

Rubi steps

$$\int \frac{1}{x \cos^{-1}(ax)^{7/2}} \, dx = \int \frac{1}{x \cos^{-1}(ax)^{7/2}} \, dx$$

Mathematica [A] time = 0.461289, size = 0, normalized size = 0.

$$\int \frac{1}{x \cos^{-1}(ax)^{7/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcCos[a*x]^(7/2)),x]
```

```
[Out] Integrate[1/(x*ArcCos[a*x]^(7/2)), x]
```

Maple [A] time = 0.102, size = 0, normalized size = 0.

$$\int \frac{1}{x} (\arccos (ax))^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccos(a*x)^(7/2),x)
```

```
[Out] int(1/x/arccos(a*x)^(7/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccos(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccos(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acos(a*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \arccos(ax)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccos(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*arccos(a*x)^(7/2)), x)
```

3.119 $\int (bx)^m \cos^{-1}(ax)^4 \, dx$

Optimal. Leaf size=64

$$\frac{4a\text{Unintegrable}\left(\frac{\cos^{-1}(ax)^3(bx)^{m+1}}{\sqrt{1-a^2x^2}},x\right)}{b(m+1)}+\frac{\cos^{-1}(ax)^4(bx)^{m+1}}{b(m+1)}$$

```
[Out] ((b*x)^(1 + m)*ArcCos[a*x]^4)/(b*(1 + m)) + (4*a*Unintegrable[((b*x)^(1 + m
)*ArcCos[a*x]^3)/Sqrt[1 - a^2*x^2], x])/(b*(1 + m))
```

Rubi [A] time = 0.116792, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (bx)^m \cos^{-1}(ax)^4 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(b*x)^m*ArcCos[a*x]^4,x]
```

```
[Out] ((b*x)^(1 + m)*ArcCos[a*x]^4)/(b*(1 + m)) + (4*a*Defer[Int][((b*x)^(1 + m)*
ArcCos[a*x]^3)/Sqrt[1 - a^2*x^2], x])/(b*(1 + m))
```

Rubi steps

$$\int (bx)^m \cos^{-1}(ax)^4 \, dx = \frac{(bx)^{1+m}\cos^{-1}(ax)^4}{b(1+m)}+\frac{(4a)\int \frac{(bx)^{1+m}\cos^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx}{b(1+m)}$$

Mathematica [A] time = 1.17904, size = 0, normalized size = 0.

$$\int (bx)^m \cos^{-1}(ax)^4 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(b*x)^m*ArcCos[a*x]^4,x]
```

```
[Out] Integrate[(b*x)^m*ArcCos[a*x]^4, x]
```

Maple [A] time = 0.842, size = 0, normalized size = 0.

$$\int (bx)^m (\arccos (ax))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m*arccos(a*x)^4,x)
```

```
[Out] int((b*x)^m*arccos(a*x)^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx)^m \arccos (ax)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*arccos(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral((b*x)^m*arccos(a*x)^4, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (bx)^m \operatorname{acos}^4 (ax) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**m*acos(a*x)**4,x)
```

```
[Out] Integral((b*x)**m*acos(a*x)**4, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (bx)^m \arccos (ax)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate((b*x)^m*arccos(a*x)^4, x)
```

3.120 $\int (bx)^m \cos^{-1}(ax)^3 \, dx$

Optimal. Leaf size=64

$$\frac{3a\text{Unintegrable}\left(\frac{\cos^{-1}(ax)^2(bx)^{m+1}}{\sqrt{1-a^2x^2}},x\right)}{b(m+1)}+\frac{\cos^{-1}(ax)^3(bx)^{m+1}}{b(m+1)}$$

```
[Out] ((b*x)^(1 + m)*ArcCos[a*x]^3)/(b*(1 + m)) + (3*a*Unintegrable[((b*x)^(1 + m
)*ArcCos[a*x]^2)/Sqrt[1 - a^2*x^2], x])/(b*(1 + m))
```

Rubi [A] time = 0.114462, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (bx)^m \cos^{-1}(ax)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(b*x)^m*ArcCos[a*x]^3,x]
```

```
[Out] ((b*x)^(1 + m)*ArcCos[a*x]^3)/(b*(1 + m)) + (3*a*Defer[Int][((b*x)^(1 + m)*
ArcCos[a*x]^2)/Sqrt[1 - a^2*x^2], x])/(b*(1 + m))
```

Rubi steps

$$\int (bx)^m \cos^{-1}(ax)^3 \, dx = \frac{(bx)^{1+m}\cos^{-1}(ax)^3}{b(1+m)}+\frac{(3a)\int \frac{(bx)^{1+m}\cos^{-1}(ax)^2}{\sqrt{1-a^2x^2}} \, dx}{b(1+m)}$$

Mathematica [A] time = 1.06303, size = 0, normalized size = 0.

$$\int (bx)^m \cos^{-1}(ax)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(b*x)^m*ArcCos[a*x]^3,x]
```

```
[Out] Integrate[(b*x)^m*ArcCos[a*x]^3, x]
```

Maple [A] time = 0.614, size = 0, normalized size = 0.

$$\int (bx)^m (\arccos (ax))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m*arccos(a*x)^3,x)
```

```
[Out] int((b*x)^m*arccos(a*x)^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*arccos(a*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx)^m \arccos (ax)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*arccos(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x)^m*arccos(a*x)^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (bx)^m \operatorname{acos}^3 (ax) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**m*acos(a*x)**3,x)
```

```
[Out] Integral((b*x)**m*acos(a*x)**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (bx)^m \arccos (ax)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*arccos(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x)^m*arccos(a*x)^3, x)
```

3.121 $\int (bx)^m \cos^{-1}(ax)^2 \, dx$

Optimal. Leaf size=150

$$\frac{2a^2(bx)^{m+3}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2}\right\},\left\{\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2}\right\},a^2x^2\right)}{b^3(m+1)(m+2)(m+3)}+\frac{2a\cos^{-1}(ax)(bx)^{m+2}\text{Hypergeometri}}{b^2(m+1)(m+}$$

```
[Out] ((b*x)^(1 + m)*ArcCos[a*x]^2)/(b*(1 + m)) + (2*a*(b*x)^(2 + m)*ArcCos[a*x]*
Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(b^2*(1 + m)*(2 + m)
) + (2*a^2*(b*x)^(3 + m)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 +
m/2, 5/2 + m/2}, a^2*x^2])/(b^3*(1 + m)*(2 + m)*(3 + m))
```

Rubi [A] time = 0.107421, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4628, 4712}

$$\frac{2a^2(bx)^{m+3}\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};a^2x^2\right)}{b^3(m+1)(m+2)(m+3)}+\frac{2a\cos^{-1}(ax)(bx)^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{b^2(m+1)(m+2)}+\frac{\cos^{-1}(ax)^2(bx)^{m}}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x)^m*ArcCos[a*x]^2,x]
```

```
[Out] ((b*x)^(1 + m)*ArcCos[a*x]^2)/(b*(1 + m)) + (2*a*(b*x)^(2 + m)*ArcCos[a*x]*
Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(b^2*(1 + m)*(2 + m)
) + (2*a^2*(b*x)^(3 + m)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 +
m/2, 5/2 + m/2}, a^2*x^2])/(b^3*(1 + m)*(2 + m)*(3 + m))
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4712

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcCos[c*x])*Hypergeomet
ric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(Sqrt[d]*f*(m + 1)), x] + Simp[
(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 +
```

```
m/2}, c^2*x^2])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d, e,
f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && !IntegerQ[m]
```

Rubi steps

$$\int (bx)^m \cos^{-1}(ax)^2\,dx = \frac{(bx)^{1+m}\cos^{-1}(ax)^2}{b(1+m)} + \frac{(2a)\int \frac{(bx)^{1+m}\cos^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{b(1+m)}$$

$$= \frac{(bx)^{1+m}\cos^{-1}(ax)^2}{b(1+m)} + \frac{2a(bx)^{2+m}\cos^{-1}(ax)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{b^2(1+m)(2+m)} + \frac{2a^2(bx)^{3+m}\,{}_3F_2\left(1,\frac{3}{2}+\frac{m}{2},\right.}{b^3(1+m)(2}$$

Mathematica [C] time = 2.17981, size = 132, normalized size = 0.88

$$\frac{x(bx)^m\left(ax\left(\sqrt{\pi}a2^{-m}x\mathrm{Gamma}(m+2)\,{}_3\tilde{F}_2\left(1,\frac{m+3}{2},\frac{m+3}{2};\frac{m+4}{2},\frac{m+5}{2};a^2x^2\right)+\frac{8\sqrt{1-a^2x^2}\cos^{-1}(ax)\mathrm{Hypergeometric2F1}\left(1,\frac{m+3}{2},\frac{m+4}{2},a^2x^2\right.}{m+2}\right.\right.}{4(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*x)^m*ArcCos[a*x]^2,x]
```

```
[Out] (x*(b*x)^m*(4*ArcCos[a*x]^2 + a*x*((8*Sqrt[1 - a^2*x^2]*ArcCos[a*x]*Hyperge
ometric2F1[1, (3 + m)/2, (4 + m)/2, a^2*x^2])/(2 + m) + (a*Sqrt[Pi]*x*Gamma
[2 + m]*HypergeometricPFQRegularized[{1, (3 + m)/2, (3 + m)/2}, {(4 + m)/2,
 (5 + m)/2}, a^2*x^2])/2^m)))/(4*(1 + m))
```

Maple [F] time = 0.808, size = 0, normalized size = 0.

$$\int (bx)^m (\arccos(ax))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m*arccos(a*x)^2,x)
```

```
[Out] int((b*x)^m*arccos(a*x)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*arccos(a*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx)^m \arccos(ax)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*arccos(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x)^m*arccos(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (bx)^m \operatorname{acos}^2(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**m*acos(a*x)**2,x)
```

```
[Out] Integral((b*x)**m*acos(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx)^m \arccos(ax)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x)^m*arccos(a*x)^2, x)
```

3.122 $\int (bx)^m \cos^{-1}(ax)\,dx$

Optimal. Leaf size=68

$$\frac{a(bx)^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},a^2x^2\right)}{b^2(m+1)(m+2)}+\frac{\cos^{-1}(ax)(bx)^{m+1}}{b(m+1)}$$

```
[Out] ((b*x)^(1 + m)*ArcCos[a*x])/(b*(1 + m)) + (a*(b*x)^(2 + m)*Hypergeometric2F
1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(b^2*(1 + m)*(2 + m))
```

Rubi [A] time = 0.0256728, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4628, 364}

$$\frac{a(bx)^{m+2}\,_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{b^2(m+1)(m+2)}+\frac{\cos^{-1}(ax)(bx)^{m+1}}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x)^m*ArcCos[a*x],x]
```

```
[Out] ((b*x)^(1 + m)*ArcCos[a*x])/(b*(1 + m)) + (a*(b*x)^(2 + m)*Hypergeometric2F
1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(b^2*(1 + m)*(2 + m))
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (bx)^m \cos^{-1}(ax)\,dx = \frac{(bx)^{1+m}\cos^{-1}(ax)}{b(1+m)} + \frac{a\int \frac{(bx)^{1+m}}{\sqrt{1-a^2x^2}}\,dx}{b(1+m)}$$
$$= \frac{(bx)^{1+m}\cos^{-1}(ax)}{b(1+m)} + \frac{a(bx)^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{b^2(1+m)(2+m)}$$

Mathematica [A] time = 0.0352791, size = 54, normalized size = 0.79

$$\frac{x(bx)^m\left(ax\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2}+1,\frac{m}{2}+2,a^2x^2\right)+(m+2)\cos^{-1}(ax)\right)}{(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x)^m*ArcCos[a*x],x]
```

```
[Out] (x*(b*x)^m*((2 + m)*ArcCos[a*x] + a*x*Hypergeometric2F1[1/2, 1 + m/2, 2 + m
/2, a^2*x^2]))/((1 + m)*(2 + m))
```

Maple [F] time = 0.866, size = 0, normalized size = 0.

$$\int (bx)^m \arccos(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m*arccos(a*x),x)
```

```
[Out] int((b*x)^m*arccos(a*x),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*arccos(a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx)^m \arccos(ax), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*arccos(a*x),x, algorithm="fricas")
```

```
[Out] integral((b*x)^m*arccos(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (bx)^m \operatorname{acos}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**m*acos(a*x),x)
```

```
[Out] Integral((b*x)**m*acos(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx)^m \arccos(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*arccos(a*x),x, algorithm="giac")
```

```
[Out] integrate((b*x)^m*arccos(a*x), x)
```

3.123 $\int \frac{(bx)^m}{\cos^{-1}(ax)}\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{(bx)^m}{\cos^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(b*x)^m/ArcCos[a*x], x]
```

Rubi [A] time = 0.0172439, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(bx)^m}{\cos^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(b*x)^m/ArcCos[a*x],x]
```

```
[Out] Defer[Int][(b*x)^m/ArcCos[a*x], x]
```

Rubi steps

$$\int \frac{(bx)^m}{\cos^{-1}(ax)}\,dx=\int \frac{(bx)^m}{\cos^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.650287, size = 0, normalized size = 0.

$$\int \frac{(bx)^m}{\cos^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(b*x)^m/ArcCos[a*x],x]
```

```
[Out] Integrate[(b*x)^m/ArcCos[a*x], x]
```

Maple [A] time = 0.536, size = 0, normalized size = 0.

$$\int \frac{(bx)^m}{\arccos(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m/arccos(a*x),x)
```

```
[Out] int((b*x)^m/arccos(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx)^m}{\arccos(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m/arccos(a*x),x, algorithm="maxima")
```

```
[Out] integrate((b*x)^m/arccos(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx)^m}{\arccos(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m/arccos(a*x),x, algorithm="fricas")
```

```
[Out] integral((b*x)^m/arccos(a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx)^m}{\operatorname{acos}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**m/acos(a*x),x)
```

```
[Out] Integral((b*x)**m/acos(a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx)^m}{\arccos(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m/arccos(a*x),x, algorithm="giac")
```

```
[Out] integrate((b*x)^m/arccos(a*x), x)
```

3.124 $\int \frac{(bx)^m}{\cos^{-1}(ax)^2} \, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{(bx)^m}{\cos^{-1}(ax)^2}, x\right)$$

```
[Out] Unintegrable[(b*x)^m/ArcCos[a*x]^2, x]
```

Rubi [A] time = 0.0151697, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(bx)^m}{\cos^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(b*x)^m/ArcCos[a*x]^2,x]
```

```
[Out] Defer[Int][(b*x)^m/ArcCos[a*x]^2, x]
```

Rubi steps

$$\int \frac{(bx)^m}{\cos^{-1}(ax)^2} \, dx = \int \frac{(bx)^m}{\cos^{-1}(ax)^2} \, dx$$

Mathematica [A] time = 0.662289, size = 0, normalized size = 0.

$$\int \frac{(bx)^m}{\cos^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(b*x)^m/ArcCos[a*x]^2,x]
```

```
[Out] Integrate[(b*x)^m/ArcCos[a*x]^2, x]
```

Maple [A] time = 0.693, size = 0, normalized size = 0.

$$\int \frac{(bx)^m}{(\arccos (ax))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m/arccos(a*x)^2,x)
```

```
[Out] int((b*x)^m/arccos(a*x)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m/arccos(a*x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx)^m}{\arccos (ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m/arccos(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x)^m/arccos(a*x)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx)^m}{\operatorname{acos}^2 (ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**m/acos(a*x)**2,x)
```

```
[Out] Integral((b*x)**m/acos(a*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx)^m}{\arccos(ax)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m/arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x)^m/arccos(a*x)^2, x)
```

3.125 $\int (bx)^m \cos^{-1}(ax)^{3/2} \, dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\cos^{-1}(ax)^{3/2}(bx)^m, x\right)$$

```
[Out] Unintegrable[(b*x)^m*ArcCos[a*x]^(3/2), x]
```

Rubi [A] time = 0.0154452, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (bx)^m \cos^{-1}(ax)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(b*x)^m*ArcCos[a*x]^(3/2),x]
```

```
[Out] Defer[Int][(b*x)^m*ArcCos[a*x]^(3/2), x]
```

Rubi steps

$$\int (bx)^m \cos^{-1}(ax)^{3/2} \, dx = \int (bx)^m \cos^{-1}(ax)^{3/2} \, dx$$

Mathematica [A] time = 5.82512, size = 0, normalized size = 0.

$$\int (bx)^m \cos^{-1}(ax)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(b*x)^m*ArcCos[a*x]^(3/2),x]
```

```
[Out] Integrate[(b*x)^m*ArcCos[a*x]^(3/2), x]
```

Maple [A] time = 0.108, size = 0, normalized size = 0.

$$\int (bx)^m (\arccos (ax))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m*arccos(a*x)^(3/2),x)
```

```
[Out] int((b*x)^m*arccos(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*arccos(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*arccos(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**m*acos(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (bx)^m \arccos{(ax)}^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*arccos(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x)^m*arccos(a*x)^(3/2), x)
```

3.126 $\int (bx)^m \sqrt{\cos^{-1}(ax)}\, dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\sqrt{\cos^{-1}(ax)}(bx)^m, x\right)$$

```
[Out] Unintegrable[(b*x)^m*Sqrt[ArcCos[a*x]], x]
```

Rubi [A] time = 0.0160559, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (bx)^m \sqrt{\cos^{-1}(ax)}\, dx$$

Verification is Not applicable to the result.

```
[In] Int[(b*x)^m*Sqrt[ArcCos[a*x]],x]
```

```
[Out] Defer[Int][(b*x)^m*Sqrt[ArcCos[a*x]], x]
```

Rubi steps

$$\int (bx)^m \sqrt{\cos^{-1}(ax)}\, dx = \int (bx)^m \sqrt{\cos^{-1}(ax)}\, dx$$

Mathematica [A] time = 6.05175, size = 0, normalized size = 0.

$$\int (bx)^m \sqrt{\cos^{-1}(ax)}\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(b*x)^m*Sqrt[ArcCos[a*x]],x]
```

```
[Out] Integrate[(b*x)^m*Sqrt[ArcCos[a*x]], x]
```

Maple [A] time = 0.1, size = 0, normalized size = 0.

$$\int (bx)^{m} \sqrt{\arccos \left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m*arccos(a*x)^(1/2),x)

[Out] int((b*x)^m*arccos(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*arccos(a*x)^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*arccos(a*x)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (bx)^{m} \sqrt{\operatorname{acos}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**m*acos(a*x)**(1/2),x)
```

```
[Out] Integral((b*x)**m*sqrt(acos(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (bx)^m \sqrt{\arccos (ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*arccos(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x)^m*sqrt(arccos(a*x)), x)
```

3.127 $\int \frac{(bx)^m}{\sqrt{\cos^{-1}(ax)}} \, dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{(bx)^m}{\sqrt{\cos^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[(b*x)^m/Sqrt[ArcCos[a*x]], x]
```

Rubi [A] time = 0.0158808, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(bx)^m}{\sqrt{\cos^{-1}(ax)}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(b*x)^m/Sqrt[ArcCos[a*x]],x]
```

```
[Out] Defer[Int][(b*x)^m/Sqrt[ArcCos[a*x]], x]
```

Rubi steps

$$\int \frac{(bx)^m}{\sqrt{\cos^{-1}(ax)}} \, dx = \int \frac{(bx)^m}{\sqrt{\cos^{-1}(ax)}} \, dx$$

Mathematica [A] time = 5.737, size = 0, normalized size = 0.

$$\int \frac{(bx)^m}{\sqrt{\cos^{-1}(ax)}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(b*x)^m/Sqrt[ArcCos[a*x]],x]
```

```
[Out] Integrate[(b*x)^m/Sqrt[ArcCos[a*x]], x]
```

Maple [A] time = 0.099, size = 0, normalized size = 0.

$$\int (bx)^m \frac{1}{\sqrt{\arccos(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m/arccos(a*x)^(1/2),x)
```

```
[Out] int((b*x)^m/arccos(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m/arccos(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m/arccos(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx)^m}{\sqrt{\operatorname{acos}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**m/acos(a*x)**(1/2),x)
```

```
[Out] Integral((b*x)**m/sqrt(acos(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx)^m}{\sqrt{\arccos(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m/arccos(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x)^m/sqrt(arccos(a*x)), x)
```

3.128 $\int \frac{(bx)^m}{\cos^{-1}(ax)^{3/2}} dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{(bx)^m}{\cos^{-1}(ax)^{3/2}}, x\right)$$

```
[Out] Unintegrable[(b*x)^m/ArcCos[a*x]^(3/2), x]
```

Rubi [A] time = 0.0150363, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(bx)^m}{\cos^{-1}(ax)^{3/2}} dx$$

Verification is Not applicable to the result.

```
[In] Int[(b*x)^m/ArcCos[a*x]^(3/2),x]
```

```
[Out] Defer[Int][(b*x)^m/ArcCos[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{(bx)^m}{\cos^{-1}(ax)^{3/2}} dx = \int \frac{(bx)^m}{\cos^{-1}(ax)^{3/2}} dx$$

Mathematica [A] time = 3.40223, size = 0, normalized size = 0.

$$\int \frac{(bx)^m}{\cos^{-1}(ax)^{3/2}} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(b*x)^m/ArcCos[a*x]^(3/2),x]
```

```
[Out] Integrate[(b*x)^m/ArcCos[a*x]^(3/2), x]
```

Maple [A] time = 0.098, size = 0, normalized size = 0.

$$\int (bx)^{m} (\arccos(ax))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m/arccos(a*x)^(3/2),x)
```

```
[Out] int((b*x)^m/arccos(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m/arccos(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m/arccos(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx)^{m}}{\operatorname{acos}^{\frac{3}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**m/acos(a*x)**(3/2),x)
```

```
[Out] Integral((b*x)**m/acos(a*x)**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx)^m}{\arccos{(ax)}^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m/arccos(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x)^m/arccos(a*x)^(3/2), x)
```

3.129 $\int (bx)^m \cos^{-1}(ax)^n \, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left((bx)^m \cos^{-1}(ax)^n, x\right)$$

```
[Out] Unintegrable[(b*x)^m*ArcCos[a*x]^n, x]
```

Rubi [A] time = 0.016163, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (bx)^m \cos^{-1}(ax)^n \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(b*x)^m*ArcCos[a*x]^n,x]
```

```
[Out] Defer[Int][(b*x)^m*ArcCos[a*x]^n, x]
```

Rubi steps

$$\int (bx)^m \cos^{-1}(ax)^n \, dx = \int (bx)^m \cos^{-1}(ax)^n \, dx$$

Mathematica [A] time = 0.98683, size = 0, normalized size = 0.

$$\int (bx)^m \cos^{-1}(ax)^n \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(b*x)^m*ArcCos[a*x]^n,x]
```

```
[Out] Integrate[(b*x)^m*ArcCos[a*x]^n, x]
```

Maple [A] time = 0.839, size = 0, normalized size = 0.

$$\int (bx)^m (\arccos (ax))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m*arccos(a*x)^n,x)
```

```
[Out] int((b*x)^m*arccos(a*x)^n,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*arccos(a*x)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx)^m \arccos (ax)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*arccos(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral((b*x)^m*arccos(a*x)^n, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (bx)^m \operatorname{acos}^n (ax) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**m*acos(a*x)**n,x)
```

```
[Out] Integral((b*x)**m*acos(a*x)**n, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (bx)^m \arccos (ax)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*arccos(a*x)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x)^m*arccos(a*x)^n, x)
```

3.130 $\int x^3 \cos^{-1}(ax)^n \, dx$

Optimal. Leaf size=165

$$\frac{2^{-n-4}\cos^{-1}(ax)^n\left(-i\cos^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-2i\cos^{-1}(ax)\right)}{a^4}+\frac{2^{-2(n+3)}\cos^{-1}(ax)^n\left(-i\cos^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+\right.}{a^4}$$

```
[Out] (2^(-4 - n)*ArcCos[a*x]^n*Gamma[1 + n, (-2*I)*ArcCos[a*x]])/(a^4*((-I)*ArcC
os[a*x])^n) + (2^(-4 - n)*ArcCos[a*x]^n*Gamma[1 + n, (2*I)*ArcCos[a*x]])/(a
^4*(I*ArcCos[a*x])^n) + (ArcCos[a*x]^n*Gamma[1 + n, (-4*I)*ArcCos[a*x]])/(2
^(2*(3 + n))*a^4*((-I)*ArcCos[a*x])^n) + (ArcCos[a*x]^n*Gamma[1 + n, (4*I)*
ArcCos[a*x]])/(2^(2*(3 + n))*a^4*(I*ArcCos[a*x])^n)
```

Rubi [A] time = 0.175624, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4636, 4406, 3308, 2181}

$$\frac{2^{-n-4}\cos^{-1}(ax)^n\left(-i\cos^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-2i\cos^{-1}(ax)\right)}{a^4}+\frac{2^{-2(n+3)}\cos^{-1}(ax)^n\left(-i\cos^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+\right.}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCos[a*x]^n,x]
```


```
[Out] (2^(-4 - n)*ArcCos[a*x]^n*Gamma[1 + n, (-2*I)*ArcCos[a*x]])/(a^4*((-I)*ArcC
os[a*x])^n) + (2^(-4 - n)*ArcCos[a*x]^n*Gamma[1 + n, (2*I)*ArcCos[a*x]])/(a
^4*(I*ArcCos[a*x])^n) + (ArcCos[a*x]^n*Gamma[1 + n, (-4*I)*ArcCos[a*x]])/(2
^(2*(3 + n))*a^4*((-I)*ArcCos[a*x])^n) + (ArcCos[a*x]^n*Gamma[1 + n, (4*I)*
ArcCos[a*x]])/(2^(2*(3 + n))*a^4*(I*ArcCos[a*x])^n)
```


Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x^3 \cos^{-1}(ax)^n\,dx = -\frac{\text{Subst}\left(\int x^n \cos^3(x)\sin(x)\,dx, x, \cos^{-1}(ax)\right)}{a^4}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{4}x^n \sin(2x) + \frac{1}{8}x^n \sin(4x)\right)dx, x, \cos^{-1}(ax)\right)}{a^4}$$
$$= -\frac{\text{Subst}\left(\int x^n \sin(4x)\,dx, x, \cos^{-1}(ax)\right)}{8a^4} - \frac{\text{Subst}\left(\int x^n \sin(2x)\,dx, x, \cos^{-1}(ax)\right)}{4a^4}$$
$$= -\frac{i\,\text{Subst}\left(\int e^{-4ix}x^n\,dx, x, \cos^{-1}(ax)\right)}{16a^4} + \frac{i\,\text{Subst}\left(\int e^{4ix}x^n\,dx, x, \cos^{-1}(ax)\right)}{16a^4} - \frac{i\,\text{Subst}\left(\int e^{-2ix}x^n\,dx,\right.}{8a^4}$$
$$= \frac{2^{-4-n}\left(-i\cos^{-1}(ax)\right)^{-n}\cos^{-1}(ax)^n\Gamma\left(1+n, -2i\cos^{-1}(ax)\right)}{a^4} + \frac{2^{-4-n}\left(i\cos^{-1}(ax)\right)^{-n}\cos^{-1}(ax)^n\Gamma\left(1+\right.}{a^4}$$

Mathematica [A] time = 0.101475, size = 130, normalized size = 0.79

$$\frac{2^{-2(n+3)}\cos^{-1}(ax)^n\left(\cos^{-1}(ax)^2\right)^{-n}\left(2^{n+2}\left(-i\cos^{-1}(ax)\right)^n \text{Gamma}\left(n+1, 2i\cos^{-1}(ax)\right) + \left(-i\cos^{-1}(ax)\right)^n \text{Gamma}\left(n+1\right.\right.}{a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCos[a*x]^n,x]
```

```
[Out] (ArcCos[a*x]^n*(2^(2 + n)*(I*ArcCos[a*x])^n*Gamma[1 + n, (-2*I)*ArcCos[a*x]
] + 2^(2 + n)*((-I)*ArcCos[a*x])^n*Gamma[1 + n, (2*I)*ArcCos[a*x]] + (I*Arc
```

```
Cos[a*x])^n*Gamma[1 + n, (-4*I)*ArcCos[a*x]] + ((-I)*ArcCos[a*x])^n*Gamma[1
 + n, (4*I)*ArcCos[a*x]]))/(2^(2*(3 + n))*a^4*(ArcCos[a*x]^2)^n)
```

Maple [C] time = 0.215, size = 287, normalized size = 1.7

$$-\frac{\sqrt{\pi}}{8\,a^4}\left(2\,\frac{(\arccos(ax))^{1+n}\sin(2\,\arccos(ax))}{\sqrt{\pi}\,(2+n)}-\frac{\sin(2\,\arccos(ax))}{\sqrt{\pi}\,(2+n)}2^{\frac{1}{2}-n}\sqrt{\arccos(ax)}\mathrm{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},2\,\arccos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccos(a*x)^n,x)
```

```
[Out] -1/8*Pi^(1/2)/a^4*(2/Pi^(1/2)/(2+n)*arccos(a*x)^(1+n)*sin(2*arccos(a*x))-2^
(1/2-n)/Pi^(1/2)/(2+n)*arccos(a*x)^(1/2)*LommelS1(n+3/2,3/2,2*arccos(a*x))*
sin(2*arccos(a*x))-3*2^(-3/2-n)/Pi^(1/2)/(2+n)/arccos(a*x)^(1/2)*(4/3+2/3*n
)*(2*arccos(a*x)*cos(2*arccos(a*x))-sin(2*arccos(a*x)))*LommelS1(n+1/2,1/2,
2*arccos(a*x)))-2^(-5-n)*Pi^(1/2)/a^4*(2^(2+n)/Pi^(1/2)/(2+n)*arccos(a*x)^(
1+n)*sin(4*arccos(a*x))-2^(1-n)/Pi^(1/2)/(2+n)*arccos(a*x)^(1/2)*LommelS1(n
+3/2,3/2,4*arccos(a*x))*sin(4*arccos(a*x))-3*2^(-2-n)/Pi^(1/2)/(2+n)/arccos
(a*x)^(1/2)*(4/3+2/3*n)*(4*arccos(a*x)*cos(4*arccos(a*x))-sin(4*arccos(a*x)
))*LommelS1(n+1/2,1/2,4*arccos(a*x)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^3\arccos(ax)^n\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(x^3*arccos(a*x)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{acos}^n(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acos(a*x)**n,x)
```

```
[Out] Integral(x**3*acos(a*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \arccos(ax)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x)^n,x, algorithm="giac")
```

```
[Out] integrate(x^3*arccos(a*x)^n, x)
```

3.131 $\int x^2 \cos^{-1}(ax)^n \, dx$

Optimal. Leaf size=163

$$\frac{\cos^{-1}(ax)^n \left(-i\cos^{-1}(ax)\right)^{-n} \text{Gamma}\left(n+1,-i\cos^{-1}(ax)\right)}{8a^3}+\frac{3^{-n-1}\cos^{-1}(ax)^n \left(-i\cos^{-1}(ax)\right)^{-n} \text{Gamma}\left(n+1,-3i\cos\right.}{8a^3}$$

```
[Out] (ArcCos[a*x]^n*Gamma[1 + n, (-I)*ArcCos[a*x]])/(8*a^3*((-I)*ArcCos[a*x])^n)
 + (ArcCos[a*x]^n*Gamma[1 + n, I*ArcCos[a*x]])/(8*a^3*(I*ArcCos[a*x])^n) +
(3^(-1 - n)*ArcCos[a*x]^n*Gamma[1 + n, (-3*I)*ArcCos[a*x]])/(8*a^3*((-I)*Ar
cCos[a*x])^n) + (3^(-1 - n)*ArcCos[a*x]^n*Gamma[1 + n, (3*I)*ArcCos[a*x]])/
(8*a^3*(I*ArcCos[a*x])^n)
```

Rubi [A] time = 0.14896, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4636, 4406, 3308, 2181}

$$\frac{\cos^{-1}(ax)^n \left(-i\cos^{-1}(ax)\right)^{-n} \text{Gamma}\left(n+1,-i\cos^{-1}(ax)\right)}{8a^3}+\frac{3^{-n-1}\cos^{-1}(ax)^n \left(-i\cos^{-1}(ax)\right)^{-n} \text{Gamma}\left(n+1,-3i\cos\right.}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCos[a*x]^n,x]
```

```
[Out] (ArcCos[a*x]^n*Gamma[1 + n, (-I)*ArcCos[a*x]])/(8*a^3*((-I)*ArcCos[a*x])^n)
 + (ArcCos[a*x]^n*Gamma[1 + n, I*ArcCos[a*x]])/(8*a^3*(I*ArcCos[a*x])^n) +
(3^(-1 - n)*ArcCos[a*x]^n*Gamma[1 + n, (-3*I)*ArcCos[a*x]])/(8*a^3*((-I)*Ar
cCos[a*x])^n) + (3^(-1 - n)*ArcCos[a*x]^n*Gamma[1 + n, (3*I)*ArcCos[a*x]])/
(8*a^3*(I*ArcCos[a*x])^n)
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^2\cos^{-1}(ax)^n\,dx &= -\frac{\operatorname{Subst}\left(\int x^n\cos^2(x)\sin(x)\,dx,x,\cos^{-1}(ax)\right)}{a^3}\\
&= -\frac{\operatorname{Subst}\left(\int\left(\frac{1}{4}x^n\sin(x)+\frac{1}{4}x^n\sin(3x)\right)dx,x,\cos^{-1}(ax)\right)}{a^3}\\
&= -\frac{\operatorname{Subst}\left(\int x^n\sin(x)\,dx,x,\cos^{-1}(ax)\right)}{4a^3}-\frac{\operatorname{Subst}\left(\int x^n\sin(3x)\,dx,x,\cos^{-1}(ax)\right)}{4a^3}\\
&= -\frac{i\operatorname{Subst}\left(\int e^{-ix}x^n\,dx,x,\cos^{-1}(ax)\right)}{8a^3}+\frac{i\operatorname{Subst}\left(\int e^{ix}x^n\,dx,x,\cos^{-1}(ax)\right)}{8a^3}-\frac{i\operatorname{Subst}\left(\int e^{-3ix}x^n\,dx,x,\right.}{8a^3}\\
&= \frac{\left(-i\cos^{-1}(ax)\right)^{-n}\cos^{-1}(ax)^n\Gamma\left(1+n,-i\cos^{-1}(ax)\right)}{8a^3}+\frac{\left(i\cos^{-1}(ax)\right)^{-n}\cos^{-1}(ax)^n\Gamma\left(1+n,i\cos^{-1}(a\right.}{8a^3}
\end{aligned}$$

Mathematica [A] time = 0.194486, size = 152, normalized size = 0.93

$$\frac{\frac{1}{8}3^{-n-1}\cos^{-1}(ax)^n\left(\cos^{-1}(ax)^2\right)^{-n}\left(\left(-i\cos^{-1}(ax)\right)^n\text{Gamma}\left(n+1,3i\cos^{-1}(ax)\right)+\left(i\cos^{-1}(ax)\right)^n\text{Gamma}\left(n+1,-3i\cos\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCos[a*x]^n,x]
```

```
[Out] (((ArcCos[a*x]^n*Gamma[1 + n, (-I)*ArcCos[a*x]])/(2*((-I)*ArcCos[a*x])^n) +
 (ArcCos[a*x]^n*Gamma[1 + n, I*ArcCos[a*x]])/(2*(I*ArcCos[a*x])^n))/4 + (3^
```

```
(-1 - n)*ArcCos[a*x]^n*((I*ArcCos[a*x])^n*Gamma[1 + n, (-3*I)*ArcCos[a*x]]
+ ((-I)*ArcCos[a*x])^n*Gamma[1 + n, (3*I)*ArcCos[a*x]]))/(8*(ArcCos[a*x]^2)
^n))/a^3
```

Maple [F] time = 0.152, size = 0, normalized size = 0.

$$\int x^{2} \left(\arccos\left(ax\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccos(a*x)^n,x)
```

```
[Out] int(x^2*arccos(a*x)^n,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^{2} \arccos\left(ax\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(x^2*arccos(a*x)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{acos}^n(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acos(a*x)**n,x)
```

```
[Out] Integral(x**2*acos(a*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \arccos(ax)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x)^n,x, algorithm="giac")
```

```
[Out] integrate(x^2*arccos(a*x)^n, x)
```

3.132 $\int x \cos^{-1}(ax)^n \, dx$

Optimal. Leaf size=83

$$\frac{2^{-n-3}\cos^{-1}(ax)^n\left(-i\cos^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-2i\cos^{-1}(ax)\right)}{a^2}+\frac{2^{-n-3}\left(i\cos^{-1}(ax)\right)^{-n}\cos^{-1}(ax)^n\text{Gamma}\left(n+1,2\right.}{a^2}$$

```
[Out] (2^(-3 - n)*ArcCos[a*x]^n*Gamma[1 + n, (-2*I)*ArcCos[a*x]])/(a^2*((-I)*ArcC
os[a*x])^n) + (2^(-3 - n)*ArcCos[a*x]^n*Gamma[1 + n, (2*I)*ArcCos[a*x]])/(a
^2*(I*ArcCos[a*x])^n)
```

Rubi [A] time = 0.0827482, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4636, 4406, 12, 3308, 2181}

$$\frac{2^{-n-3}\cos^{-1}(ax)^n\left(-i\cos^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-2i\cos^{-1}(ax)\right)}{a^2}+\frac{2^{-n-3}\left(i\cos^{-1}(ax)\right)^{-n}\cos^{-1}(ax)^n\text{Gamma}\left(n+1,2\right.}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCos[a*x]^n,x]
```

```
[Out] (2^(-3 - n)*ArcCos[a*x]^n*Gamma[1 + n, (-2*I)*ArcCos[a*x]])/(a^2*((-I)*ArcC
os[a*x])^n) + (2^(-3 - n)*ArcCos[a*x]^n*Gamma[1 + n, (2*I)*ArcCos[a*x]])/(a
^2*(I*ArcCos[a*x])^n)
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x\cos^{-1}(ax)^n\,dx &= -\frac{\text{Subst}\left(\int x^n\cos(x)\sin(x)\,dx, x, \cos^{-1}(ax)\right)}{a^2}\\ &= -\frac{\text{Subst}\left(\int \frac{1}{2}x^n\sin(2x)\,dx, x, \cos^{-1}(ax)\right)}{a^2}\\ &= -\frac{\text{Subst}\left(\int x^n\sin(2x)\,dx, x, \cos^{-1}(ax)\right)}{2a^2}\\ &= -\frac{i\,\text{Subst}\left(\int e^{-2ix}x^n\,dx, x, \cos^{-1}(ax)\right)}{4a^2}+\frac{i\,\text{Subst}\left(\int e^{2ix}x^n\,dx, x, \cos^{-1}(ax)\right)}{4a^2}\\ &= \frac{2^{-3-n}\left(-i\cos^{-1}(ax)\right)^{-n}\cos^{-1}(ax)^n\Gamma\left(1+n,-2i\cos^{-1}(ax)\right)}{a^2}+\frac{2^{-3-n}\left(i\cos^{-1}(ax)\right)^{-n}\cos^{-1}(ax)^n\Gamma\left(1+\right.}{a^2}\end{aligned}$$

Mathematica [A] time = 0.0546777, size = 74, normalized size = 0.89

$$\frac{2^{-n-3}\cos^{-1}(ax)^n\left(\cos^{-1}(ax)^2\right)^{-n}\left(\left(-i\cos^{-1}(ax)\right)^n\text{Gamma}\left(n+1,2i\cos^{-1}(ax)\right)+\left(i\cos^{-1}(ax)\right)^n\text{Gamma}\left(n+1,-2i\cos\right.\right.}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCos[a*x]^n,x]
```

```
[Out] (2^(-3 - n)*ArcCos[a*x]^n*((I*ArcCos[a*x])^n*Gamma[1 + n, (-2*I)*ArcCos[a*x
]] + ((-I)*ArcCos[a*x])^n*Gamma[1 + n, (2*I)*ArcCos[a*x]]))/(a^2*(ArcCos[a*
x]^2)^n)
```

Maple [C] time = 0.117, size = 138, normalized size = 1.7

$$-\frac{\sqrt{\pi}}{4\,a^{2}}\left(2\,\frac{(\arccos(ax))^{1+n}\sin(2\,\arccos(ax))}{\sqrt{\pi}\,(2+n)}-\frac{\sin(2\,\arccos(ax))}{\sqrt{\pi}\,(2+n)}2^{\frac{1}{2}-n}\sqrt{\arccos(ax)}\mathrm{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},2\,\arccos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccos(a*x)^n,x)
```

```
[Out] -1/4*Pi^(1/2)/a^2*(2/Pi^(1/2)/(2+n)*arccos(a*x)^(1+n)*sin(2*arccos(a*x))-2^
(1/2-n)/Pi^(1/2)/(2+n)*arccos(a*x)^(1/2)*LommelS1(n+3/2,3/2,2*arccos(a*x))*
sin(2*arccos(a*x))-3*2^(-3/2-n)/Pi^(1/2)/(2+n)/arccos(a*x)^(1/2)*(4/3+2/3*n
)*(2*arccos(a*x)*cos(2*arccos(a*x))-sin(2*arccos(a*x)))*LommelS1(n+1/2,1/2,
2*arccos(a*x)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x\arccos(ax)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(x*arccos(a*x)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{acos}^{n}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acos(a*x)**n,x)
```

```
[Out] Integral(x*acos(a*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \arccos(ax)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x)^n,x, algorithm="giac")
```

```
[Out] integrate(x*arccos(a*x)^n, x)
```

3.133 $\int \cos^{-1}(ax)^n\,dx$

Optimal. Leaf size=75

$$\frac{\cos^{-1}(ax)^n\left(-i\cos^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-i\cos^{-1}(ax)\right)}{2a}+\frac{\left(i\cos^{-1}(ax)\right)^{-n}\cos^{-1}(ax)^n\text{Gamma}\left(n+1,i\cos^{-1}(ax)\right)}{2a}$$

```
[Out] (ArcCos[a*x]^n*Gamma[1 + n, (-I)*ArcCos[a*x]])/(2*a*((-I)*ArcCos[a*x])^n) +
 (ArcCos[a*x]^n*Gamma[1 + n, I*ArcCos[a*x]])/(2*a*(I*ArcCos[a*x])^n)
```

Rubi [A] time = 0.0493503, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4624, 3308, 2181}

$$\frac{\cos^{-1}(ax)^n\left(-i\cos^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-i\cos^{-1}(ax)\right)}{2a}+\frac{\left(i\cos^{-1}(ax)\right)^{-n}\cos^{-1}(ax)^n\text{Gamma}\left(n+1,i\cos^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x]^n,x]
```

```
[Out] (ArcCos[a*x]^n*Gamma[1 + n, (-I)*ArcCos[a*x]])/(2*a*((-I)*ArcCos[a*x])^n) +
 (ArcCos[a*x]^n*Gamma[1 + n, I*ArcCos[a*x]])/(2*a*(I*ArcCos[a*x])^n)
```

Rule 4624

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Sub
st[Int[x^n*Sin[a/b - x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c,
 n}, x]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
```

```
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \cos^{-1}(ax)^n\,dx &= -\frac{\text{Subst}\left(\int x^n \sin(x)\,dx, x, \cos^{-1}(ax)\right)}{a}\\ &= -\frac{i\,\text{Subst}\left(\int e^{-ix}x^n\,dx, x, \cos^{-1}(ax)\right)}{2a} + \frac{i\,\text{Subst}\left(\int e^{ix}x^n\,dx, x, \cos^{-1}(ax)\right)}{2a}\\ &= \frac{\left(-i\cos^{-1}(ax)\right)^{-n}\cos^{-1}(ax)^n\Gamma\left(1+n, -i\cos^{-1}(ax)\right)}{2a} + \frac{\left(i\cos^{-1}(ax)\right)^{-n}\cos^{-1}(ax)^n\Gamma\left(1+n, i\cos^{-1}(ax)\right)}{2a}\end{aligned}$$

Mathematica [A] time = 0.0303296, size = 70, normalized size = 0.93

$$\frac{\cos^{-1}(ax)^n\left(\cos^{-1}(ax)^2\right)^{-n}\left(\left(-i\cos^{-1}(ax)\right)^n \text{Gamma}\left(n+1, i\cos^{-1}(ax)\right) + \left(i\cos^{-1}(ax)\right)^n \text{Gamma}\left(n+1, -i\cos^{-1}(ax)\right)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x]^n,x]
```

```
[Out] (ArcCos[a*x]^n*((I*ArcCos[a*x])^n*Gamma[1 + n, (-I)*ArcCos[a*x]] + ((-I)*Ar
cCos[a*x])^n*Gamma[1 + n, I*ArcCos[a*x]]))/(2*a*(ArcCos[a*x]^2)^n)
```

Maple [C] time = 0.095, size = 148, normalized size = 2.

$$-\frac{2^n\sqrt{\pi}}{a}\left(\frac{(\arccos(ax))^{1+n}2^{-n}}{\sqrt{\pi}(2+n)}\sqrt{-a^2x^2+1} - \frac{2^{-n}}{\sqrt{\pi}(2+n)}\sqrt{\arccos(ax)}\text{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},\arccos(ax)\right)\sqrt{-a^2x^2+1} - 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^n,x)
```

```
[Out] -2^n*Pi^(1/2)/a*(1/Pi^(1/2)/(2+n)*arccos(a*x)^(1+n)*2^(-n)*(-a^2*x^2+1)^(1/
2)-2^(-n)/Pi^(1/2)/(2+n)*arccos(a*x)^(1/2)*LommelS1(n+3/2,3/2,arccos(a*x))*
(-a^2*x^2+1)^(1/2)-3*2^(-1-n)/Pi^(1/2)/(2+n)/arccos(a*x)^(1/2)*(4/3+2/3*n)*
```

```
(a*x*arccos(a*x)-(-a^2*x^2+1)^(1/2))*LommelS1(n+1/2,1/2,arccos(a*x)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\arccos\left(ax\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acos}^{n}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**n,x)
```

```
[Out] Integral(acos(a*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \arccos\left(ax\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^n,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^n, x)
```

3.134 $\int \frac{\cos^{-1}(ax)^n}{x} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\cos^{-1}(ax)^n}{x}, x\right)$$

```
[Out] Unintegrable[ArcCos[a*x]^n/x, x]
```

Rubi [A] time = 0.0151887, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos^{-1}(ax)^n}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCos[a*x]^n/x,x]
```

```
[Out] Defer[Int][ArcCos[a*x]^n/x, x]
```

Rubi steps

$$\int \frac{\cos^{-1}(ax)^n}{x} dx = \int \frac{\cos^{-1}(ax)^n}{x} dx$$

Mathematica [A] time = 0.328122, size = 0, normalized size = 0.

$$\int \frac{\cos^{-1}(ax)^n}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCos[a*x]^n/x,x]
```

```
[Out] Integrate[ArcCos[a*x]^n/x, x]
```

Maple [A] time = 0.121, size = 0, normalized size = 0.

$$\int \frac{(\arccos(ax))^n}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^n/x,x)
```

```
[Out] int(arccos(a*x)^n/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^n/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arccos(ax)^n}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^n/x,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)^n/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acos}^n(ax)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**n/x,x)
```

```
[Out] Integral(acos(a*x)**n/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(ax)^n}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^n/x,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^n/x, x)
```

3.135 $\int \frac{\cos^{-1}(ax)^n}{x^2} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\cos^{-1}(ax)^n}{x^2},x\right)$$

```
[Out] Unintegrable[ArcCos[a*x]^n/x^2, x]
```

Rubi [A] time = 0.0152596, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos^{-1}(ax)^n}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCos[a*x]^n/x^2,x]
```

```
[Out] Defer[Int][ArcCos[a*x]^n/x^2, x]
```

Rubi steps

$$\int \frac{\cos^{-1}(ax)^n}{x^2} dx = \int \frac{\cos^{-1}(ax)^n}{x^2} dx$$

Mathematica [A] time = 0.608815, size = 0, normalized size = 0.

$$\int \frac{\cos^{-1}(ax)^n}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCos[a*x]^n/x^2,x]
```

```
[Out] Integrate[ArcCos[a*x]^n/x^2, x]
```

Maple [A] time = 0.102, size = 0, normalized size = 0.

$$\int \frac{(\arccos(ax))^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^n/x^2,x)
```

```
[Out] int(arccos(a*x)^n/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^n/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arccos(ax)^n}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^n/x^2,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)^n/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acos}^n(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**n/x**2,x)
```

```
[Out] Integral(acos(a*x)**n/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(ax)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^n/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^n/x^2, x)
```

3.136 $\int (bx)^{3/2} \cos^{-1}(ax)^n \, dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left((bx)^{3/2} \cos^{-1}(ax)^n, x\right)$$

```
[Out] Unintegrable[(b*x)^(3/2)*ArcCos[a*x]^n, x]
```

Rubi [A] time = 0.0205362, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (bx)^{3/2} \cos^{-1}(ax)^n \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(b*x)^(3/2)*ArcCos[a*x]^n,x]
```

```
[Out] Defer[Int][(b*x)^(3/2)*ArcCos[a*x]^n, x]
```

Rubi steps

$$\int (bx)^{3/2} \cos^{-1}(ax)^n \, dx = \int (bx)^{3/2} \cos^{-1}(ax)^n \, dx$$

Mathematica [A] time = 4.30827, size = 0, normalized size = 0.

$$\int (bx)^{3/2} \cos^{-1}(ax)^n \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(b*x)^(3/2)*ArcCos[a*x]^n,x]
```

```
[Out] Integrate[(b*x)^(3/2)*ArcCos[a*x]^n, x]
```

Maple [A] time = 0.169, size = 0, normalized size = 0.

$$\int (bx)^{\frac{3}{2}} (\arccos (ax))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^(3/2)*arccos(a*x)^n,x)
```

```
[Out] int((b*x)^(3/2)*arccos(a*x)^n,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(3/2)*arccos(a*x)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx}bx \arccos (ax)^n , x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(3/2)*arccos(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x)*b*x*arccos(a*x)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**(3/2)*acos(a*x)**n,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (bx)^{\frac{3}{2}} \arccos (ax)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(3/2)*arccos(a*x)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x)^(3/2)*arccos(a*x)^n, x)
```

3.137 $\int \sqrt{bx} \cos^{-1}(ax)^n \, dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\sqrt{bx} \cos^{-1}(ax)^n, x\right)$$

```
[Out] Unintegrable[Sqrt[b*x]*ArcCos[a*x]^n, x]
```

Rubi [A] time = 0.0181128, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{bx} \cos^{-1}(ax)^n \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[b*x]*ArcCos[a*x]^n,x]
```

```
[Out] Defer[Int][Sqrt[b*x]*ArcCos[a*x]^n, x]
```

Rubi steps

$$\int \sqrt{bx} \cos^{-1}(ax)^n \, dx = \int \sqrt{bx} \cos^{-1}(ax)^n \, dx$$

Mathematica [A] time = 5.4538, size = 0, normalized size = 0.

$$\int \sqrt{bx} \cos^{-1}(ax)^n \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[b*x]*ArcCos[a*x]^n,x]
```

```
[Out] Integrate[Sqrt[b*x]*ArcCos[a*x]^n, x]
```

Maple [A] time = 0.162, size = 0, normalized size = 0.

$$\int (\arccos (ax))^n \sqrt{bx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^n*(b*x)^(1/2),x)
```

```
[Out] int(arccos(a*x)^n*(b*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^n*(b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{bx} \arccos (ax)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^n*(b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x)*arccos(a*x)^n, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx} \operatorname{acos}^n (ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**n*(b*x)**(1/2),x)
```

```
[Out] Integral(sqrt(b*x)*acos(a*x)**n, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx} \arccos{(ax)}^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^n*(b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x)*arccos(a*x)^n, x)
```

3.138 $\int \frac{\cos^{-1}(ax)^n}{\sqrt{bx}}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\cos^{-1}(ax)^n}{\sqrt{bx}},x\right)$$

```
[Out] Unintegrable[ArcCos[a*x]^n/Sqrt[b*x], x]
```

Rubi [A] time = 0.0189931, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos^{-1}(ax)^n}{\sqrt{bx}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCos[a*x]^n/Sqrt[b*x],x]
```

```
[Out] Defer[Int][ArcCos[a*x]^n/Sqrt[b*x], x]
```

Rubi steps

$$\int \frac{\cos^{-1}(ax)^n}{\sqrt{bx}}\,dx = \int \frac{\cos^{-1}(ax)^n}{\sqrt{bx}}\,dx$$

Mathematica [A] time = 1.74358, size = 0, normalized size = 0.

$$\int \frac{\cos^{-1}(ax)^n}{\sqrt{bx}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCos[a*x]^n/Sqrt[b*x],x]
```

```
[Out] Integrate[ArcCos[a*x]^n/Sqrt[b*x], x]
```

Maple [A] time = 0.153, size = 0, normalized size = 0.

$$\int (\arccos (ax))^n \frac{1}{\sqrt{bx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^n/(b*x)^(1/2),x)
```

```
[Out] int(arccos(a*x)^n/(b*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^n/(b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx}\arccos (ax)^n}{bx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^n/(b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x)*arccos(a*x)^n/(b*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acos}^n (ax)}{\sqrt{bx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**n/(b*x)**(1/2),x)
```

```
[Out] Integral(acos(a*x)**n/sqrt(b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos{(ax)}^n}{\sqrt{bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^n/(b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^n/sqrt(b*x), x)
```

3.139 $\int \frac{\cos^{-1}(ax)^n}{(bx)^{3/2}}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\cos^{-1}(ax)^n}{(bx)^{3/2}},x\right)$$

```
[Out] Unintegrable[ArcCos[a*x]^n/(b*x)^(3/2), x]
```

Rubi [A] time = 0.0220214, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos^{-1}(ax)^n}{(bx)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCos[a*x]^n/(b*x)^(3/2),x]
```

```
[Out] Defer[Int][ArcCos[a*x]^n/(b*x)^(3/2), x]
```

Rubi steps

$$\int \frac{\cos^{-1}(ax)^n}{(bx)^{3/2}}\,dx = \int \frac{\cos^{-1}(ax)^n}{(bx)^{3/2}}\,dx$$

Mathematica [A] time = 1.80819, size = 0, normalized size = 0.

$$\int \frac{\cos^{-1}(ax)^n}{(bx)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCos[a*x]^n/(b*x)^(3/2),x]
```

```
[Out] Integrate[ArcCos[a*x]^n/(b*x)^(3/2), x]
```

Maple [A] time = 0.148, size = 0, normalized size = 0.

$$\int (\arccos(ax))^n (bx)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x)^n/(b*x)^(3/2),x)
```

```
[Out] int(arccos(a*x)^n/(b*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^n/(b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx}\arccos(ax)^n}{b^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^n/(b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x)*arccos(a*x)^n/(b^2*x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acos}^n(ax)}{(bx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x)**n/(b*x)**(3/2),x)
```

```
[Out] Integral(acos(a*x)**n/(b*x)**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arccos(ax)^n}{(bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x)^n/(b*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^n/(b*x)^(3/2), x)
```

3.140 $\int x^3 \left(a+b\cos^{-1}(cx)\right) dx$

Optimal. Leaf size=76

$$\frac{1}{4}x^4\left(a+b\cos^{-1}(cx)\right)-\frac{bx^3\sqrt{1-c^2x^2}}{16c}-\frac{3bx\sqrt{1-c^2x^2}}{32c^3}+\frac{3b\sin^{-1}(cx)}{32c^4}$$

```
[Out] (-3*b*x*Sqrt[1 - c^2*x^2])/(32*c^3) - (b*x^3*Sqrt[1 - c^2*x^2])/(16*c) + (x
^4*(a + b*ArcCos[c*x]))/4 + (3*b*ArcSin[c*x])/(32*c^4)
```

Rubi [A] time = 0.0355778, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4628, 321, 216}

$$\frac{1}{4}x^4\left(a+b\cos^{-1}(cx)\right)-\frac{bx^3\sqrt{1-c^2x^2}}{16c}-\frac{3bx\sqrt{1-c^2x^2}}{32c^3}+\frac{3b\sin^{-1}(cx)}{32c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcCos[c*x]),x]
```

```
[Out] (-3*b*x*Sqrt[1 - c^2*x^2])/(32*c^3) - (b*x^3*Sqrt[1 - c^2*x^2])/(16*c) + (x
^4*(a + b*ArcCos[c*x]))/4 + (3*b*ArcSin[c*x])/(32*c^4)
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+b\cos^{-1}(cx)\right)dx &= \frac{1}{4}x^4\left(a+b\cos^{-1}(cx)\right)+\frac{1}{4}(bc)\int\frac{x^4}{\sqrt{1-c^2x^2}}\,dx\\
&= -\frac{bx^3\sqrt{1-c^2x^2}}{16c}+\frac{1}{4}x^4\left(a+b\cos^{-1}(cx)\right)+\frac{(3b)\int\frac{x^2}{\sqrt{1-c^2x^2}}\,dx}{16c}\\
&= -\frac{3bx\sqrt{1-c^2x^2}}{32c^3}-\frac{bx^3\sqrt{1-c^2x^2}}{16c}+\frac{1}{4}x^4\left(a+b\cos^{-1}(cx)\right)+\frac{(3b)\int\frac{1}{\sqrt{1-c^2x^2}}\,dx}{32c^3}\\
&= -\frac{3bx\sqrt{1-c^2x^2}}{32c^3}-\frac{bx^3\sqrt{1-c^2x^2}}{16c}+\frac{1}{4}x^4\left(a+b\cos^{-1}(cx)\right)+\frac{3b\sin^{-1}(cx)}{32c^4}
\end{aligned}$$

Mathematica [A] time = 0.0474766, size = 68, normalized size = 0.89

$$\frac{ax^4}{4}+b\sqrt{1-c^2x^2}\left(-\frac{3x}{32c^3}-\frac{x^3}{16c}\right)+\frac{3b\sin^{-1}(cx)}{32c^4}+\frac{1}{4}bx^4\cos^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcCos[c*x]),x]
```

```
[Out] (a*x^4)/4 + b*Sqrt[1 - c^2*x^2]*((-3*x)/(32*c^3) - x^3/(16*c)) + (b*x^4*Arc
Cos[c*x])/4 + (3*b*ArcSin[c*x])/(32*c^4)
```

Maple [A] time = 0.006, size = 72, normalized size = 1.

$$\frac{1}{c^4}\left(\frac{c^4x^4a}{4}+b\left(\frac{c^4x^4\arccos(cx)}{4}-\frac{c^3x^3}{16}\sqrt{-c^2x^2+1}-\frac{3\,cx}{32}\sqrt{-c^2x^2+1}+\frac{3\arcsin(cx)}{32}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccos(c*x)),x)
```

```
[Out] 1/c^4*(1/4*c^4*x^4*a+b*(1/4*c^4*x^4*arccos(c*x)-1/16*c^3*x^3*(-c^2*x^2+1)^(
1/2)-3/32*c*x*(-c^2*x^2+1)^(1/2)+3/32*arcsin(c*x)))
```

Maxima [A] time = 1.44846, size = 112, normalized size = 1.47

$$\frac{1}{4}\,ax^4 + \frac{1}{32}\left(8\,x^4\arccos\left(cx\right) - \left(\frac{2\,\sqrt{-c^2x^2+1}x^3}{c^2} + \frac{3\,\sqrt{-c^2x^2+1}x}{c^4} - \frac{3\,\arcsin\left(\frac{c^2x}{\sqrt{c^2}}\right)}{\sqrt{c^2}c^4}\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccos(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/32*(8*x^4*arccos(c*x) - (2*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqr
t(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^4))*c)*b
```

Fricas [A] time = 2.45426, size = 139, normalized size = 1.83

$$\frac{8\,ac^4x^4 + \left(8\,bc^4x^4 - 3\,b\right)\arccos\left(cx\right) - \left(2\,bc^3x^3 + 3\,bcx\right)\sqrt{-c^2x^2+1}}{32\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccos(c*x)),x, algorithm="fricas")
```

```
[Out] 1/32*(8*a*c^4*x^4 + (8*b*c^4*x^4 - 3*b)*arccos(c*x) - (2*b*c^3*x^3 + 3*b*c*
x)*sqrt(-c^2*x^2 + 1))/c^4
```

Sympy [A] time = 1.28894, size = 85, normalized size = 1.12

$$\begin{cases} \frac{ax^4}{4} + \frac{bx^4\operatorname{acos}(cx)}{4} - \frac{bx^3\sqrt{-c^2x^2+1}}{16c} - \frac{3bx\sqrt{-c^2x^2+1}}{32c^3} - \frac{3b\operatorname{acos}(cx)}{32c^4} & \text{for}\ c \neq 0 \\ \frac{x^4\left(a+\frac{\pi b}{2}\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acos(c*x)),x)
```

```
[Out] Piecewise((a*x**4/4 + b*x**4*acos(c*x)/4 - b*x**3*sqrt(-c**2*x**2 + 1)/(16*
c) - 3*b*x*sqrt(-c**2*x**2 + 1)/(32*c**3) - 3*b*acos(c*x)/(32*c**4), Ne(c,
```

```
0)), (x**4*(a + pi*b/2)/4, True))
```

Giac [A] time = 1.1333, size = 90, normalized size = 1.18

$$\frac{1}{4}\,bx^4\arccos\left(cx\right)+\frac{1}{4}\,ax^4-\frac{\sqrt{-c^2x^2+1}bx^3}{16\,c}-\frac{3\,\sqrt{-c^2x^2+1}bx}{32\,c^3}-\frac{3\,b\arccos\left(cx\right)}{32\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccos(c*x)),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4*arccos(c*x) + 1/4*a*x^4 - 1/16*sqrt(-c^2*x^2 + 1)*b*x^3/c - 3/32*
sqrt(-c^2*x^2 + 1)*b*x/c^3 - 3/32*b*arccos(c*x)/c^4
```

3.141 $\int x^2 \left(a + b \cos^{-1}(cx)\right) dx$

Optimal. Leaf size=60

$$\frac{1}{3}x^3 \left(a + b\cos^{-1}(cx)\right) + \frac{b\left(1 - c^2x^2\right)^{3/2}}{9c^3} - \frac{b\sqrt{1 - c^2x^2}}{3c^3}$$

```
[Out] -(b*Sqrt[1 - c^2*x^2])/(3*c^3) + (b*(1 - c^2*x^2)^(3/2))/(9*c^3) + (x^3*(a
+ b*ArcCos[c*x]))/3
```

Rubi [A] time = 0.0418091, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4628, 266, 43}

$$\frac{1}{3}x^3 \left(a + b\cos^{-1}(cx)\right) + \frac{b\left(1 - c^2x^2\right)^{3/2}}{9c^3} - \frac{b\sqrt{1 - c^2x^2}}{3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcCos[c*x]),x]
```

```
[Out] -(b*Sqrt[1 - c^2*x^2])/(3*c^3) + (b*(1 - c^2*x^2)^(3/2))/(9*c^3) + (x^3*(a
+ b*ArcCos[c*x]))/3
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+b\cos^{-1}(cx)\right)dx &= \frac{1}{3}x^3\left(a+b\cos^{-1}(cx)\right)+\frac{1}{3}(bc)\int\frac{x^3}{\sqrt{1-c^2x^2}}\,dx\\ &= \frac{1}{3}x^3\left(a+b\cos^{-1}(cx)\right)+\frac{1}{6}(bc)\,\text{Subst}\left(\int\frac{x}{\sqrt{1-c^2x}}\,dx,x,x^2\right)\\ &= \frac{1}{3}x^3\left(a+b\cos^{-1}(cx)\right)+\frac{1}{6}(bc)\,\text{Subst}\left(\int\left(\frac{1}{c^2\sqrt{1-c^2x}}-\frac{\sqrt{1-c^2x}}{c^2}\right)dx,x,x^2\right)\\ &= -\frac{b\sqrt{1-c^2x^2}}{3c^3}+\frac{b\left(1-c^2x^2\right)^{3/2}}{9c^3}+\frac{1}{3}x^3\left(a+b\cos^{-1}(cx)\right)\end{aligned}$$

Mathematica [A] time = 0.0417632, size = 55, normalized size = 0.92

$$\frac{ax^3}{3}+b\left(-\frac{2}{9c^3}-\frac{x^2}{9c}\right)\sqrt{1-c^2x^2}+\frac{1}{3}bx^3\cos^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcCos[c*x]),x]
```

```
[Out] (a*x^3)/3 + b*(-2/(9*c^3) - x^2/(9*c))*Sqrt[1 - c^2*x^2] + (b*x^3*ArcCos[c*
x])/3
```

Maple [A] time = 0.005, size = 64, normalized size = 1.1

$$\frac{1}{c^3}\left(\frac{c^3x^3a}{3}+b\left(\frac{c^3x^3\arccos(cx)}{3}-\frac{c^2x^2}{9}\sqrt{-c^2x^2+1}-\frac{2}{9}\sqrt{-c^2x^2+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccos(c*x)),x)
```

```
[Out] 1/c^3*(1/3*c^3*x^3*a+b*(1/3*c^3*x^3*arccos(c*x)-1/9*c^2*x^2*(-c^2*x^2+1)^(1
/2)-2/9*(-c^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.4429, size = 81, normalized size = 1.35

$$\frac{1}{3}ax^3 + \frac{1}{9}\left(3x^3 \arccos(cx) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccos(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/9*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt
(-c^2*x^2 + 1)/c^4))*b
```

Fricas [A] time = 2.59659, size = 119, normalized size = 1.98

$$\frac{3bc^3x^3 \arccos(cx) + 3ac^3x^3 - (bc^2x^2 + 2b)\sqrt{-c^2x^2+1}}{9c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccos(c*x)),x, algorithm="fricas")
```

```
[Out] 1/9*(3*b*c^3*x^3*arccos(c*x) + 3*a*c^3*x^3 - (b*c^2*x^2 + 2*b)*sqrt(-c^2*x^
2 + 1))/c^3
```

Sympy [A] time = 0.642933, size = 70, normalized size = 1.17

$$\begin{cases} \frac{ax^3}{3} + \frac{bx^3 \operatorname{acos}(cx)}{3} - \frac{bx^2\sqrt{-c^2x^2+1}}{9c} - \frac{2b\sqrt{-c^2x^2+1}}{9c^3} & \text{for } c \neq 0 \\ \frac{x^3\left(a+\frac{\pi b}{2}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acos(c*x)),x)
```

```
[Out] Piecewise((a*x**3/3 + b*x**3*acos(c*x)/3 - b*x**2*sqrt(-c**2*x**2 + 1)/(9*c
) - 2*b*sqrt(-c**2*x**2 + 1)/(9*c**3), Ne(c, 0)), (x**3*(a + pi*b/2)/3, Tru
```

```
e))
```

Giac [A] time = 1.1333, size = 76, normalized size = 1.27

$$\frac{1}{3}\,bx^3\arccos\left(cx\right) + \frac{1}{3}\,ax^3 - \frac{\sqrt{-c^2x^2+1}bx^2}{9\,c} - \frac{2\,\sqrt{-c^2x^2+1}b}{9\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccos(c*x)),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*arccos(c*x) + 1/3*a*x^3 - 1/9*sqrt(-c^2*x^2 + 1)*b*x^2/c - 2/9*sq
rt(-c^2*x^2 + 1)*b/c^3
```

3.142 $\int x \left(a + b \cos^{-1}(cx)\right) dx$

Optimal. Leaf size=51

$$\frac{1}{2}x^2 \left(a + b \cos^{-1}(cx)\right) - \frac{bx\sqrt{1 - c^2x^2}}{4c} + \frac{b \sin^{-1}(cx)}{4c^2}$$

```
[Out] -(b*x*Sqrt[1 - c^2*x^2])/(4*c) + (x^2*(a + b*ArcCos[c*x]))/2 + (b*ArcSin[c*
x])/(4*c^2)
```

Rubi [A] time = 0.0191739, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4628, 321, 216}

$$\frac{1}{2}x^2 \left(a + b \cos^{-1}(cx)\right) - \frac{bx\sqrt{1 - c^2x^2}}{4c} + \frac{b \sin^{-1}(cx)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcCos[c*x]),x]
```

```
[Out] -(b*x*Sqrt[1 - c^2*x^2])/(4*c) + (x^2*(a + b*ArcCos[c*x]))/2 + (b*ArcSin[c*
x])/(4*c^2)
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\cos^{-1}(cx)\right)dx &= \frac{1}{2}x^2\left(a+b\cos^{-1}(cx)\right)+\frac{1}{2}(bc)\int\frac{x^2}{\sqrt{1-c^2x^2}}\,dx\\ &= -\frac{bx\sqrt{1-c^2x^2}}{4c}+\frac{1}{2}x^2\left(a+b\cos^{-1}(cx)\right)+\frac{b\int\frac{1}{\sqrt{1-c^2x^2}}\,dx}{4c}\\ &= -\frac{bx\sqrt{1-c^2x^2}}{4c}+\frac{1}{2}x^2\left(a+b\cos^{-1}(cx)\right)+\frac{b\sin^{-1}(cx)}{4c^2}\end{aligned}$$

Mathematica [A] time = 0.03564, size = 56, normalized size = 1.1

$$\frac{ax^2}{2}-\frac{bx\sqrt{1-c^2x^2}}{4c}+\frac{b\sin^{-1}(cx)}{4c^2}+\frac{1}{2}bx^2\cos^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcCos[c*x]),x]
```

```
[Out] (a*x^2)/2 - (b*x*Sqrt[1 - c^2*x^2])/(4*c) + (b*x^2*ArcCos[c*x])/2 + (b*ArcS
in[c*x])/(4*c^2)
```

Maple [A] time = 0.005, size = 52, normalized size = 1.

$$\frac{1}{c^2}\left(\frac{c^2x^2a}{2}+b\left(\frac{c^2x^2\arccos(cx)}{2}-\frac{cx}{4}\sqrt{-c^2x^2+1}+\frac{\arcsin(cx)}{4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccos(c*x)),x)
```

```
[Out] 1/c^2*(1/2*c^2*x^2*a+b*(1/2*c^2*x^2*arccos(c*x)-1/4*c*x*(-c^2*x^2+1)^(1/2)+
1/4*arcsin(c*x)))
```

Maxima [A] time = 1.43945, size = 84, normalized size = 1.65

$$\frac{1}{2}ax^2 + \frac{1}{4}\left(2x^2 \arccos(cx) - c\left(\frac{\sqrt{-c^2x^2+1}x}{c^2} - \frac{\arcsin\left(\frac{c^2x}{\sqrt{c^2}}\right)}{\sqrt{c^2}c^2}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccos(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/4*(2*x^2*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x/c^2 - arcsin(c
^2*x/sqrt(c^2))/(sqrt(c^2)*c^2)))*b
```

Fricas [A] time = 2.57809, size = 111, normalized size = 2.18

$$\frac{2\,ac^2x^2 - \sqrt{-c^2x^2+1}bcx + \left(2\,bc^2x^2 - b\right)\arccos(cx)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccos(c*x)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*c^2*x^2 - sqrt(-c^2*x^2 + 1)*b*c*x + (2*b*c^2*x^2 - b)*arccos(c*x)
)/c^2
```

Sympy [A] time = 0.331354, size = 60, normalized size = 1.18

$$\begin{cases} \frac{ax^2}{2} + \frac{bx^2 \operatorname{acos}(cx)}{2} - \frac{bx\sqrt{-c^2x^2+1}}{4c} - \frac{b\operatorname{acos}(cx)}{4c^2} & \text{for } c \neq 0 \\ \frac{x^2\left(a+\frac{\pi b}{2}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acos(c*x)),x)
```

```
[Out] Piecewise((a*x**2/2 + b*x**2*acos(c*x)/2 - b*x*sqrt(-c**2*x**2 + 1)/(4*c) -
 b*acos(c*x)/(4*c**2), Ne(c, 0)), (x**2*(a + pi*b/2)/2, True))
```

Giac [A] time = 1.15468, size = 62, normalized size = 1.22

$$\frac{1}{2}\,bx^2\arccos\left(cx\right)+\frac{1}{2}\,ax^2-\frac{\sqrt{-c^2x^2+1}bx}{4\,c}-\frac{b\arccos\left(cx\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccos(c*x)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2*arccos(c*x) + 1/2*a*x^2 - 1/4*sqrt(-c^2*x^2 + 1)*b*x/c - 1/4*b*ar
ccos(c*x)/c^2
```

3.143 $\int \left(a + b\cos^{-1}(cx)\right)\,dx$

Optimal. Leaf size=31

$$ax - \frac{b\sqrt{1-c^2x^2}}{c} + bx\cos^{-1}(cx)$$

```
[Out] a*x - (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcCos[c*x]
```

Rubi [A] time = 0.019312, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4620, 261}

$$ax - \frac{b\sqrt{1-c^2x^2}}{c} + bx\cos^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCos[c*x],x]
```

```
[Out] a*x - (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcCos[c*x]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\cos^{-1}(cx)\right)dx = ax + b\int \cos^{-1}(cx)\,dx$$
$$= ax + bx\cos^{-1}(cx) + (bc)\int \frac{x}{\sqrt{1-c^2x^2}}\,dx$$
$$= ax - \frac{b\sqrt{1-c^2x^2}}{c} + bx\cos^{-1}(cx)$$

Mathematica [A] time = 0.0121239, size = 31, normalized size = 1.

$$ax - \frac{b\sqrt{1-c^2x^2}}{c} + bx\cos^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcCos[c*x],x]
```

```
[Out] a*x - (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcCos[c*x]
```

Maple [A] time = 0.002, size = 32, normalized size = 1.

$$ax + \frac{b}{c}\left(cx\arccos(cx) - \sqrt{-c^2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccos(c*x),x)
```

```
[Out] a*x+b/c*(c*x*arccos(c*x)-(-c^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.43553, size = 42, normalized size = 1.35

$$ax + \frac{\left(cx\arccos(cx) - \sqrt{-c^2x^2+1}\right)b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccos(c*x),x, algorithm="maxima")
```

```
[Out] a*x + (c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*b/c
```

Fricas [A] time = 2.49453, size = 73, normalized size = 2.35

$$\frac{bcx \arccos(cx) + acx - \sqrt{-c^2x^2 + 1}b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccos(c*x),x, algorithm="fricas")
```

```
[Out] (b*c*x*arccos(c*x) + a*c*x - sqrt(-c^2*x^2 + 1)*b)/c
```

Sympy [A] time = 0.153665, size = 29, normalized size = 0.94

$$ax + b\left(\begin{cases} x \operatorname{acos}(cx) - \frac{\sqrt{-c^2x^2+1}}{c} & \text{for } c \neq 0 \\ \frac{\pi x}{2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*acos(c*x),x)
```

```
[Out] a*x + b*Piecewise((x*acos(c*x) - sqrt(-c**2*x**2 + 1)/c, Ne(c, 0)), (pi*x/2
, True))
```

Giac [A] time = 1.12345, size = 42, normalized size = 1.35

$$ax + \frac{\left(cx \arccos(cx) - \sqrt{-c^2x^2 + 1}\right)b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccos(c*x),x, algorithm="giac")
```

```
[Out] a*x + (c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*b/c
```

3.144 $\int \frac{a+b\cos^{-1}(cx)}{x}\,dx$

Optimal. Leaf size=63

$$-\frac{1}{2}ib\text{PolyLog}\left(2,-e^{2i\cos^{-1}(cx)}\right)-\frac{i\left(a+b\cos^{-1}(cx)\right)^2}{2b}+\log\left(1+e^{2i\cos^{-1}(cx)}\right)\left(a+b\cos^{-1}(cx)\right)$$

```
[Out] ((-I/2)*(a + b*ArcCos[c*x])^2)/b + (a + b*ArcCos[c*x])*Log[1 + E^((2*I)*Arc
Cos[c*x])] - (I/2)*b*PolyLog[2, -E^((2*I)*ArcCos[c*x])]
```

Rubi [A] time = 0.0716771, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4626, 3719, 2190, 2279, 2391}

$$-\frac{1}{2}ib\text{PolyLog}\left(2,-e^{2i\cos^{-1}(cx)}\right)-\frac{i\left(a+b\cos^{-1}(cx)\right)^2}{2b}+\log\left(1+e^{2i\cos^{-1}(cx)}\right)\left(a+b\cos^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])/x,x]
```

```
[Out] ((-I/2)*(a + b*ArcCos[c*x])^2)/b + (a + b*ArcCos[c*x])*Log[1 + E^((2*I)*Arc
Cos[c*x])] - (I/2)*b*PolyLog[2, -E^((2*I)*ArcCos[c*x])]
```

Rule 4626

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n/Cot[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\cos^{-1}(cx)}{x}\,dx &= -\operatorname{Subst}\left(\int (a+bx)\tan(x)\,dx, x, \cos^{-1}(cx)\right)\\
&= -\frac{i\left(a+b\cos^{-1}(cx)\right)^2}{2b} + 2i\operatorname{Subst}\left(\int \frac{e^{2ix}(a+bx)}{1+e^{2ix}}\,dx, x, \cos^{-1}(cx)\right)\\
&= -\frac{i\left(a+b\cos^{-1}(cx)\right)^2}{2b} + \left(a+b\cos^{-1}(cx)\right)\log\left(1+e^{2i\cos^{-1}(cx)}\right) - b\operatorname{Subst}\left(\int \log\left(1+e^{2ix}\right)\,dx, x\right.\\
&= -\frac{i\left(a+b\cos^{-1}(cx)\right)^2}{2b} + \left(a+b\cos^{-1}(cx)\right)\log\left(1+e^{2i\cos^{-1}(cx)}\right) + \frac{1}{2}(ib)\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,\right.\\
&= -\frac{i\left(a+b\cos^{-1}(cx)\right)^2}{2b} + \left(a+b\cos^{-1}(cx)\right)\log\left(1+e^{2i\cos^{-1}(cx)}\right) - \frac{1}{2}ib\operatorname{Li}_2\left(-e^{2i\cos^{-1}(cx)}\right)\end{aligned}$$

Mathematica [A] time = 0.0175066, size = 58, normalized size = 0.92

$$-\frac{1}{2}ib\operatorname{PolyLog}\left(2,-e^{2i\cos^{-1}(cx)}\right)+a\log(x)-\frac{1}{2}ib\cos^{-1}(cx)^2+b\cos^{-1}(cx)\log\left(1+e^{2i\cos^{-1}(cx)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[c*x])/x,x]
```

```
[Out] (-I/2)*b*ArcCos[c*x]^2 + b*ArcCos[c*x]*Log[1 + E^((2*I)*ArcCos[c*x])] + a*L
og[x] - (I/2)*b*PolyLog[2, -E^((2*I)*ArcCos[c*x])]
```

Maple [A] time = 0.06, size = 77, normalized size = 1.2

$$a\ln\left(cx\right)-\frac{i}{2}b\left(\arccos\left(cx\right)\right)^{2}+b\arccos\left(cx\right)\ln\left(1+\left(cx+i\sqrt{-c^{2}x^{2}+1}\right)^{2}\right)-\frac{i}{2}b\operatorname{polylog}\left(2,-\left(cx+i\sqrt{-c^{2}x^{2}+1}\right)^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))/x,x)
```

```
[Out] a*ln(c*x)-1/2*I*b*arccos(c*x)^2+b*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2
))^2)-1/2*I*b*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\int\frac{\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)}{x}\,dx+a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/x,x, algorithm="maxima")
```

```
[Out] b*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/x, x) + a*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b\arccos\left(cx\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arccos(c*x) + a)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\,\mathrm{acos}\,(cx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))/x,x)
```

```
[Out] Integral((a + b*acos(c*x))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\arccos\,(cx) + a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)/x, x)
```

3.145 $\int \frac{a+b\cos^{-1}(cx)}{x^2}\,dx$

Optimal. Leaf size=32

$$bc\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)-\frac{a+b\cos^{-1}(cx)}{x}$$

```
[Out] -((a + b*ArcCos[c*x])/x) + b*c*ArcTanh[Sqrt[1 - c^2*x^2]]
```

Rubi [A] time = 0.0281992, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4628, 266, 63, 208}

$$bc\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)-\frac{a+b\cos^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])/x^2,x]
```

```
[Out] -((a + b*ArcCos[c*x])/x) + b*c*ArcTanh[Sqrt[1 - c^2*x^2]]
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\cos^{-1}(cx)}{x^2}\,dx &= -\frac{a+b\cos^{-1}(cx)}{x}-(bc)\int \frac{1}{x\sqrt{1-c^2x^2}}\,dx\\ &= -\frac{a+b\cos^{-1}(cx)}{x}-\frac{1}{2}(bc)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)\\ &= -\frac{a+b\cos^{-1}(cx)}{x}+\frac{b\,\text{Subst}\left(\int \frac{1}{\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx,x,\sqrt{1-c^2x^2}\right)}{c}\\ &= -\frac{a+b\cos^{-1}(cx)}{x}+bc\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0119583, size = 43, normalized size = 1.34

$$-\frac{a}{x}+bc\log\left(\sqrt{1-c^2x^2}+1\right)-bc\log(x)-\frac{b\cos^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[c*x])/x^2,x]
```

```
[Out] -(a/x) - (b*ArcCos[c*x])/x - b*c*Log[x] + b*c*Log[1 + Sqrt[1 - c^2*x^2]]
```

Maple [A] time = 0.006, size = 41, normalized size = 1.3

$$c\left(-\frac{a}{cx}+b\left(-\frac{\arccos(cx)}{cx}+\text{Artanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))/x^2,x)
```

```
[Out] c*(-1/c/x*a+b*(-1/c/x*arccos(c*x)+arctanh(1/(-c^2*x^2+1)^(1/2))))
```

Maxima [A] time = 1.42362, size = 63, normalized size = 1.97

$$\left(c\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{\arccos(cx)}{x}\right)b-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] (c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) - arccos(c*x)/x)*b - a/x
```

Fricas [B] time = 2.74335, size = 221, normalized size = 6.91

$$\frac{bcx\log\left(\sqrt{-c^2x^2+1}+1\right)-bcx\log\left(\sqrt{-c^2x^2+1}-1\right)-2\,bx\arctan\left(\frac{\sqrt{-c^2x^2+1}cx}{c^2x^2-1}\right)+2\,(bx-b)\arccos(cx)-2\,a}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*c*x*log(sqrt(-c^2*x^2 + 1) + 1) - b*c*x*log(sqrt(-c^2*x^2 + 1) - 1)
- 2*b*x*arctan(sqrt(-c^2*x^2 + 1)*c*x/(c^2*x^2 - 1)) + 2*(b*x - b)*arccos(c
*x) - 2*a)/x
```

Sympy [A] time = 1.95527, size = 41, normalized size = 1.28

$$-\frac{a}{x}-bc\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for}\ \frac{1}{|c^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise}\end{cases}\right)-\frac{b\operatorname{acos}(cx)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))/x**2,x)
```

```
[Out] -a/x - b*c*Piecewise((-acosh(1/(c*x)), 1/Abs(c**2*x**2) > 1), (I*asin(1/(c*
x)), True)) - b*acos(c*x)/x
```

Giac [B] time = 1.52024, size = 468, normalized size = 14.62

$$-\frac{bc\arccos(cx)}{\frac{c^2x^2-1}{(cx+1)^2}+1}+\frac{bc\log\left(\left|cx+\sqrt{-c^2x^2+1}+1\right|\right)}{\frac{c^2x^2-1}{(cx+1)^2}+1}-\frac{bc\log\left(\left|-cx+\sqrt{-c^2x^2+1}-1\right|\right)}{\frac{c^2x^2-1}{(cx+1)^2}+1}-\frac{ac}{\frac{c^2x^2-1}{(cx+1)^2}+1}+\frac{(c^2x^2-1)bc\operatorname{arcco}}{(cx+1)^2\left(\frac{c^2x^2-1}{(cx+1)^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/x^2,x, algorithm="giac")
```

```
[Out] -b*c*arccos(c*x)/((c^2*x^2 - 1)/(c*x + 1)^2 + 1) + b*c*log(abs(c*x + sqrt(-
c^2*x^2 + 1) + 1))/((c^2*x^2 - 1)/(c*x + 1)^2 + 1) - b*c*log(abs(-c*x + sqr
t(-c^2*x^2 + 1) - 1))/((c^2*x^2 - 1)/(c*x + 1)^2 + 1) - a*c/((c^2*x^2 - 1)/
(c*x + 1)^2 + 1) + (c^2*x^2 - 1)*b*c*arccos(c*x)/((c*x + 1)^2*((c^2*x^2 - 1
)/(c*x + 1)^2 + 1)) + (c^2*x^2 - 1)*b*c*log(abs(c*x + sqrt(-c^2*x^2 + 1) +
1))/((c*x + 1)^2*((c^2*x^2 - 1)/(c*x + 1)^2 + 1)) - (c^2*x^2 - 1)*b*c*log(a
bs(-c*x + sqrt(-c^2*x^2 + 1) - 1))/((c*x + 1)^2*((c^2*x^2 - 1)/(c*x + 1)^2
+ 1)) + (c^2*x^2 - 1)*a*c/((c*x + 1)^2*((c^2*x^2 - 1)/(c*x + 1)^2 + 1))
```

3.146 $\int \frac{a+b\cos^{-1}(cx)}{x^3}\,dx$

Optimal. Leaf size=39

$$\frac{bc\sqrt{1-c^2x^2}}{2x}-\frac{a+b\cos^{-1}(cx)}{2x^2}$$

```
[Out] (b*c*Sqrt[1 - c^2*x^2])/(2*x) - (a + b*ArcCos[c*x])/(2*x^2)
```

Rubi [A] time = 0.0194112, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4628, 264}

$$\frac{bc\sqrt{1-c^2x^2}}{2x}-\frac{a+b\cos^{-1}(cx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])/x^3,x]
```

```
[Out] (b*c*Sqrt[1 - c^2*x^2])/(2*x) - (a + b*ArcCos[c*x])/(2*x^2)
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\cos^{-1}(cx)}{x^3}\,dx = -\frac{a+b\cos^{-1}(cx)}{2x^2} - \frac{1}{2}(bc)\int \frac{1}{x^2\sqrt{1-c^2x^2}}\,dx$$
$$= \frac{bc\sqrt{1-c^2x^2}}{2x} - \frac{a+b\cos^{-1}(cx)}{2x^2}$$

Mathematica [A] time = 0.0212276, size = 44, normalized size = 1.13

$$-\frac{a}{2x^2} + \frac{bc\sqrt{1-c^2x^2}}{2x} - \frac{b\cos^{-1}(cx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[c*x])/x^3,x]
```

```
[Out] -a/(2*x^2) + (b*c*Sqrt[1 - c^2*x^2])/(2*x) - (b*ArcCos[c*x])/(2*x^2)
```

Maple [A] time = 0.005, size = 50, normalized size = 1.3

$$c^2\left(-\frac{a}{2\,c^2x^2} + b\left(-\frac{\arccos(cx)}{2\,c^2x^2} + \frac{1}{2\,cx}\sqrt{-c^2x^2+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))/x^3,x)
```

```
[Out] c^2*(-1/2*a/c^2/x^2+b*(-1/2/c^2/x^2*arccos(c*x)+1/2/c/x*(-c^2*x^2+1)^(1/2))
)
```

Maxima [A] time = 1.45622, size = 50, normalized size = 1.28

$$\frac{1}{2}b\left(\frac{\sqrt{-c^2x^2+1}c}{x} - \frac{\arccos(cx)}{x^2}\right) - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*(sqrt(-c^2*x^2 + 1)*c/x - arccos(c*x)/x^2) - 1/2*a/x^2
```

Fricas [A] time = 2.84842, size = 86, normalized size = 2.21

$$\frac{\sqrt{-c^2x^2+1}bcx+ax^2-b\arccos(cx)-a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(-c^2*x^2 + 1)*b*c*x + a*x^2 - b*arccos(c*x) - a)/x^2
```

Sympy [A] time = 1.56326, size = 63, normalized size = 1.62

$$-\frac{a}{2x^2}-\frac{bc\left(\begin{cases}-\frac{i\sqrt{c^2x^2-1}}{x} & \text{for } \left|c^2x^2\right|>1\\ -\frac{\sqrt{-c^2x^2+1}}{x} & \text{otherwise}\end{cases}\right)}{2}-\frac{b\,\mathrm{acos}\,(cx)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))/x**3,x)
```

```
[Out] -a/(2*x**2) - b*c*Piecewise((-I*sqrt(c**2*x**2 - 1)/x, Abs(c**2*x**2) > 1),
 (-sqrt(-c**2*x**2 + 1)/x, True))/2 - b*acos(c*x)/(2*x**2)
```

Giac [B] time = 1.18139, size = 664, normalized size = 17.03

$$-\frac{bc^2\arccos(cx)}{2\left(\frac{2(c^2x^2-1)}{(cx+1)^2}+\frac{(c^2x^2-1)^2}{(cx+1)^4}+1\right)}-\frac{ac^2}{2\left(\frac{2(c^2x^2-1)}{(cx+1)^2}+\frac{(c^2x^2-1)^2}{(cx+1)^4}+1\right)}+\frac{(c^2x^2-1)bc^2\arccos(cx)}{(cx+1)^2\left(\frac{2(c^2x^2-1)}{(cx+1)^2}+\frac{(c^2x^2-1)^2}{(cx+1)^4}+1\right)}+\frac{\sqrt{-c^2x^2+1}}{(cx+1)\left(\frac{2(c^2x^2-1)}{(cx+1)^2}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/x^3,x, algorithm="giac")
```

```
[Out] -1/2*b*c^2*arccos(c*x)/(2*(c^2*x^2 - 1)/(c*x + 1)^2 + (c^2*x^2 - 1)^2/(c*x
+ 1)^4 + 1) - 1/2*a*c^2/(2*(c^2*x^2 - 1)/(c*x + 1)^2 + (c^2*x^2 - 1)^2/(c*x
 + 1)^4 + 1) + (c^2*x^2 - 1)*b*c^2*arccos(c*x)/((c*x + 1)^2*(2*(c^2*x^2 - 1
)/(c*x + 1)^2 + (c^2*x^2 - 1)^2/(c*x + 1)^4 + 1)) + sqrt(-c^2*x^2 + 1)*b*c^
2/((c*x + 1)*(2*(c^2*x^2 - 1)/(c*x + 1)^2 + (c^2*x^2 - 1)^2/(c*x + 1)^4 + 1
)) + (c^2*x^2 - 1)*a*c^2/((c*x + 1)^2*(2*(c^2*x^2 - 1)/(c*x + 1)^2 + (c^2*x
^2 - 1)^2/(c*x + 1)^4 + 1)) - 1/2*(c^2*x^2 - 1)^2*b*c^2*arccos(c*x)/((c*x +
 1)^4*(2*(c^2*x^2 - 1)/(c*x + 1)^2 + (c^2*x^2 - 1)^2/(c*x + 1)^4 + 1)) - (-
c^2*x^2 + 1)^(3/2)*b*c^2/((c*x + 1)^3*(2*(c^2*x^2 - 1)/(c*x + 1)^2 + (c^2*x
^2 - 1)^2/(c*x + 1)^4 + 1)) - 1/2*(c^2*x^2 - 1)^2*a*c^2/((c*x + 1)^4*(2*(c^
2*x^2 - 1)/(c*x + 1)^2 + (c^2*x^2 - 1)^2/(c*x + 1)^4 + 1))
```

3.147 $\int \frac{a+b\cos^{-1}(cx)}{x^4}\,dx$

Optimal. Leaf size=62

$$-\frac{a+b\cos^{-1}(cx)}{3x^3}+\frac{bc\sqrt{1-c^2x^2}}{6x^2}+\frac{1}{6}bc^3\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)$$

```
[Out] (b*c*Sqrt[1 - c^2*x^2])/(6*x^2) - (a + b*ArcCos[c*x])/(3*x^3) + (b*c^3*ArcT
anh[Sqrt[1 - c^2*x^2]])/6
```

Rubi [A] time = 0.0367287, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4628, 266, 51, 63, 208}

$$-\frac{a+b\cos^{-1}(cx)}{3x^3}+\frac{bc\sqrt{1-c^2x^2}}{6x^2}+\frac{1}{6}bc^3\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])/x^4,x]
```

```
[Out] (b*c*Sqrt[1 - c^2*x^2])/(6*x^2) - (a + b*ArcCos[c*x])/(3*x^3) + (b*c^3*ArcT
anh[Sqrt[1 - c^2*x^2]])/6
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
```

```
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\cos^{-1}(cx)}{x^4}\,dx &= -\frac{a+b\cos^{-1}(cx)}{3x^3}-\frac{1}{3}(bc)\int \frac{1}{x^3\sqrt{1-c^2x^2}}\,dx\\
&= -\frac{a+b\cos^{-1}(cx)}{3x^3}-\frac{1}{6}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-c^2x}}\,dx,x,x^2\right)\\
&= \frac{bc\sqrt{1-c^2x^2}}{6x^2}-\frac{a+b\cos^{-1}(cx)}{3x^3}-\frac{1}{12}\left(bc^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)\\
&= \frac{bc\sqrt{1-c^2x^2}}{6x^2}-\frac{a+b\cos^{-1}(cx)}{3x^3}+\frac{1}{6}(bc)\,\text{Subst}\left(\int \frac{1}{\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx,x,\sqrt{1-c^2x^2}\right)\\
&= \frac{bc\sqrt{1-c^2x^2}}{6x^2}-\frac{a+b\cos^{-1}(cx)}{3x^3}+\frac{1}{6}bc^3\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0247396, size = 79, normalized size = 1.27

$$-\frac{a}{3x^3}+\frac{bc\sqrt{1-c^2x^2}}{6x^2}+\frac{1}{6}bc^3\log\left(\sqrt{1-c^2x^2}+1\right)-\frac{1}{6}bc^3\log(x)-\frac{b\cos^{-1}(cx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[c*x])/x^4,x]
```

```
[Out] -a/(3*x^3) + (b*c*Sqrt[1 - c^2*x^2])/(6*x^2) - (b*ArcCos[c*x])/(3*x^3) - (b
*c^3*Log[x])/6 + (b*c^3*Log[1 + Sqrt[1 - c^2*x^2]])/6
```

Maple [A] time = 0.004, size = 65, normalized size = 1.1

$$c^3\left(-\frac{a}{3\,c^3x^3}+b\left(-\frac{\arccos(cx)}{3\,c^3x^3}+\frac{1}{6\,c^2x^2}\sqrt{-c^2x^2+1}+\frac{1}{6}\mathrm{Artanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))/x^4,x)
```

```
[Out] c^3*(-1/3*a/c^3/x^3+b*(-1/3/c^3/x^3*arccos(c*x)+1/6/c^2/x^2*(-c^2*x^2+1)^(1
/2)+1/6*arctanh(1/(-c^2*x^2+1)^(1/2))))
```

Maxima [A] time = 1.43471, size = 93, normalized size = 1.5

$$\frac{1}{6}\left(\left(c^2\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{\sqrt{-c^2x^2+1}}{x^2}\right)c-\frac{2\arccos(cx)}{x^3}\right)b-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*((c^2*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-c^2*x^2 + 1)/
x^2)*c - 2*arccos(c*x)/x^3)*b - 1/3*a/x^3
```

Fricas [B] time = 3.18666, size = 281, normalized size = 4.53

$$\frac{bc^3x^3\log\left(\sqrt{-c^2x^2+1}+1\right)-bc^3x^3\log\left(\sqrt{-c^2x^2+1}-1\right)-4\,bx^3\arctan\left(\frac{\sqrt{-c^2x^2+1}cx}{c^2x^2-1}\right)+2\sqrt{-c^2x^2+1}bcx+4\left(bx^3-b\right)\mathrm{ar}}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(b*c^3*x^3*log(sqrt(-c^2*x^2 + 1) + 1) - b*c^3*x^3*log(sqrt(-c^2*x^2 +
 1) - 1) - 4*b*x^3*arctan(sqrt(-c^2*x^2 + 1)*c*x/(c^2*x^2 - 1)) + 2*sqrt(-c
^2*x^2 + 1)*b*c*x + 4*(b*x^3 - b)*arccos(c*x) - 4*a)/x^3
```

Sympy [A] time = 3.2627, size = 121, normalized size = 1.95

$$-\frac{a}{3x^3}-\frac{bc\left(\begin{cases}-\frac{c^2\operatorname{acosh}\left(\frac{1}{cx}\right)}{2}-\frac{c\sqrt{-1+\frac{1}{c^2x^2}}}{2x} & \text{for } \frac{1}{|c^2x^2|}>1\\ \frac{ic^2\operatorname{asin}\left(\frac{1}{cx}\right)}{2}-\frac{ic}{2x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{i}{2cx^3\sqrt{1-\frac{1}{c^2x^2}}} & \text{otherwise}\end{cases}\right)}{3}-\frac{b\operatorname{acos}(cx)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))/x**4,x)
```

```
[Out] -a/(3*x**3) - b*c*Piecewise((-c**2*acosh(1/(c*x))/2 - c*sqrt(-1 + 1/(c**2*x
**2))/(2*x), 1/Abs(c**2*x**2) > 1), (I*c**2*asin(1/(c*x))/2 - I*c/(2*x*sqrt
(1 - 1/(c**2*x**2))) + I/(2*c*x**3*sqrt(1 - 1/(c**2*x**2))), True))/3 - b*a
cos(c*x)/(3*x**3)
```

Giac [B] time = 2.42751, size = 2206, normalized size = 35.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/x^4,x, algorithm="giac")
```

```
[Out] -1/3*b*c^3*arccos(c*x)/(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*
x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1) + 1/6*b*c^3*log(abs(c*x + sqrt(
-c^2*x^2 + 1) + 1))/(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x +
 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1) - 1/6*b*c^3*log(abs(-c*x + sqrt(-c
^2*x^2 + 1) - 1))/(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1
)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1) - 1/3*a*c^3/(3*(c^2*x^2 - 1)/(c*x +
1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1) + (
c^2*x^2 - 1)*b*c^3*arccos(c*x)/((c*x + 1)^2*(3*(c^2*x^2 - 1)/(c*x + 1)^2 +
3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) + 1/2*(c^
```

```
2*x^2 - 1)*b*c^3*log(abs(c*x + sqrt(-c^2*x^2 + 1) + 1))/((c*x + 1)^2*(3*(c^
2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c
*x + 1)^6 + 1)) - 1/2*(c^2*x^2 - 1)*b*c^3*log(abs(-c*x + sqrt(-c^2*x^2 + 1)
 - 1))/((c*x + 1)^2*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x +
 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) + 1/3*sqrt(-c^2*x^2 + 1)*b*c^3/((
c*x + 1)*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^
2*x^2 - 1)^3/(c*x + 1)^6 + 1)) + (c^2*x^2 - 1)*a*c^3/((c*x + 1)^2*(3*(c^2*x
^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x
+ 1)^6 + 1)) - (c^2*x^2 - 1)^2*b*c^3*arccos(c*x)/((c*x + 1)^4*(3*(c^2*x^2 -
 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)
^6 + 1)) + 1/2*(c^2*x^2 - 1)^2*b*c^3*log(abs(c*x + sqrt(-c^2*x^2 + 1) + 1))
/((c*x + 1)^4*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4
+ (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) - 1/2*(c^2*x^2 - 1)^2*b*c^3*log(abs(-c*
x + sqrt(-c^2*x^2 + 1) - 1))/((c*x + 1)^4*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*
(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) - (c^2*x^2
- 1)^2*a*c^3/((c*x + 1)^4*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/
(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) + 1/3*(c^2*x^2 - 1)^3*b*c^3
*arccos(c*x)/((c*x + 1)^6*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/
(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) + 1/6*(c^2*x^2 - 1)^3*b*c^3
*log(abs(c*x + sqrt(-c^2*x^2 + 1) + 1))/((c*x + 1)^6*(3*(c^2*x^2 - 1)/(c*x
+ 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1))
- 1/6*(c^2*x^2 - 1)^3*b*c^3*log(abs(-c*x + sqrt(-c^2*x^2 + 1) - 1))/((c*x +
 1)^6*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x
^2 - 1)^3/(c*x + 1)^6 + 1)) - 1/3*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*c^3/
((c*x + 1)^5*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 +
 (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) + 1/3*(c^2*x^2 - 1)^3*a*c^3/((c*x + 1)^6
*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 -
1)^3/(c*x + 1)^6 + 1))
```

3.148 $\int x^2 \left(a+b\cos^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=102

$$-\frac{2bx^2\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}{9c}-\frac{4b\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}{9c^3}+\frac{1}{3}x^3\left(a+b\cos^{-1}(cx)\right)^2-\frac{4b^2x}{9c^2}-\frac{2}{27}b^2x^3$$

```
[Out] (-4*b^2*x)/(9*c^2) - (2*b^2*x^3)/27 - (4*b*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[
c*x]))/(9*c^3) - (2*b*x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(9*c) + (x
^3*(a + b*ArcCos[c*x])^2)/3
```

Rubi [A] time = 0.15244, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4628, 4708, 4678, 8, 30}

$$-\frac{2bx^2\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}{9c}-\frac{4b\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}{9c^3}+\frac{1}{3}x^3\left(a+b\cos^{-1}(cx)\right)^2-\frac{4b^2x}{9c^2}-\frac{2}{27}b^2x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcCos[c*x])^2,x]
```

```
[Out] (-4*b^2*x)/(9*c^2) - (2*b^2*x^3)/27 - (4*b*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[
c*x]))/(9*c^3) - (2*b*x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(9*c) + (x
^3*(a + b*ArcCos[c*x])^2)/3
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

```
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+b\cos^{-1}(cx)\right)^2dx &= \frac{1}{3}x^3\left(a+b\cos^{-1}(cx)\right)^2+\frac{1}{3}(2bc)\int\frac{x^3\left(a+b\cos^{-1}(cx)\right)}{\sqrt{1-c^2x^2}}dx\\ &= -\frac{2bx^2\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}{9c}+\frac{1}{3}x^3\left(a+b\cos^{-1}(cx)\right)^2-\frac{1}{9}\left(2b^2\right)\int x^2\,dx+\frac{(4b)\int\frac{x(a}{}}{}\\ &= -\frac{2}{27}b^2x^3-\frac{4b\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}{9c^3}-\frac{2bx^2\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}{9c}+\frac{1}{3}x^3\left(a+b\right.\\ &= -\frac{4b^2x}{9c^2}-\frac{2b^2x^3}{27}-\frac{4b\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}{9c^3}-\frac{2bx^2\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}{9c}+\frac{1}{3}x^3\end{aligned}$$

Mathematica [A] time = 0.137475, size = 121, normalized size = 1.19

$$\frac{9a^2c^3x^3-6ab\sqrt{1-c^2x^2}\left(c^2x^2+2\right)-6b\cos^{-1}(cx)\left(b\sqrt{1-c^2x^2}\left(c^2x^2+2\right)-3ac^3x^3\right)-2b^2cx\left(c^2x^2+6\right)+9b^2c^3x^3\cos^{-1}(c}{27c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcCos[c*x])^2,x]
```

```
[Out] (9*a^2*c^3*x^3 - 6*a*b*Sqrt[1 - c^2*x^2]*(2 + c^2*x^2) - 2*b^2*c*x*(6 + c^2
*x^2) - 6*b*(-3*a*c^3*x^3 + b*Sqrt[1 - c^2*x^2]*(2 + c^2*x^2))*ArcCos[c*x]
+ 9*b^2*c^3*x^3*ArcCos[c*x]^2)/(27*c^3)
```

Maple [A] time = 0.052, size = 126, normalized size = 1.2

$$\frac{1}{c^3}\left(\frac{a^2c^3x^3}{3}+b^2\left(\frac{(\arccos(cx))^2c^3x^3}{3}-\frac{2\arccos(cx)\left(c^2x^2+2\right)}{9}\sqrt{-c^2x^2+1}-\frac{2c^3x^3}{27}-\frac{4cx}{9}\right)+2ab\left(1/3\,c^3x^3\arccos(cx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccos(c*x))^2,x)
```

```
[Out] 1/c^3*(1/3*a^2*c^3*x^3+b^2*(1/3*arccos(c*x)^2*c^3*x^3-2/9*arccos(c*x)*(c^2*
x^2+2)*(-c^2*x^2+1)^(1/2)-2/27*c^3*x^3-4/9*c*x)+2*a*b*(1/3*c^3*x^3*arccos(c
*x)-1/9*c^2*x^2*(-c^2*x^2+1)^(1/2)-2/9*(-c^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.45125, size = 192, normalized size = 1.88

$$\frac{1}{3}b^2x^3\arccos(cx)^2+\frac{1}{3}a^2x^3+\frac{2}{9}\left(3x^3\arccos(cx)-c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2}+\frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)ab-\frac{2}{27}\left(3c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3*arccos(c*x)^2 + 1/3*a^2*x^3 + 2/9*(3*x^3*arccos(c*x) - c*(sqrt(
-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a*b - 2/27*(3*c*(sqrt(-c
^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arccos(c*x) + (c^2*x^3 + 6*
x)/c^2)*b^2
```

Fricas [A] time = 2.79973, size = 255, normalized size = 2.5

$$\frac{9b^2c^3x^3\arccos(cx)^2+18abc^3x^3\arccos(cx)+\left(9a^2-2b^2\right)c^3x^3-12b^2cx-6\left(abc^2x^2+2ab+\left(b^2c^2x^2+2b^2\right)\arccos(\right.}{27c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/27*(9*b^2*c^3*x^3*arccos(c*x)^2 + 18*a*b*c^3*x^3*arccos(c*x) + (9*a^2 - 2
*b^2)*c^3*x^3 - 12*b^2*c*x - 6*(a*b*c^2*x^2 + 2*a*b + (b^2*c^2*x^2 + 2*b^2)
*arccos(c*x))*sqrt(-c^2*x^2 + 1))/c^3
```

Sympy [A] time = 1.48738, size = 175, normalized size = 1.72

$$\begin{cases} \frac{a^2x^3}{3} + \frac{2abx^3\operatorname{acos}(cx)}{3} - \frac{2abx^2\sqrt{-c^2x^2+1}}{9c} - \frac{4ab\sqrt{-c^2x^2+1}}{9c^3} + \frac{b^2x^3\operatorname{acos}^2(cx)}{3} - \frac{2b^2x^3}{27} - \frac{2b^2x^2\sqrt{-c^2x^2+1}\operatorname{acos}(cx)}{9c} - \frac{4b^2x}{9c^2} - \frac{4b^2\sqrt{-c^2x^2+1}\operatorname{acos}(cx)}{9c^3} \\ \frac{x^3\left(a+\frac{\pi b}{2}\right)^2}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acos(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**3/3 + 2*a*b*x**3*acos(c*x)/3 - 2*a*b*x**2*sqrt(-c**2*x**
2 + 1)/(9*c) - 4*a*b*sqrt(-c**2*x**2 + 1)/(9*c**3) + b**2*x**3*acos(c*x)**2
/3 - 2*b**2*x**3/27 - 2*b**2*x**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(9*c) - 4*
b**2*x/(9*c**2) - 4*b**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(9*c**3), Ne(c, 0))
, (x**3*(a + pi*b/2)**2/3, True))
```

Giac [A] time = 1.16179, size = 193, normalized size = 1.89

$$\frac{1}{3}\,b^2x^3\arccos(cx)^2 + \frac{2}{3}\,abx^3\arccos(cx) + \frac{1}{3}\,a^2x^3 - \frac{2}{27}\,b^2x^3 - \frac{2\,\sqrt{-c^2x^2+1}b^2x^2\arccos(cx)}{9\,c} - \frac{2\,\sqrt{-c^2x^2+1}abx^2}{9\,c} - \frac{4\,b^2}{9\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/3*b^2*x^3*arccos(c*x)^2 + 2/3*a*b*x^3*arccos(c*x) + 1/3*a^2*x^3 - 2/27*b^
2*x^3 - 2/9*sqrt(-c^2*x^2 + 1)*b^2*x^2*arccos(c*x)/c - 2/9*sqrt(-c^2*x^2 +
1)*a*b*x^2/c - 4/9*b^2*x/c^2 - 4/9*sqrt(-c^2*x^2 + 1)*b^2*arccos(c*x)/c^3 -
 4/9*sqrt(-c^2*x^2 + 1)*a*b/c^3
```

3.149 $\int x\left(a+b\cos^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=76

$$-\frac{bx\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}{2c}-\frac{\left(a+b\cos^{-1}(cx)\right)^2}{4c^2}+\frac{1}{2}x^2\left(a+b\cos^{-1}(cx)\right)^2-\frac{1}{4}b^2x^2$$

```
[Out] -(b^2*x^2)/4 - (b*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(2*c) - (a + b*A
rcCos[c*x])^2/(4*c^2) + (x^2*(a + b*ArcCos[c*x])^2)/2
```

Rubi [A] time = 0.117589, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4628, 4708, 4642, 30}

$$-\frac{bx\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}{2c}-\frac{\left(a+b\cos^{-1}(cx)\right)^2}{4c^2}+\frac{1}{2}x^2\left(a+b\cos^{-1}(cx)\right)^2-\frac{1}{4}b^2x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcCos[c*x])^2,x]
```

```
[Out] -(b^2*x^2)/4 - (b*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(2*c) - (a + b*A
rcCos[c*x])^2/(4*c^2) + (x^2*(a + b*ArcCos[c*x])^2)/2
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\cos^{-1}(cx)\right)^2dx &= \frac{1}{2}x^2\left(a+b\cos^{-1}(cx)\right)^2+(bc)\int\frac{x^2\left(a+b\cos^{-1}(cx)\right)}{\sqrt{1-c^2x^2}}dx\\ &= -\frac{bx\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}{2c}+\frac{1}{2}x^2\left(a+b\cos^{-1}(cx)\right)^2-\frac{1}{2}b^2\int x\,dx+\frac{b\int\frac{a+b\cos^{-1}(cx)}{\sqrt{1-c^2x^2}}dx}{2c}\\ &= -\frac{1}{4}b^2x^2-\frac{bx\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}{2c}-\frac{\left(a+b\cos^{-1}(cx)\right)^2}{4c^2}+\frac{1}{2}x^2\left(a+b\cos^{-1}(cx)\right)^2\end{aligned}$$

Mathematica [A] time = 0.111045, size = 104, normalized size = 1.37

$$\frac{cx\left(2a^2cx-2ab\sqrt{1-c^2x^2}-b^2cx\right)+2bcx\cos^{-1}(cx)\left(2acx-b\sqrt{1-c^2x^2}\right)+2ab\sin^{-1}(cx)+b^2\left(2c^2x^2-1\right)\cos^{-1}(cx)^2}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcCos[c*x])^2,x]
```

```
[Out] (c*x*(2*a^2*c*x - b^2*c*x - 2*a*b*Sqrt[1 - c^2*x^2]) + 2*b*c*x*(2*a*c*x - b
*Sqrt[1 - c^2*x^2])*ArcCos[c*x] + b^2*(-1 + 2*c^2*x^2)*ArcCos[c*x]^2 + 2*a*
b*ArcSin[c*x])/(4*c^2)
```

Maple [A] time = 0.051, size = 118, normalized size = 1.6

$$\frac{1}{c^2}\left(\frac{a^2c^2x^2}{2}+b^2\left(\frac{c^2x^2(\arccos(cx))^2}{2}-\frac{\arccos(cx)}{2}\left(cx\sqrt{-c^2x^2+1}+\arccos(cx)\right)+\frac{(\arccos(cx))^2}{4}-\frac{c^2x^2}{4}+\frac{1}{4}\right)+2\,ab\,(1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccos(c*x))^2,x)
```

```
[Out] 1/c^2*(1/2*a^2*c^2*x^2+b^2*(1/2*c^2*x^2*arccos(c*x)^2-1/2*arccos(c*x)*(c*x*
(-c^2*x^2+1)^(1/2)+arccos(c*x))+1/4*arccos(c*x)^2-1/4*c^2*x^2+1/4)+2*a*b*(1
/2*c^2*x^2*arccos(c*x)-1/4*c*x*(-c^2*x^2+1)^(1/2)+1/4*arcsin(c*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^2x^2 + \frac{1}{2}\left(2x^2\arccos(cx) - c\left(\frac{\sqrt{-c^2x^2+1}x}{c^2} - \frac{\arcsin\left(\frac{c^2x}{\sqrt{c^2}}\right)}{\sqrt{c^2}c^2}\right)\right)ab + \frac{1}{2}\left(x^2\arctan\left(\sqrt{cx+1}\sqrt{-cx+1}, cx\right)^2 - 2c\int \frac{\sqrt{c}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*x^2 + 1/2*(2*x^2*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x/c^2 - arcsin
(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^2)))*a*b + 1/2*(x^2*arctan2(sqrt(c*x + 1)*sq
rt(-c*x + 1), c*x)^2 - 2*c*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^2*arcta
n2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(c^2*x^2 - 1), x))*b^2
```

Fricas [A] time = 2.64273, size = 221, normalized size = 2.91

$$\frac{\left(2a^2 - b^2\right)c^2x^2 + \left(2b^2c^2x^2 - b^2\right)\arccos(cx)^2 + 2\left(2abc^2x^2 - ab\right)\arccos(cx) - 2\left(b^2cx\arccos(cx) + abcx\right)\sqrt{-c^2x^2+1}}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2 - b^2)*c^2*x^2 + (2*b^2*c^2*x^2 - b^2)*arccos(c*x)^2 + 2*(2*a*b
*c^2*x^2 - a*b)*arccos(c*x) - 2*(b^2*c*x*arccos(c*x) + a*b*c*x)*sqrt(-c^2*x
^2 + 1))/c^2
```

Sympy [A] time = 0.756254, size = 131, normalized size = 1.72

$$\begin{cases} \frac{a^2x^2}{2} + abx^2 \operatorname{acos}(cx) - \frac{abx\sqrt{-c^2x^2+1}}{2c} - \frac{ab \operatorname{acos}(cx)}{2c^2} + \frac{b^2x^2 \operatorname{acos}^2(cx)}{2} - \frac{b^2x^2}{4} - \frac{b^2x\sqrt{-c^2x^2+1} \operatorname{acos}(cx)}{2c} - \frac{b^2 \operatorname{acos}^2(cx)}{4c^2} & \text{for } c \neq 0 \\ \frac{x^2\left(a+\frac{\pi b}{2}\right)^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acos(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**2/2 + a*b*x**2*acos(c*x) - a*b*x*sqrt(-c**2*x**2 + 1)/(2
*c) - a*b*acos(c*x)/(2*c**2) + b**2*x**2*acos(c*x)**2/2 - b**2*x**2/4 - b**
2*x*sqrt(-c**2*x**2 + 1)*acos(c*x)/(2*c) - b**2*acos(c*x)**2/(4*c**2), Ne(c
, 0)), (x**2*(a + pi*b/2)**2/2, True))
```

Giac [A] time = 1.17495, size = 161, normalized size = 2.12

$$\frac{1}{2}\,b^2x^2 \arccos(cx)^2 + abx^2 \arccos(cx) + \frac{1}{2}\,a^2x^2 - \frac{1}{4}\,b^2x^2 - \frac{\sqrt{-c^2x^2+1}b^2x \arccos(cx)}{2\,c} - \frac{\sqrt{-c^2x^2+1}abx}{2\,c} - \frac{b^2 \arccos(cx}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2*arccos(c*x)^2 + a*b*x^2*arccos(c*x) + 1/2*a^2*x^2 - 1/4*b^2*x^2
 - 1/2*sqrt(-c^2*x^2 + 1)*b^2*x*arccos(c*x)/c - 1/2*sqrt(-c^2*x^2 + 1)*a*b*
x/c - 1/4*b^2*arccos(c*x)^2/c^2 - 1/2*a*b*arccos(c*x)/c^2 + 1/8*b^2/c^2
```

3.150 $\int \left(a+b\cos^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=47

$$-\frac{2b\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}{c}+x\left(a+b\cos^{-1}(cx)\right)^2-2b^2x$$

```
[Out] -2*b^2*x - (2*b*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/c + x*(a + b*ArcCos[
c*x])^2
```

Rubi [A] time = 0.0578785, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4620, 4678, 8}

$$-\frac{2b\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}{c}+x\left(a+b\cos^{-1}(cx)\right)^2-2b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])^2,x]
```

```
[Out] -2*b^2*x - (2*b*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/c + x*(a + b*ArcCos[
c*x])^2
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(a+b\cos^{-1}(cx)\right)^2\,dx = x\left(a+b\cos^{-1}(cx)\right)^2+(2bc)\int \frac{x\left(a+b\cos^{-1}(cx)\right)}{\sqrt{1-c^2x^2}}\,dx$$
$$= -\frac{2b\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}{c}+x\left(a+b\cos^{-1}(cx)\right)^2-\left(2b^2\right)\int 1\,dx$$
$$= -2b^2x-\frac{2b\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}{c}+x\left(a+b\cos^{-1}(cx)\right)^2$$

Mathematica [A] time = 0.0615706, size = 76, normalized size = 1.62

$$x\left(a^2-2b^2\right)-\frac{2ab\sqrt{1-c^2x^2}}{c}+\frac{2b\cos^{-1}(cx)\left(acx-b\sqrt{1-c^2x^2}\right)}{c}+b^2x\cos^{-1}(cx)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[c*x])^2,x]
```

```
[Out] (a^2 - 2*b^2)*x - (2*a*b*Sqrt[1 - c^2*x^2])/c + (2*b*(a*c*x - b*Sqrt[1 - c^
2*x^2])*ArcCos[c*x])/c + b^2*x*ArcCos[c*x]^2
```

Maple [A] time = 0.046, size = 74, normalized size = 1.6

$$\frac{1}{c}\left(cxa^2+b^2\left(cx\left(\arccos(cx)\right)^2-2\,cx-2\,\arccos(cx)\sqrt{-c^2x^2+1}\right)+2\,ab\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^2,x)
```

```
[Out] 1/c*(c*x*a^2+b^2*(c*x*arccos(c*x)^2-2*c*x-2*arccos(c*x)*(-c^2*x^2+1)^(1/2))
+2*a*b*(c*x*arccos(c*x)-(-c^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.43825, size = 99, normalized size = 2.11

$$b^2x\arccos(cx)^2 - 2\,b^2\left(x + \frac{\sqrt{-c^2x^2+1}\arccos(cx)}{c}\right) + a^2x + \frac{2\left(cx\arccos(cx) - \sqrt{-c^2x^2+1}\right)ab}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2,x, algorithm="maxima")
```

```
[Out] b^2*x*arccos(c*x)^2 - 2*b^2*(x + sqrt(-c^2*x^2 + 1)*arccos(c*x)/c) + a^2*x
+ 2*(c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*a*b/c
```

Fricas [A] time = 2.41873, size = 159, normalized size = 3.38

$$\frac{b^2cx\arccos(cx)^2 + 2\,abcx\arccos(cx) + \left(a^2 - 2\,b^2\right)cx - 2\,\sqrt{-c^2x^2+1}\left(b^2\arccos(cx) + ab\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2,x, algorithm="fricas")
```

```
[Out] (b^2*c*x*arccos(c*x)^2 + 2*a*b*c*x*arccos(c*x) + (a^2 - 2*b^2)*c*x - 2*sqrt
(-c^2*x^2 + 1)*(b^2*arccos(c*x) + a*b))/c
```

Sympy [A] time = 0.331679, size = 87, normalized size = 1.85

$$\begin{cases} a^2x + 2abx\operatorname{acos}(cx) - \frac{2ab\sqrt{-c^2x^2+1}}{c} + b^2x\operatorname{acos}^2(cx) - 2b^2x - \frac{2b^2\sqrt{-c^2x^2+1}\operatorname{acos}(cx)}{c} & \text{for}\ c \neq 0 \\ x\left(a + \frac{\pi b}{2}\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*x*acos(c*x) - 2*a*b*sqrt(-c**2*x**2 + 1)/c + b**2
*x*acos(c*x)**2 - 2*b**2*x - 2*b**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/c, Ne(c,
 0)), (x*(a + pi*b/2)**2, True))
```

Giac [A] time = 1.14457, size = 101, normalized size = 2.15

$$b^2x\arccos(cx)^2+2\,abx\arccos(cx)+a^2x-2\,b^2x-\frac{2\sqrt{-c^2x^2+1}b^2\arccos(cx)}{c}-\frac{2\sqrt{-c^2x^2+1}ab}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2,x, algorithm="giac")
```

```
[Out] b^2*x*arccos(c*x)^2 + 2*a*b*x*arccos(c*x) + a^2*x - 2*b^2*x - 2*sqrt(-c^2*x
^2 + 1)*b^2*arccos(c*x)/c - 2*sqrt(-c^2*x^2 + 1)*a*b/c
```

3.151 $\int \frac{\left(a+b\cos^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=92

$$-ib\text{PolyLog}\left(2,-e^{2i\cos^{-1}(cx)}\right)\left(a+b\cos^{-1}(cx)\right)+\frac{1}{2}b^2\text{PolyLog}\left(3,-e^{2i\cos^{-1}(cx)}\right)-\frac{i\left(a+b\cos^{-1}(cx)\right)^3}{3b}+\log\left(1+e^{2i\cos^{-1}\right.$$

```
[Out] ((-I/3)*(a + b*ArcCos[c*x])^3)/b + (a + b*ArcCos[c*x])^2*Log[1 + E^((2*I)*A
rcCos[c*x])] - I*b*(a + b*ArcCos[c*x])*PolyLog[2, -E^((2*I)*ArcCos[c*x])] +
 (b^2*PolyLog[3, -E^((2*I)*ArcCos[c*x])])/2
```

Rubi [A] time = 0.118095, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4626, 3719, 2190, 2531, 2282, 6589}

$$-ib\text{PolyLog}\left(2,-e^{2i\cos^{-1}(cx)}\right)\left(a+b\cos^{-1}(cx)\right)+\frac{1}{2}b^2\text{PolyLog}\left(3,-e^{2i\cos^{-1}(cx)}\right)-\frac{i\left(a+b\cos^{-1}(cx)\right)^3}{3b}+\log\left(1+e^{2i\cos^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])^2/x,x]
```

```
[Out] ((-I/3)*(a + b*ArcCos[c*x])^3)/b + (a + b*ArcCos[c*x])^2*Log[1 + E^((2*I)*A
rcCos[c*x])] - I*b*(a + b*ArcCos[c*x])*PolyLog[2, -E^((2*I)*ArcCos[c*x])] +
 (b^2*PolyLog[3, -E^((2*I)*ArcCos[c*x])])/2
```

Rule 4626

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n/Cot[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^2}{x}\,dx = -\text{Subst}\left(\int (a+bx)^2\tan(x)\,dx, x, \cos^{-1}(cx)\right)$$
$$= -\frac{i\left(a+b\cos^{-1}(cx)\right)^3}{3b} + 2i\,\text{Subst}\left(\int \frac{e^{2ix}(a+bx)^2}{1+e^{2ix}}\,dx, x, \cos^{-1}(cx)\right)$$
$$= -\frac{i\left(a+b\cos^{-1}(cx)\right)^3}{3b} + \left(a+b\cos^{-1}(cx)\right)^2\log\left(1+e^{2i\cos^{-1}(cx)}\right) - (2b)\,\text{Subst}\left(\int (a+bx)\log\right.$$
$$= -\frac{i\left(a+b\cos^{-1}(cx)\right)^3}{3b} + \left(a+b\cos^{-1}(cx)\right)^2\log\left(1+e^{2i\cos^{-1}(cx)}\right) - ib\left(a+b\cos^{-1}(cx)\right)\text{Li}_2\left(-e\right.$$
$$= -\frac{i\left(a+b\cos^{-1}(cx)\right)^3}{3b} + \left(a+b\cos^{-1}(cx)\right)^2\log\left(1+e^{2i\cos^{-1}(cx)}\right) - ib\left(a+b\cos^{-1}(cx)\right)\text{Li}_2\left(-e\right.$$
$$= -\frac{i\left(a+b\cos^{-1}(cx)\right)^3}{3b} + \left(a+b\cos^{-1}(cx)\right)^2\log\left(1+e^{2i\cos^{-1}(cx)}\right) - ib\left(a+b\cos^{-1}(cx)\right)\text{Li}_2\left(-e\right.$$

Mathematica [A] time = 0.116828, size = 128, normalized size = 1.39

$$-ib\text{PolyLog}\left(2,-e^{2i\cos^{-1}(cx)}\right)\left(a+b\cos^{-1}(cx)\right)+\frac{1}{2}b^2\text{PolyLog}\left(3,-e^{2i\cos^{-1}(cx)}\right)+a^2\log(cx)-iab\cos^{-1}(cx)^2+2ab\cos^{-}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[c*x])^2/x,x]
```

```
[Out] (-I)*a*b*ArcCos[c*x]^2 - (I/3)*b^2*ArcCos[c*x]^3 + 2*a*b*ArcCos[c*x]*Log[1
+ E^((2*I)*ArcCos[c*x])] + b^2*ArcCos[c*x]^2*Log[1 + E^((2*I)*ArcCos[c*x])]
 + a^2*Log[c*x] - I*b*(a + b*ArcCos[c*x])*PolyLog[2, -E^((2*I)*ArcCos[c*x])
] + (b^2*PolyLog[3, -E^((2*I)*ArcCos[c*x])])/2
```

Maple [A] time = 0.058, size = 194, normalized size = 2.1

$$a^2\ln(cx)-\frac{i}{3}b^2(\arccos(cx))^3+b^2(\arccos(cx))^2\ln\left(1+\left(cx+i\sqrt{-c^2x^2+1}\right)^2\right)-ib^2\arccos(cx)\,\text{polylog}\left(2,-\left(cx+i\sqrt{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^2/x,x)
```

```
[Out] a^2*ln(c*x)-1/3*I*b^2*arccos(c*x)^3+b^2*arccos(c*x)^2*ln(1+(c*x+I*(-c^2*x^2
+1)^(1/2))^2)-I*b^2*arccos(c*x)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+1/
2*b^2*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)-I*a*b*arccos(c*x)^2+2*a*b*ar
ccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-I*a*b*polylog(2,-(c*x+I*(-c^2*
x^2+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2 \log(x) + \int \frac{b^2 \arctan\left(\sqrt{cx+1}\sqrt{-cx+1}, cx\right)^2 + 2\,ab\arctan\left(\sqrt{cx+1}\sqrt{-cx+1}, cx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + integrate((b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 +
2*a*b*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \arccos(cx)^2 + 2\,ab\arccos(cx) + a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{acos}(cx))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**2/x,x)
```

```
[Out] Integral((a + b*acos(c*x))**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arccos(cx) + a)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)^2/x, x)
```

3.152 $\int \frac{\left(a+b\cos^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=89

$$2ib^2c\text{PolyLog}\left(2,-ie^{i\cos^{-1}(cx)}\right)-2ib^2c\text{PolyLog}\left(2,ie^{i\cos^{-1}(cx)}\right)-\frac{\left(a+b\cos^{-1}(cx)\right)^2}{x}-4ibc\tan^{-1}\left(e^{i\cos^{-1}(cx)}\right)\left(a+b\cos^{-1}(\right.$$

```
[Out] -((a + b*ArcCos[c*x])^2/x) - (4*I)*b*c*(a + b*ArcCos[c*x])*ArcTan[E^(I*ArcC
os[c*x])] + (2*I)*b^2*c*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - (2*I)*b^2*c*Po
lyLog[2, I*E^(I*ArcCos[c*x])]
```

Rubi [A] time = 0.127538, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4628, 4710, 4181, 2279, 2391}

$$2ib^2c\text{PolyLog}\left(2,-ie^{i\cos^{-1}(cx)}\right)-2ib^2c\text{PolyLog}\left(2,ie^{i\cos^{-1}(cx)}\right)-\frac{\left(a+b\cos^{-1}(cx)\right)^2}{x}-4ibc\tan^{-1}\left(e^{i\cos^{-1}(cx)}\right)\left(a+b\cos^{-1}(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])^2/x^2,x]
```

```
[Out] -((a + b*ArcCos[c*x])^2/x) - (4*I)*b*c*(a + b*ArcCos[c*x])*ArcTan[E^(I*ArcC
os[c*x])] + (2*I)*b^2*c*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - (2*I)*b^2*c*Po
lyLog[2, I*E^(I*ArcCos[c*x])]
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4710

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> -Dist[(c^(m + 1)*Sqrt[d])^(-1), Subst[Int[(a + b*x)^n
*Cos[x]^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\cos^{-1}(cx)\right)^2}{x^2}\,dx &= -\frac{\left(a+b\cos^{-1}(cx)\right)^2}{x} - (2bc)\int \frac{a+b\cos^{-1}(cx)}{x\sqrt{1-c^2x^2}}\,dx \\
&= -\frac{\left(a+b\cos^{-1}(cx)\right)^2}{x} + (2bc)\,\mathrm{Subst}\left(\int (a+bx)\sec(x)\,dx, x, \cos^{-1}(cx)\right) \\
&= -\frac{\left(a+b\cos^{-1}(cx)\right)^2}{x} - 4ibc\left(a+b\cos^{-1}(cx)\right)\tan^{-1}\left(e^{i\cos^{-1}(cx)}\right) - \left(2b^2c\right)\mathrm{Subst}\left(\int \log\left(1-i\right.\right. \\
&= -\frac{\left(a+b\cos^{-1}(cx)\right)^2}{x} - 4ibc\left(a+b\cos^{-1}(cx)\right)\tan^{-1}\left(e^{i\cos^{-1}(cx)}\right) + \left(2ib^2c\right)\mathrm{Subst}\left(\int \frac{\log(1-}{x}\right. \\
&= -\frac{\left(a+b\cos^{-1}(cx)\right)^2}{x} - 4ibc\left(a+b\cos^{-1}(cx)\right)\tan^{-1}\left(e^{i\cos^{-1}(cx)}\right) + 2ib^2c\,\mathrm{Li}_2\left(-ie^{i\cos^{-1}(cx)}\right) - 2i
\end{aligned}$$

Mathematica [A] time = 0.207137, size = 134, normalized size = 1.51

$$-\frac{b^2\left(\cos^{-1}(cx)^2 - 2cx\left(i\left(\mathrm{PolyLog}\left(2,-ie^{i\cos^{-1}(cx)}\right) - \mathrm{PolyLog}\left(2,ie^{i\cos^{-1}(cx)}\right)\right) + \cos^{-1}(cx)\left(\log\left(1-ie^{i\cos^{-1}(cx)}\right) - \log\left(\right.\right.\right.\right.}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[c*x])^2/x^2,x]
```

```
[Out] -((a^2 + 2*a*b*(ArcCos[c*x] - c*x*ArcTanh[Sqrt[1 - c^2*x^2]]) + b^2*(ArcCos
[c*x]^2 - 2*c*x*(ArcCos[c*x]*(Log[1 - I*E^(I*ArcCos[c*x])] - Log[1 + I*E^(I
*ArcCos[c*x])]) + I*(PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - PolyLog[2, I*E^(I
*ArcCos[c*x])]))))/x)
```

Maple [A] time = 0.053, size = 187, normalized size = 2.1

$$-\frac{a^2}{x} - \frac{b^2 (\arccos(cx))^2}{x} - 2\,cb^2 \arccos(cx) \ln\left(1 + i\left(cx + i\sqrt{-c^2x^2+1}\right)\right) + 2\,cb^2 \arccos(cx) \ln\left(1 - i\left(cx + i\sqrt{-c^2x^2+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^2/x^2,x)
```

```
[Out] -a^2/x-b^2/x*arccos(c*x)^2-2*c*b^2*arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(
1/2)))+2*c*b^2*arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+2*I*c*b^2*dil
og(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-2*I*c*b^2*dilog(1-I*(c*x+I*(-c^2*x^2+1)^
(1/2)))-2*a*b/x*arccos(c*x)+2*c*a*b*arctanh(1/(-c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2\left(c\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|} + \frac{2}{|x|}\right) - \frac{\arccos(cx)}{x}\right)ab + \frac{\left(2\,cx\int \frac{\sqrt{-cx+1}\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)}{\sqrt{cx+1}(cx-1)x}\,dx - \arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 2*(c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) - arccos(c*x)/x)*a*b + (2*
c*x*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x
+ 1), c*x)/(c^2*x^3 - x), x) - arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2
)*b^2/x - a^2/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \arccos(cx)^2 + 2\,ab \arccos(cx) + a^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\text{acos}(cx))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**2/x**2,x)
```

```
[Out] Integral((a + b*acos(c*x))**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arccos(cx) + a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)^2/x^2, x)
```

3.153 $\int x^2 \left(a+b\cos^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=178

$$-\frac{4ab^2x}{3c^2}-\frac{2}{9}b^2x^3\left(a+b\cos^{-1}(cx)\right)-\frac{bx^2\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{3c}-\frac{2b\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{3c^3}+\frac{1}{3}x^3\left(a+b\cos\right.$$

```
[Out] (-4*a*b^2*x)/(3*c^2) + (14*b^3*Sqrt[1 - c^2*x^2])/(9*c^3) - (2*b^3*(1 - c^2
*x^2)^(3/2))/(27*c^3) - (4*b^3*x*ArcCos[c*x])/(3*c^2) - (2*b^2*x^3*(a + b*A
rcCos[c*x]))/9 - (2*b*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/(3*c^3) - (b
*x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/(3*c) + (x^3*(a + b*ArcCos[c*
x])^3)/3
```

Rubi [A] time = 0.295393, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4628, 4708, 4678, 4620, 261, 266, 43}

$$-\frac{4ab^2x}{3c^2}-\frac{2}{9}b^2x^3\left(a+b\cos^{-1}(cx)\right)-\frac{bx^2\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{3c}-\frac{2b\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{3c^3}+\frac{1}{3}x^3\left(a+b\cos\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcCos[c*x])^3,x]
```


```
[Out] (-4*a*b^2*x)/(3*c^2) + (14*b^3*Sqrt[1 - c^2*x^2])/(9*c^3) - (2*b^3*(1 - c^2
*x^2)^(3/2))/(27*c^3) - (4*b^3*x*ArcCos[c*x])/(3*c^2) - (2*b^2*x^3*(a + b*A
rcCos[c*x]))/9 - (2*b*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/(3*c^3) - (b
*x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/(3*c) + (x^3*(a + b*ArcCos[c*
x])^3)/3
```


Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
```

```
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2 \left(a+b\cos^{-1}(cx)\right)^3 dx = \frac{1}{3}x^3\left(a+b\cos^{-1}(cx)\right)^3 + (bc)\int \frac{x^3\left(a+b\cos^{-1}(cx)\right)^2}{\sqrt{1-c^2x^2}}dx$$

$$= -\frac{bx^2\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{3c} + \frac{1}{3}x^3\left(a+b\cos^{-1}(cx)\right)^3 - \frac{1}{3}\left(2b^2\right)\int x^2\left(a+b\cos^{-1}(cx\right.$$

$$= -\frac{2}{9}b^2x^3\left(a+b\cos^{-1}(cx)\right) - \frac{2b\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{3c^3} - \frac{bx^2\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx\right.}{3c}$$

$$= -\frac{4ab^2x}{3c^2} - \frac{2}{9}b^2x^3\left(a+b\cos^{-1}(cx)\right) - \frac{2b\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{3c^3} - \frac{bx^2\sqrt{1-c^2x^2}\left(a+b\right.}{3c}$$

$$= -\frac{4ab^2x}{3c^2} - \frac{4b^3x\cos^{-1}(cx)}{3c^2} - \frac{2}{9}b^2x^3\left(a+b\cos^{-1}(cx)\right) - \frac{2b\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{3c^3} - \frac{bx}{}$$

$$= -\frac{4ab^2x}{3c^2} + \frac{14b^3\sqrt{1-c^2x^2}}{9c^3} - \frac{2b^3\left(1-c^2x^2\right)^{3/2}}{27c^3} - \frac{4b^3x\cos^{-1}(cx)}{3c^2} - \frac{2}{9}b^2x^3\left(a+b\cos^{-1}(cx)\right) -$$

Mathematica [A] time = 0.198356, size = 218, normalized size = 1.22

$$\frac{-3b\cos^{-1}(cx)\left(-9a^2c^3x^3+6ab\sqrt{1-c^2x^2}\left(c^2x^2+2\right)+2b^2cx\left(c^2x^2+6\right)\right)-9a^2b\sqrt{1-c^2x^2}\left(c^2x^2+2\right)+9a^3c^3x^3-6ab^2cx\left(\right.}{27c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcCos[c*x])^3,x]
```

```
[Out] (9*a^3*c^3*x^3 - 9*a^2*b*Sqrt[1 - c^2*x^2]*(2 + c^2*x^2) - 6*a*b^2*c*x*(6 +
 c^2*x^2) + 2*b^3*Sqrt[1 - c^2*x^2]*(20 + c^2*x^2) - 3*b*(-9*a^2*c^3*x^3 +
6*a*b*Sqrt[1 - c^2*x^2]*(2 + c^2*x^2) + 2*b^2*c*x*(6 + c^2*x^2))*ArcCos[c*x
] - 9*b^2*(-3*a*c^3*x^3 + b*Sqrt[1 - c^2*x^2]*(2 + c^2*x^2))*ArcCos[c*x]^2
+ 9*b^3*c^3*x^3*ArcCos[c*x]^3)/(27*c^3)
```

Maple [A] time = 0.05, size = 235, normalized size = 1.3

$$\frac{1}{c^3}\left(\frac{c^3x^3a^3}{3}+b^3\left(\frac{c^3x^3\left(\arccos(cx)\right)^3}{3}-\frac{\left(\arccos(cx)\right)^2\left(c^2x^2+2\right)}{3}\sqrt{-c^2x^2+1}+\frac{4}{3}\sqrt{-c^2x^2+1}-\frac{4cx\arccos(cx)}{3}-\frac{2c^3x^3}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccos(c*x))^3,x)
```

```
[Out] 1/c^3*(1/3*c^3*x^3*a^3+b^3*(1/3*c^3*x^3*arccos(c*x)^3-1/3*arccos(c*x)^2*(c^
2*x^2+2)*(-c^2*x^2+1)^(1/2)+4/3*(-c^2*x^2+1)^(1/2)-4/3*c*x*arccos(c*x)-2/9*
c^3*x^3*arccos(c*x)+2/27*(c^2*x^2+2)*(-c^2*x^2+1)^(1/2))+3*a*b^2*(1/3*arcco
s(c*x)^2*c^3*x^3-2/9*arccos(c*x)*(c^2*x^2+2)*(-c^2*x^2+1)^(1/2)-2/27*c^3*x^
3-4/9*c*x)+3*a^2*b*(1/3*c^3*x^3*arccos(c*x)-1/9*c^2*x^2*(-c^2*x^2+1)^(1/2)-
2/9*(-c^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.4924, size = 369, normalized size = 2.07

$$\frac{1}{3}b^3x^3\arccos(cx)^3+ab^2x^3\arccos(cx)^2+\frac{1}{3}a^3x^3+\frac{1}{3}\left(3x^3\arccos(cx)-c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2}+\frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)a^2b-\frac{2}{9}\Big(3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccos(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3*arccos(c*x)^3 + a*b^2*x^3*arccos(c*x)^2 + 1/3*a^3*x^3 + 1/3*(3*
x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)
)*a^2*b - 2/9*(3*c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*
arccos(c*x) + (c^2*x^3 + 6*x)/c^2)*a*b^2 - 1/27*(9*c*(sqrt(-c^2*x^2 + 1)*x^
2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arccos(c*x)^2 - 2*c*((sqrt(-c^2*x^2 + 1)*
x^2 + 20*sqrt(-c^2*x^2 + 1)/c^2)/c^2 - 3*(c^2*x^3 + 6*x)*arccos(c*x)/c^3))*
b^3
```

Fricas [A] time = 2.42432, size = 441, normalized size = 2.48

$$\frac{9b^3c^3x^3\arccos(cx)^3+27ab^2c^3x^3\arccos(cx)^2+3\left(3a^3-2ab^2\right)c^3x^3-36ab^2cx+3\left(\left(9a^2b-2b^3\right)c^3x^3-12b^3cx\right)\mathrm{arcc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccos(c*x))^3,x, algorithm="fricas")
```

```
[Out] 1/27*(9*b^3*c^3*x^3*arccos(c*x)^3 + 27*a*b^2*c^3*x^3*arccos(c*x)^2 + 3*(3*a
^3 - 2*a*b^2)*c^3*x^3 - 36*a*b^2*c*x + 3*((9*a^2*b - 2*b^3)*c^3*x^3 - 12*b^
3*c*x)*arccos(c*x) - ((9*a^2*b - 2*b^3)*c^2*x^2 + 18*a^2*b - 40*b^3 + 9*(b^
```

```
3*c^2*x^2 + 2*b^3)*arccos(c*x)^2 + 18*(a*b^2*c^2*x^2 + 2*a*b^2)*arccos(c*x)
)*sqrt(-c^2*x^2 + 1))/c^3
```

Sympy [A] time = 3.29767, size = 333, normalized size = 1.87

$$\begin{cases} \frac{a^3x^3}{3} + a^2bx^3\operatorname{acos}(cx) - \frac{a^2bx^2\sqrt{-c^2x^2+1}}{3c} - \frac{2a^2b\sqrt{-c^2x^2+1}}{3c^3} + ab^2x^3\operatorname{acos}^2(cx) - \frac{2ab^2x^3}{9} - \frac{2ab^2x^2\sqrt{-c^2x^2+1}\operatorname{acos}(cx)}{3c} - \frac{4ab^2x}{3c^2} - \frac{4ab^2\sqrt{-c} \\ \frac{x^3\left(a+\frac{\pi b}{2}\right)^3}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acos(c*x))**3,x)
```

```
[Out] Piecewise((a**3*x**3/3 + a**2*b*x**3*acos(c*x) - a**2*b*x**2*sqrt(-c**2*x**
2 + 1)/(3*c) - 2*a**2*b*sqrt(-c**2*x**2 + 1)/(3*c**3) + a*b**2*x**3*acos(c*
x)**2 - 2*a*b**2*x**3/9 - 2*a*b**2*x**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(3*c
) - 4*a*b**2*x/(3*c**2) - 4*a*b**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(3*c**3)
+ b**3*x**3*acos(c*x)**3/3 - 2*b**3*x**3*acos(c*x)/9 - b**3*x**2*sqrt(-c**2
*x**2 + 1)*acos(c*x)**2/(3*c) + 2*b**3*x**2*sqrt(-c**2*x**2 + 1)/(27*c) - 4
*b**3*x*acos(c*x)/(3*c**2) - 2*b**3*sqrt(-c**2*x**2 + 1)*acos(c*x)**2/(3*c*
*3) + 40*b**3*sqrt(-c**2*x**2 + 1)/(27*c**3), Ne(c, 0)), (x**3*(a + pi*b/2)
**3/3, True))
```

Giac [A] time = 1.213, size = 390, normalized size = 2.19

$$\frac{1}{3}b^3x^3\arccos(cx)^3 + ab^2x^3\arccos(cx)^2 + a^2bx^3\arccos(cx) - \frac{2}{9}b^3x^3\arccos(cx) - \frac{\sqrt{-c^2x^2+1}b^3x^2\arccos(cx)^2}{3c} + \frac{1}{3}a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccos(c*x))^3,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3*arccos(c*x)^3 + a*b^2*x^3*arccos(c*x)^2 + a^2*b*x^3*arccos(c*x)
 - 2/9*b^3*x^3*arccos(c*x) - 1/3*sqrt(-c^2*x^2 + 1)*b^3*x^2*arccos(c*x)^2/c
 + 1/3*a^3*x^3 - 2/9*a*b^2*x^3 - 2/3*sqrt(-c^2*x^2 + 1)*a*b^2*x^2*arccos(c*
x)/c - 1/3*sqrt(-c^2*x^2 + 1)*a^2*b*x^2/c + 2/27*sqrt(-c^2*x^2 + 1)*b^3*x^2
/c - 4/3*b^3*x*arccos(c*x)/c^2 - 2/3*sqrt(-c^2*x^2 + 1)*b^3*arccos(c*x)^2/c
^3 - 4/3*a*b^2*x/c^2 - 4/3*sqrt(-c^2*x^2 + 1)*a*b^2*arccos(c*x)/c^3 - 2/3*s
qrt(-c^2*x^2 + 1)*a^2*b/c^3 + 40/27*sqrt(-c^2*x^2 + 1)*b^3/c^3
```

3.154 $\int x \left(a + b \cos^{-1}(cx)\right)^3 dx$

Optimal. Leaf size=125

$$-\frac{3}{4}b^2x^2\left(a + b\cos^{-1}(cx)\right) - \frac{3bx\sqrt{1-c^2x^2}\left(a + b\cos^{-1}(cx)\right)^2}{4c} - \frac{\left(a + b\cos^{-1}(cx)\right)^3}{4c^2} + \frac{1}{2}x^2\left(a + b\cos^{-1}(cx)\right)^3 + \frac{3b^3x\sqrt{1-}}{8c}$$

```
[Out] (3*b^3*x*Sqrt[1 - c^2*x^2])/(8*c) - (3*b^2*x^2*(a + b*ArcCos[c*x]))/4 - (3*
b*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/(4*c) - (a + b*ArcCos[c*x])^3/
(4*c^2) + (x^2*(a + b*ArcCos[c*x])^3)/2 - (3*b^3*ArcSin[c*x])/(8*c^2)
```

Rubi [A] time = 0.207512, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4628, 4708, 4642, 321, 216}

$$-\frac{3}{4}b^2x^2\left(a + b\cos^{-1}(cx)\right) - \frac{3bx\sqrt{1-c^2x^2}\left(a + b\cos^{-1}(cx)\right)^2}{4c} - \frac{\left(a + b\cos^{-1}(cx)\right)^3}{4c^2} + \frac{1}{2}x^2\left(a + b\cos^{-1}(cx)\right)^3 + \frac{3b^3x\sqrt{1-}}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcCos[c*x])^3,x]
```

```
[Out] (3*b^3*x*Sqrt[1 - c^2*x^2])/(8*c) - (3*b^2*x^2*(a + b*ArcCos[c*x]))/4 - (3*
b*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/(4*c) - (a + b*ArcCos[c*x])^3/
(4*c^2) + (x^2*(a + b*ArcCos[c*x])^3)/2 - (3*b^3*ArcSin[c*x])/(8*c^2)
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

```
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int x\left(a+b\cos^{-1}(cx)\right)^3\,dx=\frac{1}{2}x^2\left(a+b\cos^{-1}(cx)\right)^3+\frac{1}{2}(3bc)\int\frac{x^2\left(a+b\cos^{-1}(cx)\right)^2}{\sqrt{1-c^2x^2}}\,dx$$

$$=-\frac{3bx\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{4c}+\frac{1}{2}x^2\left(a+b\cos^{-1}(cx)\right)^3-\frac{1}{2}\left(3b^2\right)\int x\left(a+b\cos^{-1}(cx)\right)$$

$$=-\frac{3}{4}b^2x^2\left(a+b\cos^{-1}(cx)\right)-\frac{3bx\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{4c}-\frac{\left(a+b\cos^{-1}(cx)\right)^3}{4c^2}+\frac{1}{2}x^2\left(a\right.$$

$$=\frac{3b^3x\sqrt{1-c^2x^2}}{8c}-\frac{3}{4}b^2x^2\left(a+b\cos^{-1}(cx)\right)-\frac{3bx\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{4c}-\frac{\left(a+b\cos^{-1}\right.}{4c^2}$$

$$=\frac{3b^3x\sqrt{1-c^2x^2}}{8c}-\frac{3}{4}b^2x^2\left(a+b\cos^{-1}(cx)\right)-\frac{3bx\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{4c}-\frac{\left(a+b\cos^{-1}\right.}{4c^2}$$

Mathematica [A] time = 0.167832, size = 185, normalized size = 1.48

$$\frac{cx\left(-6a^2b\sqrt{1-c^2x^2}+4a^3cx-6ab^2cx+3b^3\sqrt{1-c^2x^2}\right)-6bcx\cos^{-1}(cx)\left(-2a^2cx+2ab\sqrt{1-c^2x^2}+b^2cx\right)+\left(6a^2b-3b^3\right)}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcCos[c*x])^3,x]
```

```
[Out] (c*x*(4*a^3*c*x - 6*a*b^2*c*x - 6*a^2*b*Sqrt[1 - c^2*x^2] + 3*b^3*Sqrt[1 -
c^2*x^2]) - 6*b*c*x*(-2*a^2*c*x + b^2*c*x + 2*a*b*Sqrt[1 - c^2*x^2])*ArcCos
[c*x] - 6*b^2*(a - 2*a*c^2*x^2 + b*c*x*Sqrt[1 - c^2*x^2])*ArcCos[c*x]^2 + 2
*b^3*(-1 + 2*c^2*x^2)*ArcCos[c*x]^3 + (6*a^2*b - 3*b^3)*ArcSin[c*x])/(8*c^2
)
```

Maple [A] time = 0.052, size = 211, normalized size = 1.7

$$\frac{1}{c^2}\left(\frac{c^2x^2a^3}{2}+b^3\left(\frac{c^2x^2(\arccos(cx))^3}{2}-\frac{3(\arccos(cx))^2}{4}\left(cx\sqrt{-c^2x^2+1}+\arccos(cx)\right)-\frac{3c^2x^2\arccos(cx)}{4}+\frac{3cx}{8}\sqrt{-c^2 \cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccos(c*x))^3,x)
```

```
[Out] 1/c^2*(1/2*c^2*x^2*a^3+b^3*(1/2*c^2*x^2*arccos(c*x)^3-3/4*arccos(c*x)^2*(c*
x*(-c^2*x^2+1)^(1/2)+arccos(c*x))-3/4*c^2*x^2*arccos(c*x)+3/8*c*x*(-c^2*x^2
+1)^(1/2)+3/8*arccos(c*x)+1/2*arccos(c*x)^3)+3*a*b^2*(1/2*c^2*x^2*arccos(c*
x)^2-1/2*arccos(c*x)*(c*x*(-c^2*x^2+1)^(1/2)+arccos(c*x))+1/4*arccos(c*x)^2
-1/4*c^2*x^2+1/4)+3*a^2*b*(1/2*c^2*x^2*arccos(c*x)-1/4*c*x*(-c^2*x^2+1)^(1/
2)+1/4*arcsin(c*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}b^3x^2\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)^3+\frac{1}{2}a^3x^2+\frac{3}{4}\left(2x^2\arccos(cx)-c\left(\frac{\sqrt{-c^2x^2+1}x}{c^2}-\frac{\arcsin\left(\frac{c^2x}{\sqrt{c^2}}\right)}{\sqrt{c^2}c^2}\right)\right)a^2b-\int\frac{3(\sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccos(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^3 + 1/2*a^3*x^2 + 3/
4*(2*x^2*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x/c^2 - arcsin(c^2*x/sqrt(c^2)
)/(sqrt(c^2)*c^2)))*a^2*b - integrate(3/2*(sqrt(c*x + 1)*sqrt(-c*x + 1)*b^3
```

```
*c*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 - 2*(a*b^2*c^2*x^3 - a*
b^2*x)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2)/(c^2*x^2 - 1), x)
```

Fricas [A] time = 2.29324, size = 378, normalized size = 3.02

$$\frac{2\left(2a^3-3ab^2\right)c^2x^2+2\left(2b^3c^2x^2-b^3\right)\arccos\left(cx\right)^3+6\left(2ab^2c^2x^2-ab^2\right)\arccos\left(cx\right)^2+3\left(2\left(2a^2b-b^3\right)c^2x^2-2a^2b+b\right.}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccos(c*x))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^3 - 3*a*b^2)*c^2*x^2 + 2*(2*b^3*c^2*x^2 - b^3)*arccos(c*x)^3 +
6*(2*a*b^2*c^2*x^2 - a*b^2)*arccos(c*x)^2 + 3*(2*(2*a^2*b - b^3)*c^2*x^2 -
2*a^2*b + b^3)*arccos(c*x) - 3*(2*b^3*c*x*arccos(c*x)^2 + 4*a*b^2*c*x*arcco
s(c*x) + (2*a^2*b - b^3)*c*x)*sqrt(-c^2*x^2 + 1))/c^2
```

Sympy [A] time = 1.62723, size = 269, normalized size = 2.15

$$\begin{cases} \frac{a^3x^2}{2}+\frac{3a^2bx^2\operatorname{acos}\left(cx\right)}{2}-\frac{3a^2bx\sqrt{-c^2x^2+1}}{4c}-\frac{3a^2b\operatorname{acos}\left(cx\right)}{4c^2}+\frac{3ab^2x^2\operatorname{acos}^2\left(cx\right)}{2}-\frac{3ab^2x^2}{4}-\frac{3ab^2x\sqrt{-c^2x^2+1}\operatorname{acos}\left(cx\right)}{2c}-\frac{3ab^2\operatorname{acos}^2\left(cx\right)}{4c^2}+\frac{b^3x^2\operatorname{ac}}{2} \\ \frac{x^2\left(a+\frac{\pi b}{2}\right)^3}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acos(c*x))**3,x)
```

```
[Out] Piecewise((a**3*x**2/2 + 3*a**2*b*x**2*acos(c*x)/2 - 3*a**2*b*x*sqrt(-c**2*
x**2 + 1)/(4*c) - 3*a**2*b*acos(c*x)/(4*c**2) + 3*a*b**2*x**2*acos(c*x)**2/
2 - 3*a*b**2*x**2/4 - 3*a*b**2*x*sqrt(-c**2*x**2 + 1)*acos(c*x)/(2*c) - 3*a
*b**2*acos(c*x)**2/(4*c**2) + b**3*x**2*acos(c*x)**3/2 - 3*b**3*x**2*acos(c
*x)/4 - 3*b**3*x*sqrt(-c**2*x**2 + 1)*acos(c*x)**2/(4*c) + 3*b**3*x*sqrt(-c
**2*x**2 + 1)/(8*c) - b**3*acos(c*x)**3/(4*c**2) + 3*b**3*acos(c*x)/(8*c**2
), Ne(c, 0)), (x**2*(a + pi*b/2)**3/2, True))
```

Giac [B] time = 1.20801, size = 312, normalized size = 2.5

$$\frac{1}{2}b^3x^2\arccos\left(cx\right)^3+\frac{3}{2}ab^2x^2\arccos\left(cx\right)^2+\frac{3}{2}a^2bx^2\arccos\left(cx\right)-\frac{3}{4}b^3x^2\arccos\left(cx\right)-\frac{3\sqrt{-c^2x^2+1}b^3x\arccos\left(cx\right)^2}{4c}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccos(c*x))^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2*arccos(c*x)^3 + 3/2*a*b^2*x^2*arccos(c*x)^2 + 3/2*a^2*b*x^2*arc
cos(c*x) - 3/4*b^3*x^2*arccos(c*x) - 3/4*sqrt(-c^2*x^2 + 1)*b^3*x*arccos(c*
x)^2/c + 1/2*a^3*x^2 - 3/4*a*b^2*x^2 - 3/2*sqrt(-c^2*x^2 + 1)*a*b^2*x*arcco
s(c*x)/c - 1/4*b^3*arccos(c*x)^3/c^2 - 3/4*sqrt(-c^2*x^2 + 1)*a^2*b*x/c + 3
/8*sqrt(-c^2*x^2 + 1)*b^3*x/c - 3/4*a*b^2*arccos(c*x)^2/c^2 - 3/4*a^2*b*arc
cos(c*x)/c^2 + 3/8*b^3*arccos(c*x)/c^2 + 3/8*a*b^2/c^2
```

3.155 $\int \left(a+b\cos^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=82

$$-6ab^2x-\frac{3b\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{c}+x\left(a+b\cos^{-1}(cx)\right)^3+\frac{6b^3\sqrt{1-c^2x^2}}{c}-6b^3x\cos^{-1}(cx)$$

```
[Out] -6*a*b^2*x + (6*b^3*Sqrt[1 - c^2*x^2])/c - 6*b^3*x*ArcCos[c*x] - (3*b*Sqrt[
1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/c + x*(a + b*ArcCos[c*x])^3
```

Rubi [A] time = 0.107572, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4620, 4678, 261}

$$-6ab^2x-\frac{3b\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{c}+x\left(a+b\cos^{-1}(cx)\right)^3+\frac{6b^3\sqrt{1-c^2x^2}}{c}-6b^3x\cos^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])^3,x]
```

```
[Out] -6*a*b^2*x + (6*b^3*Sqrt[1 - c^2*x^2])/c - 6*b^3*x*ArcCos[c*x] - (3*b*Sqrt[
1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/c + x*(a + b*ArcCos[c*x])^3
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\cos^{-1}(cx)\right)^3 dx = x\left(a+b\cos^{-1}(cx)\right)^3 + (3bc)\int \frac{x\left(a+b\cos^{-1}(cx)\right)^2}{\sqrt{1-c^2x^2}}dx$$
$$= -\frac{3b\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{c} + x\left(a+b\cos^{-1}(cx)\right)^3 - \left(6b^2\right)\int \left(a+b\cos^{-1}(cx)\right)dx$$
$$= -6ab^2x - \frac{3b\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{c} + x\left(a+b\cos^{-1}(cx)\right)^3 - \left(6b^3\right)\int \cos^{-1}(cx)\,dx$$
$$= -6ab^2x - 6b^3x\cos^{-1}(cx) - \frac{3b\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{c} + x\left(a+b\cos^{-1}(cx)\right)^3 - \left(6b^3c\right)\int$$
$$= -6ab^2x + \frac{6b^3\sqrt{1-c^2x^2}}{c} - 6b^3x\cos^{-1}(cx) - \frac{3b\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{c} + x\left(a+b\cos^{-1}\right.$$

Mathematica [A] time = 0.113723, size = 128, normalized size = 1.56

$$\frac{-3b\left(a^2-2b^2\right)\sqrt{1-c^2x^2} + 3b\cos^{-1}(cx)\left(a^2cx - 2ab\sqrt{1-c^2x^2} - 2b^2cx\right) + acx\left(a^2-6b^2\right) + 3b^2\cos^{-1}(cx)^2\left(acx - b\sqrt{1-\right.}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[c*x])^3,x]
```

```
[Out] (a*(a^2 - 6*b^2)*c*x - 3*b*(a^2 - 2*b^2)*Sqrt[1 - c^2*x^2] + 3*b*(a^2*c*x -
 2*b^2*c*x - 2*a*b*Sqrt[1 - c^2*x^2])*ArcCos[c*x] + 3*b^2*(a*c*x - b*Sqrt[1
 - c^2*x^2])*ArcCos[c*x]^2 + b^3*c*x*ArcCos[c*x]^3)/c
```

Maple [A] time = 0.047, size = 134, normalized size = 1.6

$$\frac{1}{c}\left(cxa^3 + b^3\left(cx\left(\arccos(cx)\right)^3 - 3\left(\arccos(cx)\right)^2\sqrt{-c^2x^2+1} + 6\sqrt{-c^2x^2+1} - 6\,cx\arccos(cx)\right) + 3\,ab^2\left(cx\left(\arccos(c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^3,x)
```

```
[Out] 1/c*(c*x*a^3+b^3*(c*x*arccos(c*x)^3-3*arccos(c*x)^2*(-c^2*x^2+1)^(1/2)+6*(-
c^2*x^2+1)^(1/2)-6*c*x*arccos(c*x))+3*a*b^2*(c*x*arccos(c*x)^2-2*c*x-2*arcc
os(c*x)*(-c^2*x^2+1)^(1/2))+3*a^2*b*(c*x*arccos(c*x)-(-c^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.44053, size = 194, normalized size = 2.37

$$b^3 x \arccos(cx)^3 + 3\,ab^2 x \arccos(cx)^2 - 3\left(\frac{\sqrt{-c^2x^2+1}\arccos(cx)^2}{c} + \frac{2\left(cx\arccos(cx) - \sqrt{-c^2x^2+1}\right)}{c}\right)b^3 - 6\,ab^2\left(x + \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^3,x, algorithm="maxima")
```

```
[Out] b^3*x*arccos(c*x)^3 + 3*a*b^2*x*arccos(c*x)^2 - 3*(sqrt(-c^2*x^2 + 1)*arcco
s(c*x)^2/c + 2*(c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))/c)*b^3 - 6*a*b^2*(x +
 sqrt(-c^2*x^2 + 1)*arccos(c*x)/c) + a^3*x + 3*(c*x*arccos(c*x) - sqrt(-c^2
*x^2 + 1))*a^2*b/c
```

Fricas [A] time = 2.29421, size = 262, normalized size = 3.2

$$\frac{b^3 cx \arccos(cx)^3 + 3\,ab^2 cx \arccos(cx)^2 + 3\left(a^2b - 2\,b^3\right)cx\arccos(cx) + \left(a^3 - 6\,ab^2\right)cx - 3\left(b^3\arccos(cx)^2 + 2\,ab^2 \operatorname{arcc}\cdots\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^3,x, algorithm="fricas")
```

```
[Out] (b^3*c*x*arccos(c*x)^3 + 3*a*b^2*c*x*arccos(c*x)^2 + 3*(a^2*b - 2*b^3)*c*x*
arccos(c*x) + (a^3 - 6*a*b^2)*c*x - 3*(b^3*arccos(c*x)^2 + 2*a*b^2*arccos(c
*x) + a^2*b - 2*b^3)*sqrt(-c^2*x^2 + 1))/c
```

Sympy [A] time = 0.770118, size = 165, normalized size = 2.01

$$\begin{cases} a^3x + 3a^2bx\operatorname{acos}(cx) - \frac{3a^2b\sqrt{-c^2x^2+1}}{c} + 3ab^2x\operatorname{acos}^2(cx) - 6ab^2x - \frac{6ab^2\sqrt{-c^2x^2+1}\operatorname{acos}(cx)}{c} + b^3x\operatorname{acos}^3(cx) - 6b^3x\operatorname{acos}(cx \\ x\left(a + \frac{\pi b}{2}\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*x*acos(c*x) - 3*a**2*b*sqrt(-c**2*x**2 + 1)/c
+ 3*a*b**2*x*acos(c*x)**2 - 6*a*b**2*x - 6*a*b**2*sqrt(-c**2*x**2 + 1)*acos
(c*x)/c + b**3*x*acos(c*x)**3 - 6*b**3*x*acos(c*x) - 3*b**3*sqrt(-c**2*x**2
 + 1)*acos(c*x)**2/c + 6*b**3*sqrt(-c**2*x**2 + 1)/c, Ne(c, 0)), (x*(a + pi
*b/2)**3, True))
```

Giac [A] time = 1.14788, size = 203, normalized size = 2.48

$$b^3x\arccos(cx)^3 + 3\,ab^2x\arccos(cx)^2 + 3\,a^2bx\arccos(cx) - 6\,b^3x\arccos(cx) - \frac{3\,\sqrt{-c^2x^2+1}b^3\arccos(cx)^2}{c} + a^3x - 6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^3,x, algorithm="giac")
```

```
[Out] b^3*x*arccos(c*x)^3 + 3*a*b^2*x*arccos(c*x)^2 + 3*a^2*b*x*arccos(c*x) - 6*b
^3*x*arccos(c*x) - 3*sqrt(-c^2*x^2 + 1)*b^3*arccos(c*x)^2/c + a^3*x - 6*a*b
^2*x - 6*sqrt(-c^2*x^2 + 1)*a*b^2*arccos(c*x)/c - 3*sqrt(-c^2*x^2 + 1)*a^2*
b/c + 6*sqrt(-c^2*x^2 + 1)*b^3/c
```

3.156 $\int \frac{\left(a+b\cos^{-1}(cx)\right)^3}{x}\,dx$

Optimal. Leaf size=127

$$\frac{3}{2}b^2\text{PolyLog}\left(3,-e^{2i\cos^{-1}(cx)}\right)\left(a+b\cos^{-1}(cx)\right)-\frac{3}{2}ib\text{PolyLog}\left(2,-e^{2i\cos^{-1}(cx)}\right)\left(a+b\cos^{-1}(cx)\right)^2+\frac{3}{4}ib^3\text{PolyLog}\left(4,-e^{2\ldots}\right.$$

```
[Out] ((-I/4)*(a + b*ArcCos[c*x])^4)/b + (a + b*ArcCos[c*x])^3*Log[1 + E^((2*I)*A
rcCos[c*x])] - ((3*I)/2)*b*(a + b*ArcCos[c*x])^2*PolyLog[2, -E^((2*I)*ArcCo
s[c*x])] + (3*b^2*(a + b*ArcCos[c*x])*PolyLog[3, -E^((2*I)*ArcCos[c*x])])/2
 + ((3*I)/4)*b^3*PolyLog[4, -E^((2*I)*ArcCos[c*x])]
```

Rubi [A] time = 0.14096, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4626, 3719, 2190, 2531, 6609, 2282, 6589}

$$\frac{3}{2}b^2\text{PolyLog}\left(3,-e^{2i\cos^{-1}(cx)}\right)\left(a+b\cos^{-1}(cx)\right)-\frac{3}{2}ib\text{PolyLog}\left(2,-e^{2i\cos^{-1}(cx)}\right)\left(a+b\cos^{-1}(cx)\right)^2+\frac{3}{4}ib^3\text{PolyLog}\left(4,-e^{2\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])^3/x,x]
```

```
[Out] ((-I/4)*(a + b*ArcCos[c*x])^4)/b + (a + b*ArcCos[c*x])^3*Log[1 + E^((2*I)*A
rcCos[c*x])] - ((3*I)/2)*b*(a + b*ArcCos[c*x])^2*PolyLog[2, -E^((2*I)*ArcCo
s[c*x])] + (3*b^2*(a + b*ArcCos[c*x])*PolyLog[3, -E^((2*I)*ArcCos[c*x])])/2
 + ((3*I)/4)*b^3*PolyLog[4, -E^((2*I)*ArcCos[c*x])]
```

Rule 4626

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n/Cot[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^3}{x}\,dx = -\operatorname{Subst}\left(\int (a+bx)^3\tan(x)\,dx, x, \cos^{-1}(cx)\right)$$

$$= -\frac{i\left(a+b\cos^{-1}(cx)\right)^4}{4b} + 2i\operatorname{Subst}\left(\int \frac{e^{2ix}(a+bx)^3}{1+e^{2ix}}\,dx, x, \cos^{-1}(cx)\right)$$

$$= -\frac{i\left(a+b\cos^{-1}(cx)\right)^4}{4b} + \left(a+b\cos^{-1}(cx)\right)^3\log\left(1+e^{2i\cos^{-1}(cx)}\right) - (3b)\operatorname{Subst}\left(\int (a+bx)^2\log\left(\right.\right.$$

$$= -\frac{i\left(a+b\cos^{-1}(cx)\right)^4}{4b} + \left(a+b\cos^{-1}(cx)\right)^3\log\left(1+e^{2i\cos^{-1}(cx)}\right) - \frac{3}{2}ib\left(a+b\cos^{-1}(cx)\right)^2\operatorname{Li}_2\left(-\right.$$

$$= -\frac{i\left(a+b\cos^{-1}(cx)\right)^4}{4b} + \left(a+b\cos^{-1}(cx)\right)^3\log\left(1+e^{2i\cos^{-1}(cx)}\right) - \frac{3}{2}ib\left(a+b\cos^{-1}(cx)\right)^2\operatorname{Li}_2\left(-\right.$$

$$= -\frac{i\left(a+b\cos^{-1}(cx)\right)^4}{4b} + \left(a+b\cos^{-1}(cx)\right)^3\log\left(1+e^{2i\cos^{-1}(cx)}\right) - \frac{3}{2}ib\left(a+b\cos^{-1}(cx)\right)^2\operatorname{Li}_2\left(-\right.$$

$$= -\frac{i\left(a+b\cos^{-1}(cx)\right)^4}{4b} + \left(a+b\cos^{-1}(cx)\right)^3\log\left(1+e^{2i\cos^{-1}(cx)}\right) - \frac{3}{2}ib\left(a+b\cos^{-1}(cx)\right)^2\operatorname{Li}_2\left(-\right.$$

Mathematica [A] time = 0.171118, size = 204, normalized size = 1.61

$$\frac{1}{4}\left(6b^2\operatorname{PolyLog}\left(3,-e^{2i\cos^{-1}(cx)}\right)\left(a+b\cos^{-1}(cx)\right) - 6ib\operatorname{PolyLog}\left(2,-e^{2i\cos^{-1}(cx)}\right)\left(a+b\cos^{-1}(cx)\right)^2 + 3ib^3\operatorname{PolyLog}\left(4,-\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[c*x])^3/x,x]
```

```
[Out] ((-6*I)*a^2*b*ArcCos[c*x]^2 - (4*I)*a*b^2*ArcCos[c*x]^3 - I*b^3*ArcCos[c*x]
^4 + 12*a^2*b*ArcCos[c*x]*Log[1 + E^((2*I)*ArcCos[c*x])] + 12*a*b^2*ArcCos[
c*x]^2*Log[1 + E^((2*I)*ArcCos[c*x])] + 4*b^3*ArcCos[c*x]^3*Log[1 + E^((2*I
)*ArcCos[c*x])] + 4*a^3*Log[c*x] - (6*I)*b*(a + b*ArcCos[c*x])^2*PolyLog[2,
 -E^((2*I)*ArcCos[c*x])] + 6*b^2*(a + b*ArcCos[c*x])*PolyLog[3, -E^((2*I)*A
rcCos[c*x])] + (3*I)*b^3*PolyLog[4, -E^((2*I)*ArcCos[c*x])])/4
```

Maple [B] time = 0.063, size = 353, normalized size = 2.8

$$a^3\ln(cx) - \frac{i}{4}b^3(\arccos(cx))^4 + b^3(\arccos(cx))^3\ln\left(1+\left(cx+i\sqrt{-c^2x^2+1}\right)^2\right) - \frac{3i}{2}b^3(\arccos(cx))^2\operatorname{polylog}\left(2,-\left(cx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^3/x,x)
```

```
[Out] a^3*ln(c*x)-1/4*I*b^3*arccos(c*x)^4+b^3*arccos(c*x)^3*ln(1+(c*x+I*(-c^2*x^2
+1)^(1/2))^2)-3/2*I*b^3*arccos(c*x)^2*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))
^2)+3/2*b^3*arccos(c*x)*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+3/4*I*b^3*
polylog(4,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)-I*a*b^2*arccos(c*x)^3+3*a*b^2*arcc
os(c*x)^2*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-3*I*a*b^2*arccos(c*x)*polylog(
2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+3/2*a*b^2*polylog(3,-(c*x+I*(-c^2*x^2+1)^(
1/2))^2)-3/2*I*a^2*b*arccos(c*x)^2+3*a^2*b*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^
2+1)^(1/2))^2)-3/2*I*a^2*b*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^3 \log(x) + \int \frac{b^3 \arctan\left(\sqrt{cx+1}\sqrt{-cx+1}, cx\right)^3 + 3\,ab^2 \arctan\left(\sqrt{cx+1}\sqrt{-cx+1}, cx\right)^2 + 3\,a^2 b \arctan\left(\sqrt{cx+1}\sqrt{-cx\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + integrate((b^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^3 +
3*a*b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 3*a^2*b*arctan2(sqrt
(c*x + 1)*sqrt(-c*x + 1), c*x))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \arccos(cx)^3 + 3\,ab^2 \arccos(cx)^2 + 3\,a^2 b \arccos(cx) + a^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x) +
 a^3)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{acos}\,(cx))^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**3/x,x)
```

```
[Out] Integral((a + b*acos(c*x))**3/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arccos(cx) + a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)^3/x, x)
```

3.157 $\int \frac{\left(a+b\cos^{-1}(cx)\right)^3}{x^2}\,dx$

Optimal. Leaf size=151

$$6ib^2c\text{PolyLog}\left(2,-ie^{i\cos^{-1}(cx)}\right)\left(a+b\cos^{-1}(cx)\right)-6ib^2c\text{PolyLog}\left(2,ie^{i\cos^{-1}(cx)}\right)\left(a+b\cos^{-1}(cx)\right)-6b^3c\text{PolyLog}\left(3,-i\right.$$

```
[Out] -((a + b*ArcCos[c*x])^3/x) - (6*I)*b*c*(a + b*ArcCos[c*x])^2*ArcTan[E^(I*Ar
cCos[c*x])] + (6*I)*b^2*c*(a + b*ArcCos[c*x])*PolyLog[2, (-I)*E^(I*ArcCos[c
*x])] - (6*I)*b^2*c*(a + b*ArcCos[c*x])*PolyLog[2, I*E^(I*ArcCos[c*x])] - 6
*b^3*c*PolyLog[3, (-I)*E^(I*ArcCos[c*x])] + 6*b^3*c*PolyLog[3, I*E^(I*ArcCo
s[c*x])]
```

Rubi [A] time = 0.213895, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4628, 4710, 4181, 2531, 2282, 6589}

$$6ib^2c\text{PolyLog}\left(2,-ie^{i\cos^{-1}(cx)}\right)\left(a+b\cos^{-1}(cx)\right)-6ib^2c\text{PolyLog}\left(2,ie^{i\cos^{-1}(cx)}\right)\left(a+b\cos^{-1}(cx)\right)-6b^3c\text{PolyLog}\left(3,-i\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])^3/x^2,x]
```

```
[Out] -((a + b*ArcCos[c*x])^3/x) - (6*I)*b*c*(a + b*ArcCos[c*x])^2*ArcTan[E^(I*Ar
cCos[c*x])] + (6*I)*b^2*c*(a + b*ArcCos[c*x])*PolyLog[2, (-I)*E^(I*ArcCos[c
*x])] - (6*I)*b^2*c*(a + b*ArcCos[c*x])*PolyLog[2, I*E^(I*ArcCos[c*x])] - 6
*b^3*c*PolyLog[3, (-I)*E^(I*ArcCos[c*x])] + 6*b^3*c*PolyLog[3, I*E^(I*ArcCo
s[c*x])]
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4710

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> -Dist[(c^(m + 1)*Sqrt[d])^(-1), Subst[Int[(a + b*x)^n
```

```
*Cos[x]^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^3}{x^2}\,dx = -\frac{\left(a+b\cos^{-1}(cx)\right)^3}{x} - (3bc)\int \frac{\left(a+b\cos^{-1}(cx)\right)^2}{x\sqrt{1-c^2x^2}}\,dx$$
$$= -\frac{\left(a+b\cos^{-1}(cx)\right)^3}{x} + (3bc)\,\mathrm{Subst}\left(\int (a+bx)^2\sec(x)\,dx, x, \cos^{-1}(cx)\right)$$
$$= -\frac{\left(a+b\cos^{-1}(cx)\right)^3}{x} - 6ibc\left(a+b\cos^{-1}(cx)\right)^2\tan^{-1}\left(e^{i\cos^{-1}(cx)}\right) - \left(6b^2c\right)\mathrm{Subst}\left(\int (a+bx)\right.$$
$$= -\frac{\left(a+b\cos^{-1}(cx)\right)^3}{x} - 6ibc\left(a+b\cos^{-1}(cx)\right)^2\tan^{-1}\left(e^{i\cos^{-1}(cx)}\right) + 6ib^2c\left(a+b\cos^{-1}(cx)\right)\mathrm{Li}$$
$$= -\frac{\left(a+b\cos^{-1}(cx)\right)^3}{x} - 6ibc\left(a+b\cos^{-1}(cx)\right)^2\tan^{-1}\left(e^{i\cos^{-1}(cx)}\right) + 6ib^2c\left(a+b\cos^{-1}(cx)\right)\mathrm{Li}$$
$$= -\frac{\left(a+b\cos^{-1}(cx)\right)^3}{x} - 6ibc\left(a+b\cos^{-1}(cx)\right)^2\tan^{-1}\left(e^{i\cos^{-1}(cx)}\right) + 6ib^2c\left(a+b\cos^{-1}(cx)\right)\mathrm{Li}$$

Mathematica [B] time = 0.291687, size = 308, normalized size = 2.04

$$3ab^2c\left(-\frac{\cos^{-1}(cx)^2}{cx} + 2\left(i\left(\mathrm{PolyLog}\left(2, -ie^{i\cos^{-1}(cx)}\right) - \mathrm{PolyLog}\left(2, ie^{i\cos^{-1}(cx)}\right)\right) + \cos^{-1}(cx)\left(\log\left(1-ie^{i\cos^{-1}(cx)}\right) - \log\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[c*x])^3/x^2,x]
```

```
[Out] -(a^3/x) - (3*a^2*b*ArcCos[c*x])/x - 3*a^2*b*c*Log[x] + 3*a^2*b*c*Log[1 + S
qrt[1 - c^2*x^2]] + 3*a*b^2*c*(-(ArcCos[c*x]^2/(c*x)) + 2*(ArcCos[c*x]*(Log
[1 - I*E^(I*ArcCos[c*x])] - Log[1 + I*E^(I*ArcCos[c*x])]) + I*(PolyLog[2, (
-I)*E^(I*ArcCos[c*x])] - PolyLog[2, I*E^(I*ArcCos[c*x])]))) + b^3*c*(-(ArcC
os[c*x]^3/(c*x)) + 3*(ArcCos[c*x]^2*(Log[1 - I*E^(I*ArcCos[c*x])] - Log[1 +
 I*E^(I*ArcCos[c*x])]) + (2*I)*ArcCos[c*x]*(PolyLog[2, (-I)*E^(I*ArcCos[c*x
])] - PolyLog[2, I*E^(I*ArcCos[c*x])]) - 2*(PolyLog[3, (-I)*E^(I*ArcCos[c*x
])] - PolyLog[3, I*E^(I*ArcCos[c*x])])))
```

Maple [F] time = 0.161, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\arccos(cx)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^3/x^2,x)
```

```
[Out] int((a+b*arccos(c*x))^3/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$3\left(c\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{\arccos(cx)}{x}\right)a^2b-\frac{a^3}{x}-\frac{b^3\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)^3-3x\int\frac{\sqrt{cx+1}\sqrt{-cx+1}b^3cx\arctan(\cdots}{\cdots}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^3/x^2,x, algorithm="maxima")
```

```
[Out] 3*(c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) - arccos(c*x)/x)*a^2*b - a
^3/x - (b^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^3 - x*integrate(3*(s
qrt(c*x + 1)*sqrt(-c*x + 1)*b^3*c*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c
*x)^2 + (a*b^2*c^2*x^2 - a*b^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^
2)/(c^2*x^4 - x^2), x))/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^3\arccos(cx)^3+3ab^2\arccos(cx)^2+3a^2b\arccos(cx)+a^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x) +
 a^3)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+b\operatorname{acos}(cx))^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**3/x**2,x)
```

```
[Out] Integral((a + b*acos(c*x))**3/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arccos(cx) + a)^3}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)^3/x^2, x)
```

3.158 $\int \frac{x^2}{a+b\cos^{-1}(cx)}\,dx$

Optimal. Leaf size=121

$$\frac{\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\cos^{-1}(cx)}{b}\right)}{4bc^3}+\frac{\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3\left(a+b\cos^{-1}(cx)\right)}{b}\right)}{4bc^3}-\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\cos^{-1}(cx)}{b}\right)}{4bc^3}-\frac{\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a}{}\right.}{4bc^3}$$

```
[Out] (CosIntegral[(a + b*ArcCos[c*x])/b]*Sin[a/b])/(4*b*c^3) + (CosIntegral[(3*(
a + b*ArcCos[c*x]))/b]*Sin[(3*a)/b])/(4*b*c^3) - (Cos[a/b]*SinIntegral[(a +
 b*ArcCos[c*x])/b])/(4*b*c^3) - (Cos[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos[
c*x]))/b])/(4*b*c^3)
```

Rubi [A] time = 0.21562, antiderivative size = 117, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4636, 4406, 3303, 3299, 3302}

$$\frac{\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\cos^{-1}(cx)\right)}{4bc^3}+\frac{\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3a}{b}+3\cos^{-1}(cx)\right)}{4bc^3}-\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\cos^{-1}(cx)\right)}{4bc^3}-\frac{\cos\left(\frac{3a}{b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*ArcCos[c*x]),x]
```

```
[Out] (CosIntegral[a/b + ArcCos[c*x]]*Sin[a/b])/(4*b*c^3) + (CosIntegral[(3*a)/b
+ 3*ArcCos[c*x]]*Sin[(3*a)/b])/(4*b*c^3) - (Cos[a/b]*SinIntegral[a/b + ArcC
os[c*x]])/(4*b*c^3) - (Cos[(3*a)/b]*SinIntegral[(3*a)/b + 3*ArcCos[c*x]])/(
4*b*c^3)
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x^2}{a+b\cos^{-1}(cx)}\,dx = -\frac{\text{Subst}\left(\int \frac{\cos^2(x)\sin(x)}{a+bx}\,dx, x, \cos^{-1}(cx)\right)}{c^3}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{\sin(x)}{4(a+bx)}+\frac{\sin(3x)}{4(a+bx)}\right)dx, x, \cos^{-1}(cx)\right)}{c^3}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sin(x)}{a+bx}\,dx, x, \cos^{-1}(cx)\right)}{4c^3} - \frac{\text{Subst}\left(\int \frac{\sin(3x)}{a+bx}\,dx, x, \cos^{-1}(cx)\right)}{4c^3}$$

$$= -\frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cos^{-1}(cx)\right)}{4c^3} - \frac{\cos\left(\frac{3a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx, x, \cos^{-1}(cx)\right)}{4c^3} +$$

$$= \frac{\text{Ci}\left(\frac{a}{b}+\cos^{-1}(cx)\right)\sin\left(\frac{a}{b}\right)}{4bc^3} + \frac{\text{Ci}\left(\frac{3a}{b}+3\cos^{-1}(cx)\right)\sin\left(\frac{3a}{b}\right)}{4bc^3} - \frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\cos^{-1}(cx)\right)}{4bc^3} -$$

Mathematica [A] time = 0.160972, size = 91, normalized size = 0.75

$$-\frac{\sin\left(\frac{a}{b}\right)\left(-\text{CosIntegral}\left(\frac{a}{b}+\cos^{-1}(cx)\right)\right) - \sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(3\left(\frac{a}{b}+\cos^{-1}(cx)\right)\right) + \cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\cos^{-1}(cx)\right) + \text{c}}{4bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*ArcCos[c*x]),x]
```

```
[Out] -(-(CosIntegral[a/b + ArcCos[c*x]]*Sin[a/b]) - CosIntegral[3*(a/b + ArcCos[
c*x])]*Sin[(3*a)/b] + Cos[a/b]*SinIntegral[a/b + ArcCos[c*x]] + Cos[(3*a)/b
]*SinIntegral[3*(a/b + ArcCos[c*x])])/(4*b*c^3)
```

Maple [A] time = 0.05, size = 102, normalized size = 0.8

$$\frac{1}{c^3}\left(-\frac{1}{4b}\mathrm{Si}\left(3\arccos(cx)+3\frac{a}{b}\right)\cos\left(3\frac{a}{b}\right)+\frac{1}{4b}\mathrm{Ci}\left(3\arccos(cx)+3\frac{a}{b}\right)\sin\left(3\frac{a}{b}\right)-\frac{1}{4b}\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arccos(c*x)),x)
```

```
[Out] 1/c^3*(-1/4*Si(3*arccos(c*x)+3*a/b)*cos(3*a/b)/b+1/4*Ci(3*arccos(c*x)+3*a/b
)*sin(3*a/b)/b-1/4*Si(arccos(c*x)+a/b)*cos(a/b)/b+1/4*Ci(arccos(c*x)+a/b)*s
in(a/b)/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{b\arccos(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccos(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*arccos(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2}{b\arccos(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccos(c*x)),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b*arccos(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{a + b\,\mathrm{acos}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*acos(c*x)),x)
```

```
[Out] Integral(x**2/(a + b*acos(c*x)), x)
```

Giac [A] time = 1.20722, size = 232, normalized size = 1.92

$$\frac{\cos\left(\frac{a}{b}\right)^2 \mathrm{Ci}\left(\frac{3a}{b} + 3\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{bc^3} - \frac{\cos\left(\frac{a}{b}\right)^3 \mathrm{Si}\left(\frac{3a}{b} + 3\arccos(cx)\right)}{bc^3} - \frac{\mathrm{Ci}\left(\frac{3a}{b} + 3\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{4bc^3} + \frac{\mathrm{Ci}\left(\frac{a}{b}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccos(c*x)),x, algorithm="giac")
```

```
[Out] cos(a/b)^2*cos_integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b*c^3) - cos(a/b)^
3*sin_integral(3*a/b + 3*arccos(c*x))/(b*c^3) - 1/4*cos_integral(3*a/b + 3*
arccos(c*x))*sin(a/b)/(b*c^3) + 1/4*cos_integral(a/b + arccos(c*x))*sin(a/b
)/(b*c^3) + 3/4*cos(a/b)*sin_integral(3*a/b + 3*arccos(c*x))/(b*c^3) - 1/4*
cos(a/b)*sin_integral(a/b + arccos(c*x))/(b*c^3)
```

3.159 $\int \frac{x}{a+b\cos^{-1}(cx)}\,dx$

Optimal. Leaf size=63

$$\frac{\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2\left(a+b\cos^{-1}(cx)\right)}{b}\right)}{2bc^2}-\frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2\left(a+b\cos^{-1}(cx)\right)}{b}\right)}{2bc^2}$$

```
[Out] (CosIntegral[(2*(a + b*ArcCos[c*x]))/b]*Sin[(2*a)/b])/(2*b*c^2) - (Cos[(2*a
)/b]*SinIntegral[(2*(a + b*ArcCos[c*x]))/b])/(2*b*c^2)
```

Rubi [A] time = 0.120868, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4636, 4406, 12, 3303, 3299, 3302}

$$\frac{\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2a}{b}+2\cos^{-1}(cx)\right)}{2bc^2}-\frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\cos^{-1}(cx)\right)}{2bc^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*ArcCos[c*x]),x]
```

```
[Out] (CosIntegral[(2*a)/b + 2*ArcCos[c*x]]*Sin[(2*a)/b])/(2*b*c^2) - (Cos[(2*a)/
b]*SinIntegral[(2*a)/b + 2*ArcCos[c*x]])/(2*b*c^2)
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a+b\cos^{-1}(cx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{a+bx}\,dx, x, \cos^{-1}(cx)\right)}{c^2}\\ &= -\frac{\text{Subst}\left(\int \frac{\sin(2x)}{2(a+bx)}\,dx, x, \cos^{-1}(cx)\right)}{c^2}\\ &= -\frac{\text{Subst}\left(\int \frac{\sin(2x)}{a+bx}\,dx, x, \cos^{-1}(cx)\right)}{2c^2}\\ &= -\frac{\cos\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \cos^{-1}(cx)\right)}{2c^2}+\frac{\sin\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \cos^{-1}(cx)\right)}{2c^2}\\ &= \frac{\text{Ci}\left(\frac{2a}{b}+2\cos^{-1}(cx)\right)\sin\left(\frac{2a}{b}\right)}{2bc^2}-\frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\cos^{-1}(cx)\right)}{2bc^2}\end{aligned}$$

Mathematica [A] time = 0.067942, size = 56, normalized size = 0.89

$$-\frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\cos^{-1}(cx)\right)-\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2a}{b}+2\cos^{-1}(cx)\right)}{2bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*ArcCos[c*x]),x]
```

```
[Out] -(-(CosIntegral[(2*a)/b + 2*ArcCos[c*x]]*Sin[(2*a)/b]) + Cos[(2*a)/b]*SinIn
tegral[(2*a)/b + 2*ArcCos[c*x]])/(2*b*c^2)
```

Maple [A] time = 0.046, size = 58, normalized size = 0.9

$$\frac{1}{c^2}\left(-\frac{1}{2b}\operatorname{Si}\left(2\arccos(cx)+2\frac{a}{b}\right)\cos\left(2\frac{a}{b}\right)+\frac{1}{2b}\operatorname{Ci}\left(2\arccos(cx)+2\frac{a}{b}\right)\sin\left(2\frac{a}{b}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arccos(c*x)),x)
```

```
[Out] 1/c^2*(-1/2*Si(2*arccos(c*x)+2*a/b)*cos(2*a/b)/b+1/2*Ci(2*arccos(c*x)+2*a/b
)*sin(2*a/b)/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b\arccos(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccos(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arccos(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x}{b\arccos(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccos(c*x)),x, algorithm="fricas")
```

```
[Out] integral(x/(b*arccos(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{a+b\,\mathrm{acos}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*acos(c*x)),x)
```

```
[Out] Integral(x/(a + b*acos(c*x)), x)
```

Giac [A] time = 1.17329, size = 116, normalized size = 1.84

$$\frac{\cos\left(\frac{a}{b}\right)\mathrm{Ci}\left(\frac{2\,a}{b}+2\,\arccos\left(cx\right)\right)\sin\left(\frac{a}{b}\right)}{bc^2}-\frac{\cos\left(\frac{a}{b}\right)^2\mathrm{Si}\left(\frac{2\,a}{b}+2\,\arccos\left(cx\right)\right)}{bc^2}+\frac{\mathrm{Si}\left(\frac{2\,a}{b}+2\,\arccos\left(cx\right)\right)}{2\,bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccos(c*x)),x, algorithm="giac")
```

```
[Out] cos(a/b)*cos_integral(2*a/b + 2*arccos(c*x))*sin(a/b)/(b*c^2) - cos(a/b)^2*
sin_integral(2*a/b + 2*arccos(c*x))/(b*c^2) + 1/2*sin_integral(2*a/b + 2*ar
ccos(c*x))/(b*c^2)
```

3.160 $\int \frac{1}{a+b\cos^{-1}(cx)}\,dx$

Optimal. Leaf size=54

$$\frac{\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\cos^{-1}(cx)}{b}\right)}{bc}-\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\cos^{-1}(cx)}{b}\right)}{bc}$$

```
[Out] (CosIntegral[(a + b*ArcCos[c*x])/b]*Sin[a/b])/(b*c) - (Cos[a/b]*SinIntegral
[(a + b*ArcCos[c*x])/b])/(b*c)
```

Rubi [A] time = 0.0619756, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4624, 3303, 3299, 3302}

$$\frac{\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\cos^{-1}(cx)}{b}\right)}{bc}-\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\cos^{-1}(cx)}{b}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])^(-1),x]
```

```
[Out] (CosIntegral[(a + b*ArcCos[c*x])/b]*Sin[a/b])/(b*c) - (Cos[a/b]*SinIntegral
[(a + b*ArcCos[c*x])/b])/(b*c)
```

Rule 4624

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Sub
st[Int[x^n*Sin[a/b - x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c,
 n}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\cos^{-1}(cx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\cos^{-1}(cx)\right)}{bc}\\ &= -\frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\cos^{-1}(cx)\right)}{bc}+\frac{\sin\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\cos^{-1}(cx\right.}{bc}\\ &= \frac{\text{Ci}\left(\frac{a+b\cos^{-1}(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{bc}-\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\cos^{-1}(cx)}{b}\right)}{bc}\end{aligned}$$

Mathematica [A] time = 0.0619107, size = 46, normalized size = 0.85

$$-\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\cos^{-1}(cx)\right)-\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\cos^{-1}(cx)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[c*x])^(-1),x]
```

```
[Out] -((-(CosIntegral[a/b + ArcCos[c*x]]*Sin[a/b]) + Cos[a/b]*SinIntegral[a/b +
ArcCos[c*x]])/(b*c))
```

Maple [A] time = 0.049, size = 49, normalized size = 0.9

$$\frac{1}{c}\left(-\frac{1}{b}\text{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)+\frac{1}{b}\text{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(c*x)),x)
```

```
[Out] 1/c*(-Si(arccos(c*x)+a/b)*cos(a/b)/b+Ci(arccos(c*x)+a/b)*sin(a/b)/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \arccos(cx) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arccos(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b \arccos(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*arccos(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a + b \operatorname{acos}(cx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(c*x)),x)
```

```
[Out] Integral(1/(a + b*acos(c*x)), x)
```

Giac [A] time = 1.14686, size = 68, normalized size = 1.26

$$\frac{\operatorname{Ci}\left(\frac{a}{b} + \arccos\left(cx\right)\right)\sin\left(\frac{a}{b}\right)}{bc} - \frac{\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b} + \arccos\left(cx\right)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(c*x)),x, algorithm="giac")
```

```
[Out] cos_integral(a/b + arccos(c*x))*sin(a/b)/(b*c) - cos(a/b)*sin_integral(a/b
+ arccos(c*x))/(b*c)
```

3.161 $\int \frac{1}{x\left(a+b\cos^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{x\left(a+b\cos^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/(x*(a + b*ArcCos[c*x])), x]
```

Rubi [A] time = 0.0232703, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(a+b\cos^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + b*ArcCos[c*x])),x]
```

```
[Out] Defer[Int][1/(x*(a + b*ArcCos[c*x])), x]
```

Rubi steps

$$\int \frac{1}{x\left(a+b\cos^{-1}(cx)\right)}\,dx=\int \frac{1}{x\left(a+b\cos^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.301794, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\cos^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + b*ArcCos[c*x])),x]
```

```
[Out] Integrate[1/(x*(a + b*ArcCos[c*x])), x]
```

Maple [A] time = 0.263, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\arccos\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*arccos(c*x)),x)
```

```
[Out] int(1/x/(a+b*arccos(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\arccos\left(cx\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccos(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arccos(c*x) + a)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{bx\arccos\left(cx\right)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccos(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x*arccos(c*x) + a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\,\text{acos}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*acos(c*x)),x)
```

```
[Out] Integral(1/(x*(a + b*acos(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\arccos\left(cx\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccos(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*arccos(c*x) + a)*x), x)
```

3.162 $\int \frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/(x^2*(a + b*ArcCos[c*x])), x]
```

Rubi [A] time = 0.0244692, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(a + b*ArcCos[c*x])),x]

[Out] Defer[Int][1/(x^2*(a + b*ArcCos[c*x])), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)}\,dx=\int \frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 3.58985, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(a + b*ArcCos[c*x])),x]
```

```
[Out] Integrate[1/(x^2*(a + b*ArcCos[c*x])), x]
```

Maple [A] time = 0.18, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a + b \arccos (cx))} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arccos(c*x)),x)
```

```
[Out] int(1/x^2/(a+b*arccos(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccos (cx) + a)x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccos(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arccos(c*x) + a)*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bx^2 \arccos (cx) + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccos(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x^2*arccos(c*x) + a*x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a + b \operatorname{acos}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*acos(c*x)),x)
```

```
[Out] Integral(1/(x**2*(a + b*acos(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccos(cx) + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccos(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*arccos(c*x) + a)*x^2), x)
```

3.163 $\int \frac{x^2}{\left(a+b\cos^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=155

$$-\frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\cos^{-1}(cx)}{b}\right)}{4b^2c^3}-\frac{3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3\left(a+b\cos^{-1}(cx)\right)}{b}\right)}{4b^2c^3}-\frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\cos^{-1}(cx)}{b}\right)}{4b^2c^3}-\frac{3\sin\left(\frac{3a}{b}\right)\text{Si}}{4}$$

```
[Out] (x^2*Sqrt[1 - c^2*x^2])/(b*c*(a + b*ArcCos[c*x])) - (Cos[a/b]*CosIntegral[(
a + b*ArcCos[c*x])/b])/(4*b^2*c^3) - (3*Cos[(3*a)/b]*CosIntegral[(3*(a + b*
ArcCos[c*x]))/b])/(4*b^2*c^3) - (Sin[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b
])/(4*b^2*c^3) - (3*Sin[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/(4
*b^2*c^3)
```

Rubi [A] time = 0.182654, antiderivative size = 151, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4632, 3303, 3299, 3302}

$$-\frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\cos^{-1}(cx)\right)}{4b^2c^3}-\frac{3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3a}{b}+3\cos^{-1}(cx)\right)}{4b^2c^3}-\frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\cos^{-1}(cx)\right)}{4b^2c^3}-\frac{3\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*ArcCos[c*x])^2,x]
```

```
[Out] (x^2*Sqrt[1 - c^2*x^2])/(b*c*(a + b*ArcCos[c*x])) - (Cos[a/b]*CosIntegral[a
/b + ArcCos[c*x]])/(4*b^2*c^3) - (3*Cos[(3*a)/b]*CosIntegral[(3*a)/b + 3*Ar
cCos[c*x]])/(4*b^2*c^3) - (Sin[a/b]*SinIntegral[a/b + ArcCos[c*x]])/(4*b^2*
c^3) - (3*Sin[(3*a)/b]*SinIntegral[(3*a)/b + 3*ArcCos[c*x]])/(4*b^2*c^3)
```

Rule 4632

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dis
t[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), Co
s[x]^(m - 1)*(m - (m + 1)*Cos[x]^2), x], x], x, ArcCos[c*x]], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+b\cos^{-1}(cx)\right)^2}\,dx = \frac{x^2\sqrt{1-c^2x^2}}{bc\left(a+b\cos^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int\left(-\frac{\cos(x)}{4(a+bx)}-\frac{3\cos(3x)}{4(a+bx)}\right)dx,x,\cos^{-1}(cx)\right)}{bc^3}$$

$$= \frac{x^2\sqrt{1-c^2x^2}}{bc\left(a+b\cos^{-1}(cx)\right)} - \frac{\text{Subst}\left(\int\frac{\cos(x)}{a+bx}dx,x,\cos^{-1}(cx)\right)}{4bc^3} - \frac{3\,\text{Subst}\left(\int\frac{\cos(3x)}{a+bx}dx,x,\cos^{-1}(cx)\right.}{4bc^3}$$

$$= \frac{x^2\sqrt{1-c^2x^2}}{bc\left(a+b\cos^{-1}(cx)\right)} - \frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int\frac{\cos\left(\frac{a}{b}+x\right)}{a+bx}dx,x,\cos^{-1}(cx)\right)}{4bc^3} - \frac{\left(3\cos\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{x^2\sqrt{1-c^2x^2}}{bc\left(a+b\cos^{-1}(cx)\right)} - \frac{\cos\left(\frac{a}{b}\right)\text{Ci}\left(\frac{a}{b}+\cos^{-1}(cx)\right)}{4b^2c^3} - \frac{3\cos\left(\frac{3a}{b}\right)\text{Ci}\left(\frac{3a}{b}+3\cos^{-1}(cx)\right)}{4b^2c^3} - \frac{\sin(}{}$$

Mathematica [A] time = 0.565015, size = 124, normalized size = 0.8

$$-\frac{-\frac{4bc^2x^2\sqrt{1-c^2x^2}}{a+b\cos^{-1}(cx)}+\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\cos^{-1}(cx)\right)+3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(3\left(\frac{a}{b}+\cos^{-1}(cx)\right)\right)+\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\right.}{4b^2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*ArcCos[c*x])^2,x]
```

```
[Out] -((-4*b*c^2*x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]) + Cos[a/b]*CosIntegr
al[a/b + ArcCos[c*x]] + 3*Cos[(3*a)/b]*CosIntegral[3*(a/b + ArcCos[c*x])] +
 Sin[a/b]*SinIntegral[a/b + ArcCos[c*x]] + 3*Sin[(3*a)/b]*SinIntegral[3*(a/
b + ArcCos[c*x])])/(4*b^2*c^3)
```

Maple [A] time = 0.056, size = 147, normalized size = 1.

$$\frac{1}{c^3}\left(\frac{\sin(3\arccos(cx))}{(4a+4b\arccos(cx))b}-\frac{3}{4b^2}\left(\mathrm{Si}\left(3\arccos(cx)+3\frac{a}{b}\right)\sin\left(3\frac{a}{b}\right)+\mathrm{Ci}\left(3\arccos(cx)+3\frac{a}{b}\right)\cos\left(3\frac{a}{b}\right)\right)+\frac{}{(4a+4b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arccos(c*x))^2,x)
```

```
[Out] 1/c^3*(1/4*sin(3*arccos(c*x))/(a+b*arccos(c*x))/b-3/4*(Si(3*arccos(c*x)+3*a
/b)*sin(3*a/b)+Ci(3*arccos(c*x)+3*a/b)*cos(3*a/b))/b^2+1/4*(-c^2*x^2+1)^(1/
2)/(a+b*arccos(c*x))/b-1/4*(Si(arccos(c*x)+a/b)*sin(a/b)+Ci(arccos(c*x)+a/b
)*cos(a/b))/b^2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2}{b^2\arccos(cx)^2+2ab\arccos(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(x^2/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a + b\operatorname{acos}(cx))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*acos(c*x))**2,x)
```

```
[Out] Integral(x**2/(a + b*acos(c*x))**2, x)
```

Giac [B] time = 1.31066, size = 830, normalized size = 5.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

```
[Out] -3*b*arccos(c*x)*cos(a/b)^3*cos_integral(3*a/b + 3*arccos(c*x))/(b^3*c^3*ar
ccos(c*x) + a*b^2*c^3) - 3*b*arccos(c*x)*cos(a/b)^2*sin(a/b)*sin_integral(3
*a/b + 3*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) + sqrt(-c^2*x^2 + 1
)*b*c^2*x^2/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 3*a*cos(a/b)^3*cos_integral
(3*a/b + 3*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 3*a*cos(a/b)^2*
sin(a/b)*sin_integral(3*a/b + 3*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c
^3) + 9/4*b*arccos(c*x)*cos(a/b)*cos_integral(3*a/b + 3*arccos(c*x))/(b^3*c
^3*arccos(c*x) + a*b^2*c^3) - 1/4*b*arccos(c*x)*cos(a/b)*cos_integral(a/b +
 arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) + 3/4*b*arccos(c*x)*sin(a/b
)*sin_integral(3*a/b + 3*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 1
/4*b*arccos(c*x)*sin(a/b)*sin_integral(a/b + arccos(c*x))/(b^3*c^3*arccos(c
*x) + a*b^2*c^3) + 9/4*a*cos(a/b)*cos_integral(3*a/b + 3*arccos(c*x))/(b^3*
c^3*arccos(c*x) + a*b^2*c^3) - 1/4*a*cos(a/b)*cos_integral(a/b + arccos(c*x
))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) + 3/4*a*sin(a/b)*sin_integral(3*a/b +
3*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 1/4*a*sin(a/b)*sin_integ
ral(a/b + arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3)
```

3.164 $\int \frac{x}{\left(a+b\cos^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=91

$$-\frac{\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2\left(a+b\cos^{-1}(cx)\right)}{b}\right)}{b^2c^2}-\frac{\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2\left(a+b\cos^{-1}(cx)\right)}{b}\right)}{b^2c^2}+\frac{x\sqrt{1-c^2x^2}}{bc\left(a+b\cos^{-1}(cx)\right)}$$

```
[Out] (x*Sqrt[1 - c^2*x^2])/(b*c*(a + b*ArcCos[c*x])) - (Cos[(2*a)/b]*CosIntegral
[(2*(a + b*ArcCos[c*x]))/b])/(b^2*c^2) - (Sin[(2*a)/b]*SinIntegral[(2*(a +
b*ArcCos[c*x]))/b])/(b^2*c^2)
```

Rubi [A] time = 0.0968336, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4632, 3303, 3299, 3302}

$$-\frac{\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2a}{b}+2\cos^{-1}(cx)\right)}{b^2c^2}-\frac{\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\cos^{-1}(cx)\right)}{b^2c^2}+\frac{x\sqrt{1-c^2x^2}}{bc\left(a+b\cos^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*ArcCos[c*x])^2,x]
```

```
[Out] (x*Sqrt[1 - c^2*x^2])/(b*c*(a + b*ArcCos[c*x])) - (Cos[(2*a)/b]*CosIntegral
[(2*a)/b + 2*ArcCos[c*x]])/(b^2*c^2) - (Sin[(2*a)/b]*SinIntegral[(2*a)/b +
2*ArcCos[c*x]])/(b^2*c^2)
```

Rule 4632

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dis
t[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), Co
s[x]^(m - 1)*(m - (m + 1)*Cos[x]^2), x], x], x, ArcCos[c*x]], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+b\cos^{-1}(cx)\right)^2}\,dx = \frac{x\sqrt{1-c^2x^2}}{bc\left(a+b\cos^{-1}(cx)\right)} - \frac{\text{Subst}\left(\int \frac{\cos(2x)}{a+bx}\,dx, x, \cos^{-1}(cx)\right)}{bc^2}$$

$$= \frac{x\sqrt{1-c^2x^2}}{bc\left(a+b\cos^{-1}(cx)\right)} - \frac{\cos\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \cos^{-1}(cx)\right)}{bc^2} - \frac{\sin\left(\frac{2a}{b}\right)\text{Subst}\left(\int -\right.}{}$$

$$= \frac{x\sqrt{1-c^2x^2}}{bc\left(a+b\cos^{-1}(cx)\right)} - \frac{\cos\left(\frac{2a}{b}\right)\text{Ci}\left(\frac{2a}{b}+2\cos^{-1}(cx)\right)}{b^2c^2} - \frac{\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\cos^{-1}(cx)\right)}{b^2c^2}$$

Mathematica [A] time = 0.257513, size = 80, normalized size = 0.88

$$\frac{\frac{bcx\sqrt{1-c^2x^2}}{a+b\cos^{-1}(cx)} - \cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(2\left(\frac{a}{b}+\cos^{-1}(cx)\right)\right) - \sin\left(\frac{2a}{b}\right)\text{Si}\left(2\left(\frac{a}{b}+\cos^{-1}(cx)\right)\right)}{b^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*ArcCos[c*x])^2,x]
```

```
[Out] ((b*c*x*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]) - Cos[(2*a)/b]*CosIntegral[2
*(a/b + ArcCos[c*x])] - Sin[(2*a)/b]*SinIntegral[2*(a/b + ArcCos[c*x])])/(b
^2*c^2)
```

Maple [A] time = 0.052, size = 78, normalized size = 0.9

$$\frac{1}{c^2}\left(\frac{\sin(2\arccos(cx))}{(2a+2b\arccos(cx))b}-\frac{1}{b^2}\left(\mathrm{Si}\left(2\arccos(cx)+2\frac{a}{b}\right)\sin\left(2\frac{a}{b}\right)+\mathrm{Ci}\left(2\arccos(cx)+2\frac{a}{b}\right)\cos\left(2\frac{a}{b}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arccos(c*x))^2,x)
```

```
[Out] 1/c^2*(1/2*sin(2*arccos(c*x))/(a+b*arccos(c*x))/b-(Si(2*arccos(c*x)+2*a/b)*
sin(2*a/b)+Ci(2*arccos(c*x)+2*a/b)*cos(2*a/b))/b^2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x}{b^2\arccos(cx)^2+2ab\arccos(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(x/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{(a+b\operatorname{acos}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*acos(c*x))**2,x)
```

```
[Out] Integral(x/(a + b*acos(c*x))**2, x)
```

Giac [B] time = 1.21624, size = 436, normalized size = 4.79

$$-\frac{2\,b\arccos\left(cx\right)\cos\left(\frac{a}{b}\right)^{2}\operatorname{Ci}\left(\frac{2\,a}{b}+2\,\arccos\left(cx\right)\right)}{b^{3}c^{2}\arccos\left(cx\right)+ab^{2}c^{2}}-\frac{2\,b\arccos\left(cx\right)\cos\left(\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{2\,a}{b}+2\,\arccos\left(cx\right)\right)}{b^{3}c^{2}\arccos\left(cx\right)+ab^{2}c^{2}}-\frac{2\,a\cos\left(\frac{a}{b}\right.}{b^{3}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

```
[Out] -2*b*arccos(c*x)*cos(a/b)^2*cos_integral(2*a/b + 2*arccos(c*x))/(b^3*c^2*ar
ccos(c*x) + a*b^2*c^2) - 2*b*arccos(c*x)*cos(a/b)*sin(a/b)*sin_integral(2*a
/b + 2*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 2*a*cos(a/b)^2*cos_
integral(2*a/b + 2*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 2*a*cos
(a/b)*sin(a/b)*sin_integral(2*a/b + 2*arccos(c*x))/(b^3*c^2*arccos(c*x) + a
*b^2*c^2) + sqrt(-c^2*x^2 + 1)*b*c*x/(b^3*c^2*arccos(c*x) + a*b^2*c^2) + b*
arccos(c*x)*cos_integral(2*a/b + 2*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^
2*c^2) + a*cos_integral(2*a/b + 2*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^2
*c^2)
```

3.165 $\int \frac{1}{\left(a+b\cos^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=86

$$-\frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\cos^{-1}(cx)}{b}\right)}{b^2c}-\frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\cos^{-1}(cx)}{b}\right)}{b^2c}+\frac{\sqrt{1-c^2x^2}}{bc\left(a+b\cos^{-1}(cx)\right)}$$

```
[Out] Sqrt[1 - c^2*x^2]/(b*c*(a + b*ArcCos[c*x])) - (Cos[a/b]*CosIntegral[(a + b*
ArcCos[c*x])/b])/(b^2*c) - (Sin[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/(b
^2*c)
```

Rubi [A] time = 0.16794, antiderivative size = 82, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4622, 4724, 3303, 3299, 3302}

$$-\frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\cos^{-1}(cx)\right)}{b^2c}-\frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\cos^{-1}(cx)\right)}{b^2c}+\frac{\sqrt{1-c^2x^2}}{bc\left(a+b\cos^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])^(-2),x]
```

```
[Out] Sqrt[1 - c^2*x^2]/(b*c*(a + b*ArcCos[c*x])) - (Cos[a/b]*CosIntegral[a/b + A
rcCos[c*x]])/(b^2*c) - (Sin[a/b]*SinIntegral[a/b + ArcCos[c*x]])/(b^2*c)
```

Rule 4622

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(Sqrt[1 - c
^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1))
, Int[(x*(a + b*ArcCos[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && LtQ[n, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}}{bc\left(a+b\cos^{-1}(cx)\right)} + \frac{c\int \frac{x}{\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)}\,dx}{b}$$

$$= \frac{\sqrt{1-c^2x^2}}{bc\left(a+b\cos^{-1}(cx)\right)} - \frac{\text{Subst}\left(\int \frac{\cos(x)}{a+bx}\,dx, x, \cos^{-1}(cx)\right)}{bc}$$

$$= \frac{\sqrt{1-c^2x^2}}{bc\left(a+b\cos^{-1}(cx)\right)} - \frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cos^{-1}(cx)\right)}{bc} - \frac{\sin\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}-\right.}{a+b}\right.}{bc}$$

$$= \frac{\sqrt{1-c^2x^2}}{bc\left(a+b\cos^{-1}(cx)\right)} - \frac{\cos\left(\frac{a}{b}\right)\text{Ci}\left(\frac{a}{b}+\cos^{-1}(cx)\right)}{b^2c} - \frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\cos^{-1}(cx)\right)}{b^2c}$$

Mathematica [A] time = 0.141575, size = 72, normalized size = 0.84

$$\frac{\frac{b\sqrt{1-c^2x^2}}{a+b\cos^{-1}(cx)} - \cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\cos^{-1}(cx)\right) - \sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\cos^{-1}(cx)\right)}{b^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[c*x])^(-2),x]
```

```
[Out] ((b*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]) - Cos[a/b]*CosIntegral[a/b + Arc
Cos[c*x]] - Sin[a/b]*SinIntegral[a/b + ArcCos[c*x]])/(b^2*c)
```

Maple [A] time = 0.052, size = 74, normalized size = 0.9

$$\frac{1}{c}\left(\frac{1}{\left(a+b\arccos\left(cx\right)\right)b}\sqrt{-c^2x^2+1}-\frac{1}{b^2}\left(\mathrm{Si}\left(\arccos\left(cx\right)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)+\mathrm{Ci}\left(\arccos\left(cx\right)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(c*x))^2,x)
```

```
[Out] 1/c*((-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))/b-(Si(arccos(c*x)+a/b)*sin(a/b)+C
i(arccos(c*x)+a/b)*cos(a/b))/b^2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{b^2\arccos\left(cx\right)^2+2\,ab\arccos\left(cx\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\,\mathrm{acos}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(c*x))**2,x)
```

```
[Out] Integral((a + b*acos(c*x))**(-2), x)
```

Giac [B] time = 1.16975, size = 261, normalized size = 3.03

$$-\frac{b\arccos(cx)\cos\left(\frac{a}{b}\right)\mathrm{Ci}\left(\frac{a}{b}+\arccos(cx)\right)}{b^3c\arccos(cx)+ab^2c}-\frac{b\arccos(cx)\sin\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{b^3c\arccos(cx)+ab^2c}-\frac{a\cos\left(\frac{a}{b}\right)\mathrm{Ci}\left(\frac{a}{b}+\arccos(cx\right)}{b^3c\arccos(cx)+ab^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

```
[Out] -b*arccos(c*x)*cos(a/b)*cos_integral(a/b + arccos(c*x))/(b^3*c*arccos(c*x)
+ a*b^2*c) - b*arccos(c*x)*sin(a/b)*sin_integral(a/b + arccos(c*x))/(b^3*c*
arccos(c*x) + a*b^2*c) - a*cos(a/b)*cos_integral(a/b + arccos(c*x))/(b^3*c*
arccos(c*x) + a*b^2*c) - a*sin(a/b)*sin_integral(a/b + arccos(c*x))/(b^3*c*
arccos(c*x) + a*b^2*c) + sqrt(-c^2*x^2 + 1)*b/(b^3*c*arccos(c*x) + a*b^2*c)
```

3.166 $\int \frac{1}{x\left(a+b\cos^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{x\left(a+b\cos^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/(x*(a + b*ArcCos[c*x])^2), x]
```

Rubi [A] time = 0.0213703, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(a+b\cos^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + b*ArcCos[c*x])^2),x]
```

```
[Out] Defer[Int][1/(x*(a + b*ArcCos[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{x\left(a+b\cos^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x\left(a+b\cos^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 6.4151, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\cos^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + b*ArcCos[c*x])^2),x]
```

```
[Out] Integrate[1/(x*(a + b*ArcCos[c*x])^2), x]
```

Maple [A] time = 0.247, size = 0, normalized size = 0.

$$\int \frac{1}{x \left(a + b \arccos \left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*arccos(c*x))^2,x)
```

```
[Out] int(1/x/(a+b*arccos(c*x))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^{2} x \arccos \left(cx\right)^{2} + 2\,abx \arccos \left(cx\right) + a^{2} x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*x*arccos(c*x)^2 + 2*a*b*x*arccos(c*x) + a^2*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\,\mathrm{acos}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*acos(c*x))**2,x)
```

```
[Out] Integral(1/(x*(a + b*acos(c*x))**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\arccos\left(cx\right)+a\right)^{2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*arccos(c*x) + a)^2*x), x)
```

3.167 $\int \frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/(x^2*(a + b*ArcCos[c*x])^2), x]
```

Rubi [A] time = 0.0218532, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(a + b*ArcCos[c*x])^2),x]
```

```
[Out] Defer[Int][1/(x^2*(a + b*ArcCos[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)^2}\,dx = \int \frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 51.281, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(a + b*ArcCos[c*x])^2),x]
```

```
[Out] Integrate[1/(x^2*(a + b*ArcCos[c*x])^2), x]
```

Maple [A] time = 0.198, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(a + b \arccos (cx)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arccos(c*x))^2,x)
```

```
[Out] int(1/x^2/(a+b*arccos(c*x))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^2x^2 \arccos (cx)^2 + 2\,abx^2 \arccos (cx) + a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*x^2*arccos(c*x)^2 + 2*a*b*x^2*arccos(c*x) + a^2*x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a + b \operatorname{acos}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*acos(c*x))**2,x)
```

```
[Out] Integral(1/(x**2*(a + b*acos(c*x))**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccos(cx) + a)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*arccos(c*x) + a)^2*x^2), x)
```

3.168 $\int \frac{x^2}{\left(a+b\cos^{-1}(cx)\right)^3}\,dx$

Optimal. Leaf size=197

$$-\frac{\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\cos^{-1}(cx)}{b}\right)}{8b^3c^3}-\frac{9\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3\left(a+b\cos^{-1}(cx)\right)}{b}\right)}{8b^3c^3}+\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\cos^{-1}(cx)}{b}\right)}{8b^3c^3}+\frac{9\cos\left(\frac{3a}{b}\right)\text{S}}{8}$$

```
[Out] (x^2*Sqrt[1 - c^2*x^2])/(2*b*c*(a + b*ArcCos[c*x])^2) - x/(b^2*c^2*(a + b*A
rcCos[c*x])) + (3*x^3)/(2*b^2*(a + b*ArcCos[c*x])) - (CosIntegral[(a + b*Ar
cCos[c*x])/b]*Sin[a/b])/(8*b^3*c^3) - (9*CosIntegral[(3*(a + b*ArcCos[c*x])
)/b]*Sin[(3*a)/b])/(8*b^3*c^3) + (Cos[a/b]*SinIntegral[(a + b*ArcCos[c*x])/
b])/(8*b^3*c^3) + (9*Cos[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/(
8*b^3*c^3)
```

Rubi [A] time = 0.534519, antiderivative size = 246, normalized size of antiderivative = 1.25, number of steps used = 16, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4634, 4720, 4636, 4406, 3303, 3299, 3302, 4624}

$$-\frac{9\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\cos^{-1}(cx)\right)}{8b^3c^3}+\frac{\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\cos^{-1}(cx)}{b}\right)}{b^3c^3}-\frac{9\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3a}{b}+3\cos^{-1}(c\right.}{8b^3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*ArcCos[c*x])^3,x]
```

```
[Out] (x^2*Sqrt[1 - c^2*x^2])/(2*b*c*(a + b*ArcCos[c*x])^2) - x/(b^2*c^2*(a + b*A
rcCos[c*x])) + (3*x^3)/(2*b^2*(a + b*ArcCos[c*x])) - (9*CosIntegral[a/b + A
rcCos[c*x]]*Sin[a/b])/(8*b^3*c^3) + (CosIntegral[(a + b*ArcCos[c*x])/b]*Sin
[a/b])/(b^3*c^3) - (9*CosIntegral[(3*a)/b + 3*ArcCos[c*x]]*Sin[(3*a)/b])/(8
*b^3*c^3) + (9*Cos[a/b]*SinIntegral[a/b + ArcCos[c*x]])/(8*b^3*c^3) + (9*Co
s[(3*a)/b]*SinIntegral[(3*a)/b + 3*ArcCos[c*x]])/(8*b^3*c^3) - (Cos[a/b]*Si
nIntegral[(a + b*ArcCos[c*x])/b])/(b^3*c^3)
```

Rule 4634

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-D
ist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCos[c*x])^(n + 1))/Sq
```

```
rt[1 - c^2*x^2], x], x] + Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*ArcCo
s[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[
m, 0] && LtQ[n, -2]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4624

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Sub
st[Int[x^n*Sin[a/b - x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c,
 n}, x]
```

Rubi steps

$$\int \frac{x^2}{\left(a+b\cos^{-1}(cx)\right)^3}\,dx = \frac{x^2\sqrt{1-c^2x^2}}{2bc\left(a+b\cos^{-1}(cx)\right)^2} - \frac{\int \frac{x}{\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}\,dx}{bc} + \frac{(3c)\int \frac{x^3}{\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}\,dx}{2b}$$

$$= \frac{x^2\sqrt{1-c^2x^2}}{2bc\left(a+b\cos^{-1}(cx)\right)^2} - \frac{x}{b^2c^2\left(a+b\cos^{-1}(cx)\right)} + \frac{3x^3}{2b^2\left(a+b\cos^{-1}(cx)\right)} - \frac{9\int \frac{x^2}{a+b\cos^{-1}(cx)}\,dx}{2b^2}$$

$$= \frac{x^2\sqrt{1-c^2x^2}}{2bc\left(a+b\cos^{-1}(cx)\right)^2} - \frac{x}{b^2c^2\left(a+b\cos^{-1}(cx)\right)} + \frac{3x^3}{2b^2\left(a+b\cos^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\right.}{b}$$

$$= \frac{x^2\sqrt{1-c^2x^2}}{2bc\left(a+b\cos^{-1}(cx)\right)^2} - \frac{x}{b^2c^2\left(a+b\cos^{-1}(cx)\right)} + \frac{3x^3}{2b^2\left(a+b\cos^{-1}(cx)\right)} + \frac{9\,\text{Subst}\left(\int\left(\frac{\sin(x)}{4(a+bx)}\right.\right.}{}$$

$$= \frac{x^2\sqrt{1-c^2x^2}}{2bc\left(a+b\cos^{-1}(cx)\right)^2} - \frac{x}{b^2c^2\left(a+b\cos^{-1}(cx)\right)} + \frac{3x^3}{2b^2\left(a+b\cos^{-1}(cx)\right)} + \frac{\text{Ci}\left(\frac{a+b\cos^{-1}(cx)}{b}\right)\sin}{b^3c^3}$$

$$= \frac{x^2\sqrt{1-c^2x^2}}{2bc\left(a+b\cos^{-1}(cx)\right)^2} - \frac{x}{b^2c^2\left(a+b\cos^{-1}(cx)\right)} + \frac{3x^3}{2b^2\left(a+b\cos^{-1}(cx)\right)} + \frac{\text{Ci}\left(\frac{a+b\cos^{-1}(cx)}{b}\right)\sin}{b^3c^3}$$

$$= \frac{x^2\sqrt{1-c^2x^2}}{2bc\left(a+b\cos^{-1}(cx)\right)^2} - \frac{x}{b^2c^2\left(a+b\cos^{-1}(cx)\right)} + \frac{3x^3}{2b^2\left(a+b\cos^{-1}(cx)\right)} - \frac{9\text{Ci}\left(\frac{a}{b}+\cos^{-1}(cx)\right.}{8b^3c^3}$$

Mathematica [A] time = 0.455213, size = 169, normalized size = 0.86

$$\frac{\frac{4b^2x^2\sqrt{1-c^2x^2}}{c\left(a+b\cos^{-1}(cx)\right)^2} - \frac{\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\cos^{-1}(cx)\right)}{c^3} - \frac{9\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(3\left(\frac{a}{b}+\cos^{-1}(cx)\right)\right)}{c^3} + \frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\cos^{-1}(cx)\right)}{c^3} + \frac{9\cos\left(\frac{3a}{b}\right)\text{Si}\left(3\left(\frac{a}{b}+\cos^{-1}(\right.\right.}{c^3}}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*ArcCos[c*x])^3,x]
```

```
[Out] ((4*b^2*x^2*Sqrt[1 - c^2*x^2])/(c*(a + b*ArcCos[c*x])^2) - (8*b*x)/(c^2*(a
+ b*ArcCos[c*x])) + (12*b*x^3)/(a + b*ArcCos[c*x]) - (CosIntegral[a/b + Arc
Cos[c*x]]*Sin[a/b])/c^3 - (9*CosIntegral[3*(a/b + ArcCos[c*x])]*Sin[(3*a)/b
])/c^3 + (Cos[a/b]*SinIntegral[a/b + ArcCos[c*x]])/c^3 + (9*Cos[(3*a)/b]*Si
nIntegral[3*(a/b + ArcCos[c*x])])/c^3)/(8*b^3)
```

Maple [A] time = 0.06, size = 291, normalized size = 1.5

$$\frac{1}{c^3}\left(\frac{\sin(3\arccos(cx))}{8\,(a+b\arccos(cx))^2\,b}-\frac{3}{(8\,a+8\,b\arccos(cx))\,b^3}\left(3\arccos(cx)\sin\left(3\,\frac{a}{b}\right)\mathrm{Ci}\left(3\arccos(cx)+3\,\frac{a}{b}\right)b-3\arccos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arccos(c*x))^3,x)
```

```
[Out] 1/c^3*(1/8*sin(3*arccos(c*x))/(a+b*arccos(c*x))^2/b-3/8*(3*arccos(c*x)*sin(
3*a/b)*Ci(3*arccos(c*x)+3*a/b)*b-3*arccos(c*x)*Si(3*arccos(c*x)+3*a/b)*cos(
3*a/b)*b+3*sin(3*a/b)*Ci(3*arccos(c*x)+3*a/b)*a-3*Si(3*arccos(c*x)+3*a/b)*c
os(3*a/b)*a-cos(3*arccos(c*x))*b)/(a+b*arccos(c*x))/b^3+1/8*(-c^2*x^2+1)^(1
/2)/(a+b*arccos(c*x))^2/b+1/8*(arccos(c*x)*Si(arccos(c*x)+a/b)*cos(a/b)*b-a
rccos(c*x)*Ci(arccos(c*x)+a/b)*sin(a/b)*b+Si(arccos(c*x)+a/b)*cos(a/b)*a-Ci
(arccos(c*x)+a/b)*sin(a/b)*a+x*b*c)/(a+b*arccos(c*x))/b^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\,ac^2x^3+\sqrt{cx+1}\sqrt{-cx+1}bcx^2-2\,ax+\left(3\,bc^2x^3-2\,bx\right)\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)-\left(b^4c^2\arctan\left(\sqrt{cx+1}\sqrt{-cx\right.\right.}{2\left(b^4c^2\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)^2+2\,ab^3c^2\arctan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccos(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(3*a*c^2*x^3 + sqrt(c*x + 1)*sqrt(-c*x + 1)*b*c*x^2 - 2*a*x + (3*b*c^2*
x^3 - 2*b*x)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - 2*(b^4*c^2*arctan
2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b^3*c^2*arctan2(sqrt(c*x + 1)*
sqrt(-c*x + 1), c*x) + a^2*b^2*c^2)*integrate(1/2*(9*c^2*x^2 - 2)/(b^3*c^2*
arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b^2*c^2), x))/(b^4*c^2*arcta
n2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b^3*c^2*arctan2(sqrt(c*x + 1)
```

```
*sqrt(-c*x + 1), c*x) + a^2*b^2*c^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{b^3 \arccos{(cx)}^3 + 3\,ab^2 \arccos{(cx)}^2 + 3\,a^2b \arccos{(cx)} + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccos(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(x^2/(b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*
x) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a + b \operatorname{acos}{(cx)})^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*acos(c*x))**3,x)
```

```
[Out] Integral(x**2/(a + b*acos(c*x))**3, x)
```

Giac [B] time = 1.44992, size = 1997, normalized size = 10.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccos(c*x))^3,x, algorithm="giac")
```

```
[Out] 3/2*b^2*c^3*x^3*arccos(c*x)/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x
) + a^2*b^3*c^3) + 3/2*a*b*c^3*x^3/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arc
cos(c*x) + a^2*b^3*c^3) - 9/2*b^2*arccos(c*x)^2*cos(a/b)^2*cos_integral(3*a
/b + 3*arccos(c*x))*sin(a/b)/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*
```

```
x) + a^2*b^3*c^3) + 9/2*b^2*arccos(c*x)^2*cos(a/b)^3*sin_integral(3*a/b + 3
*arccos(c*x))/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^
3) - 9*a*b*arccos(c*x)*cos(a/b)^2*cos_integral(3*a/b + 3*arccos(c*x))*sin(a
/b)/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3) + 9*a*b
*arccos(c*x)*cos(a/b)^3*sin_integral(3*a/b + 3*arccos(c*x))/(b^5*c^3*arccos
(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3) + 1/2*sqrt(-c^2*x^2 + 1)*b
^2*c^2*x^2/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3)
+ 9/8*b^2*arccos(c*x)^2*cos_integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b^5*c
^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3) - 9/2*a^2*cos(a/b
)^2*cos_integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b^5*c^3*arccos(c*x)^2 + 2
*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3) - 1/8*b^2*arccos(c*x)^2*cos_integral(
a/b + arccos(c*x))*sin(a/b)/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x
) + a^2*b^3*c^3) - 27/8*b^2*arccos(c*x)^2*cos(a/b)*sin_integral(3*a/b + 3*a
rccos(c*x))/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3)
 + 9/2*a^2*cos(a/b)^3*sin_integral(3*a/b + 3*arccos(c*x))/(b^5*c^3*arccos(c
*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3) + 1/8*b^2*arccos(c*x)^2*cos(
a/b)*sin_integral(a/b + arccos(c*x))/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*a
rccos(c*x) + a^2*b^3*c^3) - b^2*c*x*arccos(c*x)/(b^5*c^3*arccos(c*x)^2 + 2*
a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3) + 9/4*a*b*arccos(c*x)*cos_integral(3*a
/b + 3*arccos(c*x))*sin(a/b)/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*
x) + a^2*b^3*c^3) - 1/4*a*b*arccos(c*x)*cos_integral(a/b + arccos(c*x))*sin
(a/b)/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3) - 27/
4*a*b*arccos(c*x)*cos(a/b)*sin_integral(3*a/b + 3*arccos(c*x))/(b^5*c^3*arc
cos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3) + 1/4*a*b*arccos(c*x)*c
os(a/b)*sin_integral(a/b + arccos(c*x))/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^
3*arccos(c*x) + a^2*b^3*c^3) - a*b*c*x/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3
*arccos(c*x) + a^2*b^3*c^3) + 9/8*a^2*cos_integral(3*a/b + 3*arccos(c*x))*s
in(a/b)/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3) - 1
/8*a^2*cos_integral(a/b + arccos(c*x))*sin(a/b)/(b^5*c^3*arccos(c*x)^2 + 2*
a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3) - 27/8*a^2*cos(a/b)*sin_integral(3*a/b
 + 3*arccos(c*x))/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^
3*c^3) + 1/8*a^2*cos(a/b)*sin_integral(a/b + arccos(c*x))/(b^5*c^3*arccos(c
*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3)
```

3.169 $\int \frac{x}{\left(a+b\cos^{-1}(cx)\right)^3}\,dx$

Optimal. Leaf size=130

$$-\frac{\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2\left(a+b\cos^{-1}(cx)\right)}{b}\right)}{b^3c^2}+\frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2\left(a+b\cos^{-1}(cx)\right)}{b}\right)}{b^3c^2}-\frac{1}{2b^2c^2\left(a+b\cos^{-1}(cx)\right)}+\frac{x^2}{b^2\left(a+b\cos^{-1}(cx)\right)}+$$

```
[Out] (x*Sqrt[1 - c^2*x^2])/(2*b*c*(a + b*ArcCos[c*x])^2) - 1/(2*b^2*c^2*(a + b*A
rcCos[c*x])) + x^2/(b^2*(a + b*ArcCos[c*x])) - (CosIntegral[(2*(a + b*ArcCo
s[c*x]))/b]*Sin[(2*a)/b])/(b^3*c^2) + (Cos[(2*a)/b]*SinIntegral[(2*(a + b*A
rcCos[c*x]))/b])/(b^3*c^2)
```

Rubi [A] time = 0.311036, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {4634, 4720, 4636, 4406, 12, 3303, 3299, 3302, 4642}

$$-\frac{\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2a}{b}+2\cos^{-1}(cx)\right)}{b^3c^2}+\frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\cos^{-1}(cx)\right)}{b^3c^2}-\frac{1}{2b^2c^2\left(a+b\cos^{-1}(cx)\right)}+\frac{x^2}{b^2\left(a+b\cos^{-1}(c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*ArcCos[c*x])^3,x]
```

```
[Out] (x*Sqrt[1 - c^2*x^2])/(2*b*c*(a + b*ArcCos[c*x])^2) - 1/(2*b^2*c^2*(a + b*A
rcCos[c*x])) + x^2/(b^2*(a + b*ArcCos[c*x])) - (CosIntegral[(2*a)/b + 2*Arc
Cos[c*x]]*Sin[(2*a)/b])/(b^3*c^2) + (Cos[(2*a)/b]*SinIntegral[(2*a)/b + 2*A
rcCos[c*x]])/(b^3*c^2)
```

Rule 4634

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-D
ist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCos[c*x])^(n + 1))/Sq
rt[1 - c^2*x^2], x], x] + Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*ArcCo
s[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[
m, 0] && LtQ[n, -2]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+b\cos^{-1}(cx)\right)^3}\,dx = \frac{x\sqrt{1-c^2x^2}}{2bc\left(a+b\cos^{-1}(cx)\right)^2} - \frac{\int \frac{1}{\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}\,dx}{2bc} + \frac{c\int \frac{x^2}{\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}\,dx}{b}$$

$$= \frac{x\sqrt{1-c^2x^2}}{2bc\left(a+b\cos^{-1}(cx)\right)^2} - \frac{1}{2b^2c^2\left(a+b\cos^{-1}(cx)\right)} + \frac{x^2}{b^2\left(a+b\cos^{-1}(cx)\right)} - \frac{2\int \frac{x}{a+b\cos^{-1}(cx)}\,dx}{b^2}$$

$$= \frac{x\sqrt{1-c^2x^2}}{2bc\left(a+b\cos^{-1}(cx)\right)^2} - \frac{1}{2b^2c^2\left(a+b\cos^{-1}(cx)\right)} + \frac{x^2}{b^2\left(a+b\cos^{-1}(cx)\right)} + \frac{2\,\mathrm{Subst}\left(\int \frac{\cos(x)\sin}{a+bx}\right.}{b^2}$$

$$= \frac{x\sqrt{1-c^2x^2}}{2bc\left(a+b\cos^{-1}(cx)\right)^2} - \frac{1}{2b^2c^2\left(a+b\cos^{-1}(cx)\right)} + \frac{x^2}{b^2\left(a+b\cos^{-1}(cx)\right)} + \frac{2\,\mathrm{Subst}\left(\int \frac{\sin(2x)}{2(a+bx)}\right.}{b^2c}$$

$$= \frac{x\sqrt{1-c^2x^2}}{2bc\left(a+b\cos^{-1}(cx)\right)^2} - \frac{1}{2b^2c^2\left(a+b\cos^{-1}(cx)\right)} + \frac{x^2}{b^2\left(a+b\cos^{-1}(cx)\right)} + \frac{\mathrm{Subst}\left(\int \frac{\sin(2x)}{a+bx}\,dx\right.}{b^2c^2}$$

$$= \frac{x\sqrt{1-c^2x^2}}{2bc\left(a+b\cos^{-1}(cx)\right)^2} - \frac{1}{2b^2c^2\left(a+b\cos^{-1}(cx)\right)} + \frac{x^2}{b^2\left(a+b\cos^{-1}(cx)\right)} + \frac{\cos\left(\frac{2a}{b}\right)\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{x\sqrt{1-c^2x^2}}{2bc\left(a+b\cos^{-1}(cx)\right)^2} - \frac{1}{2b^2c^2\left(a+b\cos^{-1}(cx)\right)} + \frac{x^2}{b^2\left(a+b\cos^{-1}(cx)\right)} - \frac{\mathrm{Ci}\left(\frac{2a}{b}+2\cos^{-1}(cx\right.}{b^3c^2}$$

Mathematica [A] time = 0.278308, size = 107, normalized size = 0.82

$$\frac{\frac{b^2cx\sqrt{1-c^2x^2}}{\left(a+b\cos^{-1}(cx)\right)^2} + \frac{b\left(2c^2x^2-1\right)}{a+b\cos^{-1}(cx)} - 2\sin\left(\frac{2a}{b}\right)\mathrm{CosIntegral}\left(2\left(\frac{a}{b}+\cos^{-1}(cx)\right)\right) + 2\cos\left(\frac{2a}{b}\right)\mathrm{Si}\left(2\left(\frac{a}{b}+\cos^{-1}(cx)\right)\right)}{2b^3c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*ArcCos[c*x])^3,x]
```

```
[Out] ((b^2*c*x*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x])^2 + (b*(-1 + 2*c^2*x^2))/(
a + b*ArcCos[c*x]) - 2*CosIntegral[2*(a/b + ArcCos[c*x])]*Sin[(2*a)/b] + 2*
Cos[(2*a)/b]*SinIntegral[2*(a/b + ArcCos[c*x])])/(2*b^3*c^2)
```

Maple [A] time = 0.053, size = 157, normalized size = 1.2

$$\frac{1}{c^2}\left(\frac{\sin(2\arccos(cx))}{4\,(a+b\arccos(cx))^2\,b}+\frac{1}{(2\,a+2\,b\arccos(cx))\,b^3}\left(2\arccos(cx)\,\mathrm{Si}\left(2\arccos(cx)+2\,\frac{a}{b}\right)\cos\left(2\,\frac{a}{b}\right)b-2\arccos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arccos(c*x))^3,x)
```

```
[Out] 1/c^2*(1/4*sin(2*arccos(c*x))/(a+b*arccos(c*x))^2/b+1/2*(2*arccos(c*x)*Si(2
*arccos(c*x)+2*a/b)*cos(2*a/b)*b-2*arccos(c*x)*Ci(2*arccos(c*x)+2*a/b)*sin(
2*a/b)*b+2*Si(2*arccos(c*x)+2*a/b)*cos(2*a/b)*a-2*Ci(2*arccos(c*x)+2*a/b)*s
in(2*a/b)*a+cos(2*arccos(c*x))*b)/(a+b*arccos(c*x))/b^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,ac^2x^2+\sqrt{cx+1}\sqrt{-cx+1}bcx+\left(2\,bc^2x^2-b\right)\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)-a-\frac{4\left(b^4c^2\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)^2+2\,ab^3c^2\,\mathrm{a}\right.}{}}{2\left(b^4c^2\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)^2+2\,ab^3c^2\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccos(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*c^2*x^2 + sqrt(c*x + 1)*sqrt(-c*x + 1)*b*c*x + (2*b*c^2*x^2 - b)*a
rctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - 4*(b^4*c^2*arctan2(sqrt(c*x + 1
)*sqrt(-c*x + 1), c*x)^2 + 2*a*b^3*c^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1)
, c*x) + a^2*b^2*c^2)*integrate(x/(b^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1)
, c*x) + a*b^2), x) - a)/(b^4*c^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x
)^2 + 2*a*b^3*c^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a^2*b^2*c^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{b^3 \arccos{(cx)}^3 + 3\,ab^2 \arccos{(cx)}^2 + 3\,a^2b \arccos{(cx)} + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccos(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(x/(b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x)
 + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a + b\,\text{acos}\,(cx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*acos(c*x))**3,x)
```

```
[Out] Integral(x/(a + b*acos(c*x))**3, x)
```

Giac [B] time = 1.33554, size = 1161, normalized size = 8.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccos(c*x))^3,x, algorithm="giac")
```

```
[Out] b^2*c^2*x^2*arccos(c*x)/(b^5*c^2*arccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) +
a^2*b^3*c^2) - 2*b^2*arccos(c*x)^2*cos(a/b)*cos_integral(2*a/b + 2*arccos(c
*x))*sin(a/b)/(b^5*c^2*arccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*b^3*c^
2) + 2*b^2*arccos(c*x)^2*cos(a/b)^2*sin_integral(2*a/b + 2*arccos(c*x))/(b^
5*c^2*arccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*b^3*c^2) + a*b*c^2*x^2/
(b^5*c^2*arccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*b^3*c^2) - 4*a*b*arc
cos(c*x)*cos(a/b)*cos_integral(2*a/b + 2*arccos(c*x))*sin(a/b)/(b^5*c^2*arc
cos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*b^3*c^2) + 4*a*b*arccos(c*x)*cos
```

```
(a/b)^2*sin_integral(2*a/b + 2*arccos(c*x))/(b^5*c^2*arccos(c*x)^2 + 2*a*b^
4*c^2*arccos(c*x) + a^2*b^3*c^2) - 2*a^2*cos(a/b)*cos_integral(2*a/b + 2*ar
ccos(c*x))*sin(a/b)/(b^5*c^2*arccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*
b^3*c^2) - b^2*arccos(c*x)^2*sin_integral(2*a/b + 2*arccos(c*x))/(b^5*c^2*a
rccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*b^3*c^2) + 2*a^2*cos(a/b)^2*si
n_integral(2*a/b + 2*arccos(c*x))/(b^5*c^2*arccos(c*x)^2 + 2*a*b^4*c^2*arcc
os(c*x) + a^2*b^3*c^2) + 1/2*sqrt(-c^2*x^2 + 1)*b^2*c*x/(b^5*c^2*arccos(c*x
)^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*b^3*c^2) - 2*a*b*arccos(c*x)*sin_integr
al(2*a/b + 2*arccos(c*x))/(b^5*c^2*arccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x)
+ a^2*b^3*c^2) - 1/2*b^2*arccos(c*x)/(b^5*c^2*arccos(c*x)^2 + 2*a*b^4*c^2*a
rccos(c*x) + a^2*b^3*c^2) - a^2*sin_integral(2*a/b + 2*arccos(c*x))/(b^5*c^
2*arccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*b^3*c^2) - 1/2*a*b/(b^5*c^2
*arccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*b^3*c^2)
```

3.170 $\int \frac{1}{\left(a+b\cos^{-1}(cx)\right)^3}\,dx$

Optimal. Leaf size=111

$$-\frac{\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\cos^{-1}(cx)}{b}\right)}{2b^3c}+\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\cos^{-1}(cx)}{b}\right)}{2b^3c}+\frac{x}{2b^2\left(a+b\cos^{-1}(cx)\right)}+\frac{\sqrt{1-c^2x^2}}{2bc\left(a+b\cos^{-1}(cx)\right)^2}$$

```
[Out] Sqrt[1 - c^2*x^2]/(2*b*c*(a + b*ArcCos[c*x])^2) + x/(2*b^2*(a + b*ArcCos[c*
x])) - (CosIntegral[(a + b*ArcCos[c*x])/b]*Sin[a/b])/(2*b^3*c) + (Cos[a/b]*
SinIntegral[(a + b*ArcCos[c*x])/b])/(2*b^3*c)
```

Rubi [A] time = 0.165866, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4622, 4720, 4624, 3303, 3299, 3302}

$$-\frac{\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\cos^{-1}(cx)}{b}\right)}{2b^3c}+\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\cos^{-1}(cx)}{b}\right)}{2b^3c}+\frac{x}{2b^2\left(a+b\cos^{-1}(cx)\right)}+\frac{\sqrt{1-c^2x^2}}{2bc\left(a+b\cos^{-1}(cx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])^(-3),x]
```



```
[Out] Sqrt[1 - c^2*x^2]/(2*b*c*(a + b*ArcCos[c*x])^2) + x/(2*b^2*(a + b*ArcCos[c*
x])) - (CosIntegral[(a + b*ArcCos[c*x])/b]*Sin[a/b])/(2*b^3*c) + (Cos[a/b]*
SinIntegral[(a + b*ArcCos[c*x])/b])/(2*b^3*c)
```


Rule 4622

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(Sqrt[1 - c
^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1))
, Int[(x*(a + b*ArcCos[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && LtQ[n, -1]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
```

```
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4624

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Sub
st[Int[x^n*Sin[a/b - x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c,
 n}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^{-1}(cx)\right)^3}\,dx = \frac{\sqrt{1-c^2x^2}}{2bc\left(a+b\cos^{-1}(cx)\right)^2} + \frac{c\int \frac{x}{\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}\,dx}{2b}$$

$$= \frac{\sqrt{1-c^2x^2}}{2bc\left(a+b\cos^{-1}(cx)\right)^2} + \frac{x}{2b^2\left(a+b\cos^{-1}(cx)\right)} - \frac{\int \frac{1}{a+b\cos^{-1}(cx)}\,dx}{2b^2}$$

$$= \frac{\sqrt{1-c^2x^2}}{2bc\left(a+b\cos^{-1}(cx)\right)^2} + \frac{x}{2b^2\left(a+b\cos^{-1}(cx)\right)} - \frac{\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\cos^{-1}(cx)\right)}{2b^3c}$$

$$= \frac{\sqrt{1-c^2x^2}}{2bc\left(a+b\cos^{-1}(cx)\right)^2} + \frac{x}{2b^2\left(a+b\cos^{-1}(cx)\right)} + \frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\cos^{-1}(cx\right.}{2b^3c}$$

$$= \frac{\sqrt{1-c^2x^2}}{2bc\left(a+b\cos^{-1}(cx)\right)^2} + \frac{x}{2b^2\left(a+b\cos^{-1}(cx)\right)} - \frac{\text{Ci}\left(\frac{a+b\cos^{-1}(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{2b^3c} + \frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\,\text{co}}{b}\right.}{2b^3c}$$

Mathematica [A] time = 0.233061, size = 89, normalized size = 0.8

$$\frac{\frac{b\left(acx+b\sqrt{1-c^2x^2}+bcx\cos^{-1}(cx)\right)}{\left(a+b\cos^{-1}(cx)\right)^2} - \sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\cos^{-1}(cx)\right) + \cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\cos^{-1}(cx)\right)}{2b^3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[c*x])^(-3),x]
```

```
[Out] ((b*(a*c*x + b*Sqrt[1 - c^2*x^2] + b*c*x*ArcCos[c*x]))/(a + b*ArcCos[c*x])^
2 - CosIntegral[a/b + ArcCos[c*x]]*Sin[a/b] + Cos[a/b]*SinIntegral[a/b + Ar
cCos[c*x]])/(2*b^3*c)
```

Maple [A] time = 0.058, size = 139, normalized size = 1.3

$$\frac{1}{c}\left(\frac{1}{2\left(a+b\arccos(cx)\right)^2 b}\sqrt{-c^2x^2+1} + \frac{1}{\left(2a+2b\arccos(cx)\right)b^3}\left(\arccos(cx)\,\text{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)b - \arccos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(c*x))^3,x)
```

```
[Out] 1/c*(1/2*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2/b+1/2*(arccos(c*x)*Si(arcco
s(c*x)+a/b)*cos(a/b)*b-arccos(c*x)*Ci(arccos(c*x)+a/b)*sin(a/b)*b+Si(arccos
(c*x)+a/b)*cos(a/b)*a-Ci(arccos(c*x)+a/b)*sin(a/b)*a+x*b*c)/(a+b*arccos(c*x
))/b^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bcx\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)+acx+\sqrt{cx+1}\sqrt{-cx+1}b-\left(b^4c\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)^2+2\,ab^3c\arctan\left(\right.\right.}{2\left(b^4c\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)^2+2\,ab^3c\arctan\left(\sqrt{cx+1}\sqrt{-cx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*c*x + sqrt(c*x +
1)*sqrt(-c*x + 1)*b - 2*(b^4*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2
 + 2*a*b^3*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a^2*b^2*c)*integr
ate(1/2/(b^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b^2), x))/(b^4*
c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b^3*c*arctan2(sqrt(c*x
 + 1)*sqrt(-c*x + 1), c*x) + a^2*b^2*c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^3\arccos\left(cx\right)^3+3\,ab^2\arccos\left(cx\right)^2+3\,a^2b\arccos\left(cx\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x)
 + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + b\operatorname{acos}\left(cx\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(c*x))**3,x)
```

```
[Out] Integral((a + b*acos(c*x))**(-3), x)
```

Giac [B] time = 1.17704, size = 649, normalized size = 5.85

$$-\frac{b^{2}\arccos\left(cx\right)^{2}\operatorname{Ci}\left(\frac{a}{b}+\arccos\left(cx\right)\right)\sin\left(\frac{a}{b}\right)}{2\left(b^{5}c\arccos\left(cx\right)^{2}+2\,ab^{4}c\arccos\left(cx\right)+a^{2}b^{3}c\right)}+\frac{b^{2}\arccos\left(cx\right)^{2}\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos\left(cx\right)\right)}{2\left(b^{5}c\arccos\left(cx\right)^{2}+2\,ab^{4}c\arccos\left(cx\right)+a^{2}b^{3}c\right)}+\frac{}{2\left(b^{5}c\arccos(c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(c*x))^3,x, algorithm="giac")
```

```
[Out] -1/2*b^2*arccos(c*x)^2*cos_integral(a/b + arccos(c*x))*sin(a/b)/(b^5*c*arcc
os(c*x)^2 + 2*a*b^4*c*arccos(c*x) + a^2*b^3*c) + 1/2*b^2*arccos(c*x)^2*cos(
a/b)*sin_integral(a/b + arccos(c*x))/(b^5*c*arccos(c*x)^2 + 2*a*b^4*c*arcco
s(c*x) + a^2*b^3*c) + 1/2*b^2*c*x*arccos(c*x)/(b^5*c*arccos(c*x)^2 + 2*a*b^
4*c*arccos(c*x) + a^2*b^3*c) - a*b*arccos(c*x)*cos_integral(a/b + arccos(c*
x))*sin(a/b)/(b^5*c*arccos(c*x)^2 + 2*a*b^4*c*arccos(c*x) + a^2*b^3*c) + a*
b*arccos(c*x)*cos(a/b)*sin_integral(a/b + arccos(c*x))/(b^5*c*arccos(c*x)^2
 + 2*a*b^4*c*arccos(c*x) + a^2*b^3*c) + 1/2*a*b*c*x/(b^5*c*arccos(c*x)^2 +
2*a*b^4*c*arccos(c*x) + a^2*b^3*c) - 1/2*a^2*cos_integral(a/b + arccos(c*x)
)*sin(a/b)/(b^5*c*arccos(c*x)^2 + 2*a*b^4*c*arccos(c*x) + a^2*b^3*c) + 1/2*
a^2*cos(a/b)*sin_integral(a/b + arccos(c*x))/(b^5*c*arccos(c*x)^2 + 2*a*b^4
*c*arccos(c*x) + a^2*b^3*c) + 1/2*sqrt(-c^2*x^2 + 1)*b^2/(b^5*c*arccos(c*x)
^2 + 2*a*b^4*c*arccos(c*x) + a^2*b^3*c)
```

3.171 $\int \frac{1}{x(a+b\cos^{-1}(cx))^3}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{x\left(a+b\cos^{-1}(cx)\right)^3},x\right)$$

```
[Out] Unintegrable[1/(x*(a + b*ArcCos[c*x])^3), x]
```

Rubi [A] time = 0.0226184, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(a+b\cos^{-1}(cx)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + b*ArcCos[c*x])^3),x]
```

```
[Out] Defer[Int][1/(x*(a + b*ArcCos[c*x])^3), x]
```

Rubi steps

$$\int \frac{1}{x\left(a+b\cos^{-1}(cx)\right)^3}\,dx=\int \frac{1}{x\left(a+b\cos^{-1}(cx)\right)^3}\,dx$$

Mathematica [A] time = 3.37358, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\cos^{-1}(cx)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + b*ArcCos[c*x])^3),x]
```

```
[Out] Integrate[1/(x*(a + b*ArcCos[c*x])^3), x]
```

Maple [A] time = 0.317, size = 0, normalized size = 0.

$$\int \frac{1}{x \left(a + b \arccos \left(cx\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*arccos(c*x))^3,x)
```

```
[Out] int(1/x/(a+b*arccos(c*x))^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{cx+1}\sqrt{-cx+1}bcx + b\arctan\left(\sqrt{cx+1}\sqrt{-cx+1}, cx\right) + a + \frac{2\left(b^4c^2x^2\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)^2 + 2\,ab^3c^2x^2\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)\right.}{b^2c^2}}{2\left(b^4c^2x^2\arctan\left(\sqrt{cx+1}\sqrt{-cx+1}, cx\right)^2 + 2\,ab^3c^2x^2\arctan\left(\sqrt{cx+1}\sqrt{-cx+1}, cx\right) + a^2b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccos(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(c*x + 1)*sqrt(-c*x + 1)*b*c*x + b*arctan2(sqrt(c*x + 1)*sqrt(-c*x
 + 1), c*x) + 2*(b^4*c^2*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 +
 2*a*b^3*c^2*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a^2*b^2*c^2*x
^2)*integrate(1/(b^3*c^2*x^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a
*b^2*c^2*x^3), x) + a)/(b^4*c^2*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c
*x)^2 + 2*a*b^3*c^2*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a^2*b^
2*c^2*x^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{1}{b^{3} x \arccos \left(cx\right)^{3} + 3\, a b^{2} x \arccos \left(cx\right)^{2} + 3\, a^{2} b x \arccos \left(cx\right) + a^{3} x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccos(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*x*arccos(c*x)^3 + 3*a*b^2*x*arccos(c*x)^2 + 3*a^2*b*x*arcco
s(c*x) + a^3*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\,\mathrm{acos}\,(cx)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*acos(c*x))**3,x)
```

```
[Out] Integral(1/(x*(a + b*acos(c*x))**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\arccos\,(cx)+a\right)^{3}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccos(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*arccos(c*x) + a)^3*x), x)
```

3.172 $\int \frac{1}{x^2(a+b\cos^{-1}(cx))^3}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)^3},x\right)$$

```
[Out] Unintegrable[1/(x^2*(a + b*ArcCos[c*x])^3), x]
```

Rubi [A] time = 0.0238722, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(a + b*ArcCos[c*x])^3),x]
```

```
[Out] Defer[Int][1/(x^2*(a + b*ArcCos[c*x])^3), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)^3}\,dx=\int \frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)^3}\,dx$$

Mathematica [A] time = 31.5442, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(a + b*ArcCos[c*x])^3),x]
```

```
[Out] Integrate[1/(x^2*(a + b*ArcCos[c*x])^3), x]
```

Maple [A] time = 0.51, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\arccos\left(cx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arccos(c*x))^3,x)
```

```
[Out] int(1/x^2/(a+b*arccos(c*x))^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{ac^2x^2-\sqrt{cx+1}\sqrt{-cx+1}bcx+\left(bc^2x^2-2\,b\right)\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)+\left(b^4c^2x^3\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)^2\right.}{2\left(b^4c^2x^3\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)^2+2\,ab^3c^2x^3\,\mathrm{arc}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a*c^2*x^2 - sqrt(c*x + 1)*sqrt(-c*x + 1)*b*c*x + (b*c^2*x^2 - 2*b)*ar
ctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + 2*(b^4*c^2*x^3*arctan2(sqrt(c*x
+ 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b^3*c^2*x^3*arctan2(sqrt(c*x + 1)*sqrt(-c
*x + 1), c*x) + a^2*b^2*c^2*x^3)*integrate(1/2*(c^2*x^2 - 6)/(b^3*c^2*x^4*a
rctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b^2*c^2*x^4), x) - 2*a)/(b^4*
c^2*x^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b^3*c^2*x^3*arct
an2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a^2*b^2*c^2*x^3)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{b^3x^2\arccos\left(cx\right)^3+3\,ab^2x^2\arccos\left(cx\right)^2+3\,a^2bx^2\arccos\left(cx\right)+a^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*x^2*arccos(c*x)^3 + 3*a*b^2*x^2*arccos(c*x)^2 + 3*a^2*b*x^2
*arccos(c*x) + a^3*x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(a + b \operatorname{acos}\left(cx\right)\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*acos(c*x))**3,x)
```

```
[Out] Integral(1/(x**2*(a + b*acos(c*x))**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \arccos\left(cx\right) + a\right)^3 x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*arccos(c*x) + a)^3*x^2), x)
```

3.173 $\int x^2\sqrt{a+b\cos^{-1}(cx)}\,dx$

Optimal. Leaf size=242

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{4c^3}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{b}\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{12c^3}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\sin\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+}}{}\right.}{4c^3}$$

```
[Out] (x^3*Sqrt[a + b*ArcCos[c*x]])/3 - (Sqrt[b]*Sqrt[Pi/2]*Cos[a/b]*FresnelC[(Sq
rt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(4*c^3) - (Sqrt[b]*Sqrt[Pi/6]*C
os[(3*a)/b]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(12*c^3
) - (Sqrt[b]*Sqrt[Pi/2]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[
b]]*Sin[a/b])/(4*c^3) - (Sqrt[b]*Sqrt[Pi/6]*FresnelS[(Sqrt[6/Pi]*Sqrt[a + b
*ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/(12*c^3)
```

Rubi [A] time = 0.702423, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4630, 4724, 3312, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{4c^3}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{b}\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{12c^3}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\sin\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+}}{}\right.}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*ArcCos[c*x]],x]
```

```
[Out] (x^3*Sqrt[a + b*ArcCos[c*x]])/3 - (Sqrt[b]*Sqrt[Pi/2]*Cos[a/b]*FresnelC[(Sq
rt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(4*c^3) - (Sqrt[b]*Sqrt[Pi/6]*C
os[(3*a)/b]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(12*c^3
) - (Sqrt[b]*Sqrt[Pi/2]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[
b]]*Sin[a/b])/(4*c^3) - (Sqrt[b]*Sqrt[Pi/6]*FresnelS[(Sqrt[6/Pi]*Sqrt[a + b
*ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/(12*c^3)
```

Rule 4630

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
x^(m + 1)*(a + b*ArcCos[c*x])^n)/(m + 1), x] + Dist[(b*c*n)/(m + 1), Int[(x
^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int x^2\sqrt{a+b\cos^{-1}(cx)}\,dx = \frac{1}{3}x^3\sqrt{a+b\cos^{-1}(cx)} + \frac{1}{6}(bc)\int \frac{x^3}{\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}\,dx$$

$$= \frac{1}{3}x^3\sqrt{a+b\cos^{-1}(cx)} - \frac{b\,\mathrm{Subst}\left(\int \frac{\cos^3(x)}{\sqrt{a+bx}}\,dx, x, \cos^{-1}(cx)\right)}{6c^3}$$

$$= \frac{1}{3}x^3\sqrt{a+b\cos^{-1}(cx)} - \frac{b\,\mathrm{Subst}\left(\int \left(\frac{3\cos(x)}{4\sqrt{a+bx}} + \frac{\cos(3x)}{4\sqrt{a+bx}}\right) dx, x, \cos^{-1}(cx)\right)}{6c^3}$$

$$= \frac{1}{3}x^3\sqrt{a+b\cos^{-1}(cx)} - \frac{b\,\mathrm{Subst}\left(\int \frac{\cos(3x)}{\sqrt{a+bx}}\,dx, x, \cos^{-1}(cx)\right)}{24c^3} - \frac{b\,\mathrm{Subst}\left(\int \frac{\cos(x)}{\sqrt{a+bx}}\,dx, x, \cos^{-1}\right.}{8c^3}$$

$$= \frac{1}{3}x^3\sqrt{a+b\cos^{-1}(cx)} - \frac{\left(b\cos\left(\frac{a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\cos\left(\frac{a}{b}+x\right)}{\sqrt{a+bx}}\,dx, x, \cos^{-1}(cx)\right)}{8c^3} - \frac{\left(b\cos\left(\frac{3a}{b}\right)\right)\mathrm{Subs}}{}$$

$$= \frac{1}{3}x^3\sqrt{a+b\cos^{-1}(cx)} - \frac{\cos\left(\frac{a}{b}\right)\mathrm{Subst}\left(\int \cos\left(\frac{x^2}{b}\right) dx, x, \sqrt{a+b\cos^{-1}(cx)}\right)}{4c^3} - \frac{\cos\left(\frac{3a}{b}\right)\mathrm{Sub}}{}$$

$$= \frac{1}{3}x^3\sqrt{a+b\cos^{-1}(cx)} - \frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)C\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{4c^3} - \frac{\sqrt{b}\sqrt{\frac{\pi}{6}}\cos\left(\frac{3a}{b}\right)C\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\,\mathrm{co}}}{\sqrt{b}}\right.}{12c^3}$$

Mathematica [C] time = 0.516004, size = 243, normalized size = 1.

$$\frac{e^{-\frac{3ia}{b}}\sqrt{a+b\cos^{-1}(cx)}\left(9e^{\frac{2ia}{b}}\sqrt{\frac{i\left(a+b\cos^{-1}(cx)\right)}{b}}\mathrm{Gamma}\left(\frac{3}{2}, -\frac{i\left(a+b\cos^{-1}(cx)\right)}{b}\right) + 9e^{\frac{4ia}{b}}\sqrt{-\frac{i\left(a+b\cos^{-1}(cx)\right)}{b}}\mathrm{Gamma}\left(\frac{3}{2}, \frac{i\left(a+b\cos^{-1}\right.}{b}\right.\right.}{72c^3\sqrt{\frac{\left(a+b\cos\right.}{b^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[a + b*ArcCos[c*x]],x]
```

```
[Out] (Sqrt[a + b*ArcCos[c*x]]*(9*E^(((2*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]
*Gamma[3/2, ((-I)*(a + b*ArcCos[c*x]))/b] + 9*E^(((4*I)*a)/b)*Sqrt[((-I)*(a
 + b*ArcCos[c*x]))/b]*Gamma[3/2, (I*(a + b*ArcCos[c*x]))/b] + Sqrt[3]*(Sqrt
[(I*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((-3*I)*(a + b*ArcCos[c*x]))/b] + E^
(((6*I)*a)/b)*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((3*I)*(a + b*A
rcCos[c*x]))/b])))/(72*c^3*E^(((3*I)*a)/b)*Sqrt[(a + b*ArcCos[c*x])^2/b^2])
```

Maple [A] time = 0.134, size = 357, normalized size = 1.5

$$\frac{1}{72\,c^{3}}\left(-\sqrt{3}\sqrt{2}\sqrt{\pi}\sqrt{b^{-1}}\sqrt{a+b\arccos(cx)}\mathrm{FresnelS}\left(\frac{\sqrt{3}\sqrt{2}}{\sqrt{\pi}b}\sqrt{a+b\arccos(cx)}\frac{1}{\sqrt{b^{-1}}}\right)\sin\left(3\,\frac{a}{b}\right)b-\sqrt{3}\sqrt{2}\sqrt{\pi}\sqrt{b^{-1}}\sqrt{a+b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccos(c*x))^(1/2),x)
```

```
[Out] 1/72/c^3/(a+b*arccos(c*x))^(1/2)*(-3^(1/2)*2^(1/2)*Pi^(1/2)*(1/b)^(1/2)*(a+
b*arccos(c*x))^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(1/b)^(1/2)*(a+b*arc
cos(c*x))^(1/2)/b)*sin(3*a/b)*b-3^(1/2)*2^(1/2)*Pi^(1/2)*(1/b)^(1/2)*(a+b*a
rccos(c*x))^(1/2)*cos(3*a/b)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(1/b)^(1/2)*
(a+b*arccos(c*x))^(1/2)/b)*b-9*2^(1/2)*Pi^(1/2)*(1/b)^(1/2)*(a+b*arccos(c*x
))^(1/2)*cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(
1/2)/b)*b-9*2^(1/2)*Pi^(1/2)*(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(a/b)*F
resnelS(2^(1/2)/Pi^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b+18*arccos
(c*x)*cos((a+b*arccos(c*x))/b-a/b)*b+18*cos((a+b*arccos(c*x))/b-a/b)*a+6*ar
ccos(c*x)*cos(3*(a+b*arccos(c*x))/b-3*a/b)*b+6*cos(3*(a+b*arccos(c*x))/b-3*
a/b)*a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\arccos(cx)+a}x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccos(c*x) + a)*x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{a+b\operatorname{acos}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acos(c*x))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*acos(c*x)), x)
```

Giac [B] time = 2.00748, size = 549, normalized size = 2.27

$$\frac{\sqrt{2}\sqrt{\pi}b\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{b\arccos(cx)+a}i}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{b\arccos(cx)+a}\sqrt{|b|}}{2b}\right)e^{\left(\frac{ai}{b}\right)}}{16\left(\frac{bi}{\sqrt{|b|}}+\sqrt{|b|}\right)c^3}-\frac{\sqrt{2}\sqrt{\pi}b\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b\arccos(cx)+a}i}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{b\arccos(cx)+a}\sqrt{|b|}}{2b}\right)e^{\left(-\frac{ai}{b}\right)}}{16\left(\frac{bi}{\sqrt{|b|}}-\sqrt{|b|}\right)c^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*sqrt(pi)*b*erf(-1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/sqrt(abs
(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(a*i/b)/((b*i/
sqrt(abs(b)) + sqrt(abs(b)))*c^3) - 1/16*sqrt(2)*sqrt(pi)*b*erf(1/2*sqrt(2)
*sqrt(b*arccos(c*x) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) +
a)*sqrt(abs(b))/b)*e^(-a*i/b)/((b*i/sqrt(abs(b)) - sqrt(abs(b)))*c^3) + 1/2
4*sqrt(pi)*sqrt(b)*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b
) - 1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b))*e^(3*a*i/b)/((sqrt(6)*b*i/
abs(b) + sqrt(6))*c^3) - 1/24*sqrt(pi)*sqrt(b)*erf(1/2*sqrt(6)*sqrt(b*arcco
s(c*x) + a)*sqrt(b)*i/abs(b) - 1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b))
*e^(-3*a*i/b)/((sqrt(6)*b*i/abs(b) - sqrt(6))*c^3) + 1/24*sqrt(b*arccos(c*x
) + a)*e^(3*i*arccos(c*x))/c^3 + 1/8*sqrt(b*arccos(c*x) + a)*e^(i*arccos(c*
x))/c^3 + 1/8*sqrt(b*arccos(c*x) + a)*e^(-i*arccos(c*x))/c^3 + 1/24*sqrt(b*
arccos(c*x) + a)*e^(-3*i*arccos(c*x))/c^3
```

3.174 $\int x\sqrt{a+b\cos^{-1}(cx)}\,dx$

Optimal. Leaf size=137

$$-\frac{\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{\pi}\sqrt{b}}\right)}{8c^2}-\frac{\sqrt{\pi}\sqrt{b}\sin\left(\frac{2a}{b}\right)S\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{8c^2}-\frac{\sqrt{a+b\cos^{-1}(cx)}}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\cos^{-1}(c$$

```
[Out] -Sqrt[a + b*ArcCos[c*x]]/(4*c^2) + (x^2*Sqrt[a + b*ArcCos[c*x]])/2 - (Sqrt[
b]*Sqrt[Pi]*Cos[(2*a)/b]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt
[Pi])])/(8*c^2) - (Sqrt[b]*Sqrt[Pi]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(S
qrt[b]*Sqrt[Pi])]*Sin[(2*a)/b])/(8*c^2)
```

Rubi [A] time = 0.39969, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4630, 4724, 3312, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{\pi}\sqrt{b}}\right)}{8c^2}-\frac{\sqrt{\pi}\sqrt{b}\sin\left(\frac{2a}{b}\right)S\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{8c^2}-\frac{\sqrt{a+b\cos^{-1}(cx)}}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\cos^{-1}(c$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*ArcCos[c*x]],x]
```

```
[Out] -Sqrt[a + b*ArcCos[c*x]]/(4*c^2) + (x^2*Sqrt[a + b*ArcCos[c*x]])/2 - (Sqrt[
b]*Sqrt[Pi]*Cos[(2*a)/b]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt
[Pi])])/(8*c^2) - (Sqrt[b]*Sqrt[Pi]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(S
qrt[b]*Sqrt[Pi])]*Sin[(2*a)/b])/(8*c^2)
```

Rule 4630

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
x^(m + 1)*(a + b*ArcCos[c*x])^n)/(m + 1), x] + Dist[(b*c*n)/(m + 1), Int[(x
^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
```

```
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int x\sqrt{a+b\cos^{-1}(cx)}\,dx = \frac{1}{2}x^2\sqrt{a+b\cos^{-1}(cx)} + \frac{1}{4}(bc)\int \frac{x^2}{\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}\,dx$$
$$= \frac{1}{2}x^2\sqrt{a+b\cos^{-1}(cx)} - \frac{b\,\text{Subst}\left(\int \frac{\cos^2(x)}{\sqrt{a+bx}}\,dx, x, \cos^{-1}(cx)\right)}{4c^2}$$
$$= \frac{1}{2}x^2\sqrt{a+b\cos^{-1}(cx)} - \frac{b\,\text{Subst}\left(\int \left(\frac{1}{2\sqrt{a+bx}} + \frac{\cos(2x)}{2\sqrt{a+bx}}\right)dx, x, \cos^{-1}(cx)\right)}{4c^2}$$
$$= -\frac{\sqrt{a+b\cos^{-1}(cx)}}{4c^2} + \frac{1}{2}x^2\sqrt{a+b\cos^{-1}(cx)} - \frac{b\,\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{a+bx}}\,dx, x, \cos^{-1}(cx)\right)}{8c^2}$$
$$= -\frac{\sqrt{a+b\cos^{-1}(cx)}}{4c^2} + \frac{1}{2}x^2\sqrt{a+b\cos^{-1}(cx)} - \frac{\left(b\cos\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{2a}{b}+2x\right)}{\sqrt{a+bx}}\,dx, x, \cos^{-1}(cx)\right.}{8c^2}$$
$$= -\frac{\sqrt{a+b\cos^{-1}(cx)}}{4c^2} + \frac{1}{2}x^2\sqrt{a+b\cos^{-1}(cx)} - \frac{\cos\left(\frac{2a}{b}\right)\text{Subst}\left(\int \cos\left(\frac{2x^2}{b}\right)dx, x, \sqrt{a+b\cos^{-1}\right.}{4c^2}$$
$$= -\frac{\sqrt{a+b\cos^{-1}(cx)}}{4c^2} + \frac{1}{2}x^2\sqrt{a+b\cos^{-1}(cx)} - \frac{\sqrt{b}\sqrt{\pi}\cos\left(\frac{2a}{b}\right)C\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{8c^2} - \frac{\sqrt{b}\sqrt{\pi}S\left(\frac{2\sqrt{}}{}\right.}{}$$

Mathematica [A] time = 0.200957, size = 123, normalized size = 0.9

$$-\frac{\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{\pi}}\right) + \sqrt{\pi}\sin\left(\frac{2a}{b}\right)S\left(\frac{2\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{\pi}}\right) - 2\sqrt{\frac{1}{b}}\cos\left(2\cos^{-1}(cx)\right)\sqrt{a+b\cos^{-1}(cx)}}{8\sqrt{\frac{1}{b}}c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*ArcCos[c*x]],x]
```

```
[Out] -(-2*Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[c*x]]*Cos[2*ArcCos[c*x]] + Sqrt[Pi]*Cos
[(2*a)/b]*FresnelC[(2*Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[Pi]] + Sqr
t[Pi]*FresnelS[(2*Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[Pi]]*Sin[(2*a)
/b])/(8*Sqrt[b^(-1)]*c^2)
```

Maple [A] time = 0.102, size = 173, normalized size = 1.3

$$\frac{1}{8c^2}\left(-\text{FresnelC}\left(2\frac{\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\sqrt{b^{-1}}b}\right)\cos\left(2\frac{a}{b}\right)\sqrt{\pi}\sqrt{b^{-1}}\sqrt{a+b\arccos(cx)}b - \text{FresnelS}\left(2\frac{\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\sqrt{b^{-1}}b}\right)\sin\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccos(c*x))^(1/2),x)
```

```
[Out] 1/8/c^2/(a+b*arccos(c*x))^(1/2)*(-FresnelC(2/Pi^(1/2)/(1/b)^(1/2)*(a+b*arcc
os(c*x))^(1/2)/b)*cos(2*a/b)*Pi^(1/2)*(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*b
-FresnelS(2/Pi^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*sin(2*a/b)*Pi^(
1/2)*(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*b+2*arccos(c*x)*cos(2*(a+b*arccos(
c*x))/b-2*a/b)*b+2*cos(2*(a+b*arccos(c*x))/b-2*a/b)*a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \arccos(cx) + a}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccos(c*x) + a)*x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{a + b\operatorname{acos}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acos(c*x))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*acos(c*x)), x)
```

Giac [A] time = 1.57599, size = 243, normalized size = 1.77

$$\frac{\sqrt{\pi}\sqrt{b}\,\mathrm{erf}\left(-\frac{\sqrt{b\arccos(cx)+a}\sqrt{b}i}{|b|}-\frac{\sqrt{b\arccos(cx)+a}}{\sqrt{b}}\right)e^{\left(\frac{2ai}{b}\right)}}{16\,c^2\left(\frac{bi}{|b|}+1\right)}-\frac{\sqrt{\pi}\sqrt{b}\,\mathrm{erf}\left(\frac{\sqrt{b\arccos(cx)+a}\sqrt{b}i}{|b|}-\frac{\sqrt{b\arccos(cx)+a}}{\sqrt{b}}\right)e^{\left(-\frac{2ai}{b}\right)}}{16\,c^2\left(\frac{bi}{|b|}-1\right)}+\frac{\sqrt{b\arccos}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(pi)*sqrt(b)*erf(-sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - sqrt(
b*arccos(c*x) + a)/sqrt(b))*e^(2*a*i/b)/(c^2*(b*i/abs(b) + 1)) - 1/16*sqrt(
pi)*sqrt(b)*erf(sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - sqrt(b*arccos(c*
x) + a)/sqrt(b))*e^(-2*a*i/b)/(c^2*(b*i/abs(b) - 1)) + 1/8*sqrt(b*arccos(c*
x) + a)*e^(2*i*arccos(c*x))/c^2 + 1/8*sqrt(b*arccos(c*x) + a)*e^(-2*i*arcco
s(c*x))/c^2
```

3.175 $\int \sqrt{a+b\cos^{-1}(cx)}\,dx$

Optimal. Leaf size=121

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{c}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\sin\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{c}+x\sqrt{a+b\cos^{-1}(cx)}$$

```
[Out] x*Sqrt[a + b*ArcCos[c*x]] - (Sqrt[b]*Sqrt[Pi/2]*Cos[a/b]*FresnelC[(Sqrt[2/P
i]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/c - (Sqrt[b]*Sqrt[Pi/2]*FresnelS[(Sqr
t[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/c
```

Rubi [A] time = 0.278333, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {4620, 4724, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{c}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\sin\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{c}+x\sqrt{a+b\cos^{-1}(cx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcCos[c*x]],x]
```

```
[Out] x*Sqrt[a + b*ArcCos[c*x]] - (Sqrt[b]*Sqrt[Pi/2]*Cos[a/b]*FresnelC[(Sqrt[2/P
i]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/c - (Sqrt[b]*Sqrt[Pi/2]*FresnelS[(Sqr
t[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/c
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt{a+b\cos^{-1}(cx)}\,dx = x\sqrt{a+b\cos^{-1}(cx)}+\frac{1}{2}(bc)\int \frac{x}{\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}\,dx$$
$$= x\sqrt{a+b\cos^{-1}(cx)}-\frac{b\,\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{a+bx}}\,dx,x,\cos^{-1}(cx)\right)}{2c}$$
$$= x\sqrt{a+b\cos^{-1}(cx)}-\frac{\left(b\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{a}{b}+x\right)}{\sqrt{a+bx}}\,dx,x,\cos^{-1}(cx)\right)}{2c}-\frac{\left(b\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\text{si}}{}\right.}{2}$$
$$= x\sqrt{a+b\cos^{-1}(cx)}-\frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int \cos\left(\frac{x^2}{b}\right)dx,x,\sqrt{a+b\cos^{-1}(cx)}\right)}{c}-\frac{\sin\left(\frac{a}{b}\right)\text{Subst}\left(\int \text{s}\right.}{}$$
$$= x\sqrt{a+b\cos^{-1}(cx)}-\frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)C\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{c}-\frac{\sqrt{b}\sqrt{\frac{\pi}{2}}S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{c}$$

Mathematica [C] time = 0.183199, size = 120, normalized size = 0.99

$$-\frac{e^{-\frac{ia}{b}}\sqrt{a+b\cos^{-1}(cx)}\left(-\frac{\text{Gamma}\left(\frac{3}{2},-\frac{i(a+b\cos^{-1}(cx))}{b}\right)}{\sqrt{-\frac{i(a+b\cos^{-1}(cx))}{b}}}-\frac{e^{\frac{2ia}{b}}\text{Gamma}\left(\frac{3}{2},\frac{i(a+b\cos^{-1}(cx))}{b}\right)}{\sqrt{\frac{i(a+b\cos^{-1}(cx))}{b}}}\right)}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*ArcCos[c*x]],x]
```

```
[Out] -(Sqrt[a + b*ArcCos[c*x]]*(-(Gamma[3/2, ((-I)*(a + b*ArcCos[c*x]))/b]/Sqrt[
((-I)*(a + b*ArcCos[c*x]))/b]) - (E^(((2*I)*a)/b)*Gamma[3/2, (I*(a + b*ArcC
os[c*x]))/b])/Sqrt[(I*(a + b*ArcCos[c*x]))/b]))/(2*c*E^((I*a)/b))
```

Maple [A] time = 0.099, size = 179, normalized size = 1.5

$$\frac{1}{2c}\left(-\sqrt{2}\sqrt{\pi}\sqrt{b^{-1}}\sqrt{a+b\arccos(cx)}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}b}\sqrt{a+b\arccos(cx)}\frac{1}{\sqrt{b^{-1}}}\right)b-\sqrt{2}\sqrt{\pi}\sqrt{b^{-1}}\sqrt{a+b\arccos(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^(1/2),x)
```

```
[Out] 1/2/c/(a+b*arccos(c*x))^(1/2)*(-2^(1/2)*Pi^(1/2)*(1/b)^(1/2)*(a+b*arccos(c*
x))^(1/2)*cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^
(1/2)/b)*b-2^(1/2)*Pi^(1/2)*(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(a/b)*Fr
esnelS(2^(1/2)/Pi^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b+2*arccos(c
*x)*cos((a+b*arccos(c*x))/b-a/b)*b+2*cos((a+b*arccos(c*x))/b-a/b)*a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \arccos (cx) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccos(c*x) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \operatorname{acos}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*acos(c*x)), x)
```

Giac [B] time = 1.40676, size = 274, normalized size = 2.26

$$\frac{\sqrt{2}\sqrt{\pi}b\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{b\arccos(cx)+a}i}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{b\arccos(cx)+a}\sqrt{|b|}}{2b}\right)e^{\left(\frac{ai}{b}\right)}}{4\left(\frac{bi}{\sqrt{|b|}}+\sqrt{|b|}\right)c}-\frac{\sqrt{2}\sqrt{\pi}b\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{b\arccos(cx)+a}i}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{b\arccos(cx)+a}\sqrt{|b|}}{2b}\right)e^{\left(-\frac{ai}{b}\right)}}{4\left(\frac{bi}{\sqrt{|b|}}-\sqrt{|b|}\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*sqrt(pi)*b*erf(-1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/sqrt(abs(
b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(a*i/b)/((b*i/s
qrt(abs(b)) + sqrt(abs(b)))*c) - 1/4*sqrt(2)*sqrt(pi)*b*erf(1/2*sqrt(2)*sqr
t(b*arccos(c*x) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*s
qrt(abs(b))/b)*e^(-a*i/b)/((b*i/sqrt(abs(b)) - sqrt(abs(b)))*c) + 1/2*sqrt(
b*arccos(c*x) + a)*e^(i*arccos(c*x))/c + 1/2*sqrt(b*arccos(c*x) + a)*e^(-i*
arccos(c*x))/c
```

3.176 $\int \frac{\sqrt{a+b\cos^{-1}(cx)}}{x}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\sqrt{a+b\cos^{-1}(cx)}}{x},x\right)$$

```
[Out] Unintegrable[Sqrt[a + b*ArcCos[c*x]]/x, x]
```

Rubi [A] time = 0.0341507, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{a+b\cos^{-1}(cx)}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*ArcCos[c*x]]/x,x]
```

```
[Out] Defer[Int][Sqrt[a + b*ArcCos[c*x]]/x, x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos^{-1}(cx)}}{x}\,dx=\int \frac{\sqrt{a+b\cos^{-1}(cx)}}{x}\,dx$$

Mathematica [A] time = 3.58934, size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\cos^{-1}(cx)}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*ArcCos[c*x]]/x,x]
```

```
[Out] Integrate[Sqrt[a + b*ArcCos[c*x]]/x, x]
```

Maple [A] time = 0.209, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{a+b\arccos\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^(1/2)/x,x)
```

```
[Out] int((a+b*arccos(c*x))^(1/2)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\arccos\left(cx\right)+a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccos(c*x) + a)/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\operatorname{acos}\left(cx\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*acos(c*x))/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b \arccos(cx) + a}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arccos(c*x) + a)/x, x)
```

3.177 $\int \frac{\sqrt{a+b\cos^{-1}(cx)}}{x^2}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\sqrt{a+b\cos^{-1}(cx)}}{x^2},x\right)$$

```
[Out] Unintegrable[Sqrt[a + b*ArcCos[c*x]]/x^2, x]
```

Rubi [A] time = 0.0336354, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sqrt{a+b\cos^{-1}(cx)}}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*ArcCos[c*x]]/x^2,x]
```

```
[Out] Defer[Int][Sqrt[a + b*ArcCos[c*x]]/x^2, x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos^{-1}(cx)}}{x^2}\,dx = \int \frac{\sqrt{a+b\cos^{-1}(cx)}}{x^2}\,dx$$

Mathematica [A] time = 9.94041, size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\cos^{-1}(cx)}}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*ArcCos[c*x]]/x^2,x]
```

```
[Out] Integrate[Sqrt[a + b*ArcCos[c*x]]/x^2, x]
```

Maple [A] time = 0.274, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt{a+b\arccos\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^(1/2)/x^2,x)
```

```
[Out] int((a+b*arccos(c*x))^(1/2)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\arccos\left(cx\right)+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccos(c*x) + a)/x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\operatorname{acos}\left(cx\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a + b*acos(c*x))/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b \arccos (cx) + a}}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arccos(c*x) + a)/x^2, x)
```

3.178 $\int x^2 \left(a + b \cos^{-1}(cx)\right)^{3/2} \, dx$

Optimal. Leaf size=313

$$-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{8c^3}-\frac{\sqrt{\frac{\pi}{6}}b^{3/2}\sin\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{24c^3}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\cos\left(\frac{a}{b}\right)S\left(\sqrt{\frac{2}{\pi}}\right.}{8c^3}$$

```
[Out] -(b*Sqrt[1 - c^2*x^2]*Sqrt[a + b*ArcCos[c*x]])/(3*c^3) - (b*x^2*Sqrt[1 - c^
2*x^2]*Sqrt[a + b*ArcCos[c*x]])/(6*c) + (x^3*(a + b*ArcCos[c*x])^(3/2))/3 +
 (3*b^(3/2)*Sqrt[Pi/2]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]
])/Sqrt[b]])/(8*c^3) + (b^(3/2)*Sqrt[Pi/6]*Cos[(3*a)/b]*FresnelS[(Sqrt[6/Pi
]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(24*c^3) - (3*b^(3/2)*Sqrt[Pi/2]*Fresn
elC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(8*c^3) - (b^(3
/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(
3*a)/b])/(24*c^3)
```

Rubi [A] time = 0.948978, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {4630, 4708, 4678, 4624, 3306, 3305, 3351, 3304, 3352, 4636, 4406}

$$-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{8c^3}-\frac{\sqrt{\frac{\pi}{6}}b^{3/2}\sin\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{24c^3}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\cos\left(\frac{a}{b}\right)S\left(\sqrt{\frac{2}{\pi}}\right.}{8c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcCos[c*x])^(3/2),x]
```

```
[Out] -(b*Sqrt[1 - c^2*x^2]*Sqrt[a + b*ArcCos[c*x]])/(3*c^3) - (b*x^2*Sqrt[1 - c^
2*x^2]*Sqrt[a + b*ArcCos[c*x]])/(6*c) + (x^3*(a + b*ArcCos[c*x])^(3/2))/3 +
 (3*b^(3/2)*Sqrt[Pi/2]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]
])/Sqrt[b]])/(8*c^3) + (b^(3/2)*Sqrt[Pi/6]*Cos[(3*a)/b]*FresnelS[(Sqrt[6/Pi
]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(24*c^3) - (3*b^(3/2)*Sqrt[Pi/2]*Fresn
elC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(8*c^3) - (b^(3
/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(
3*a)/b])/(24*c^3)
```

Rule 4630

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
x^(m + 1)*(a + b*ArcCos[c*x])^n)/(m + 1), x] + Dist[(b*c*n)/(m + 1), Int[(x
^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4624

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Sub
st[Int[x^n*Sin[a/b - x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c,
 n}, x]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int x^2 \left(a+b\cos^{-1}(cx)\right)^{3/2} dx = \frac{1}{3}x^3\left(a+b\cos^{-1}(cx)\right)^{3/2} + \frac{1}{2}(bc)\int \frac{x^3\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{1-c^2x^2}}\,dx$$

$$= -\frac{bx^2\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{6c} + \frac{1}{3}x^3\left(a+b\cos^{-1}(cx)\right)^{3/2} - \frac{1}{12}b^2\int \frac{x^2}{\sqrt{a+b\cos^{-1}(cx)}}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{3c^3} - \frac{bx^2\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{6c} + \frac{1}{3}x^3\left(a+b\cos^{-1}(cx)\right)$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{3c^3} - \frac{bx^2\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{6c} + \frac{1}{3}x^3\left(a+b\cos^{-1}(cx)\right)$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{3c^3} - \frac{bx^2\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{6c} + \frac{1}{3}x^3\left(a+b\cos^{-1}(cx)\right)$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{3c^3} - \frac{bx^2\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{6c} + \frac{1}{3}x^3\left(a+b\cos^{-1}(cx)\right)$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{3c^3} - \frac{bx^2\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{6c} + \frac{1}{3}x^3\left(a+b\cos^{-1}(cx)\right)$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{3c^3} - \frac{bx^2\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{6c} + \frac{1}{3}x^3\left(a+b\cos^{-1}(cx)\right)$$

Mathematica [C] time = 9.59482, size = 589, normalized size = 1.88

$$\frac{b\left(-18\left(3\sqrt{1-c^2x^2}-2cx\cos^{-1}(cx)\right)\sqrt{a+b\cos^{-1}(cx)} - 9\sqrt{2\pi}\sqrt{\frac{1}{b}}\left(3b\sin\left(\frac{a}{b}\right)-2a\cos\left(\frac{a}{b}\right)\right)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{b}}\sqrt{a+b\,\text{c}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcCos[c*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a + b*ArcCos[c*x]]*(9*E^(((2*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/
b]*Gamma[3/2, ((-I)*(a + b*ArcCos[c*x]))/b] + 9*E^(((4*I)*a)/b)*Sqrt[((-I)*
(a + b*ArcCos[c*x]))/b]*Gamma[3/2, (I*(a + b*ArcCos[c*x]))/b] + Sqrt[3]*(Sq
rt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((-3*I)*(a + b*ArcCos[c*x]))/b] +
```

```
E^(((6*I)*a)/b)*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((3*I)*(a + b
*ArcCos[c*x]))/b])))/(72*c^3*E^(((3*I)*a)/b)*Sqrt[(a + b*ArcCos[c*x])^2/b^2
]) + (b*(-18*Sqrt[a + b*ArcCos[c*x]]*(3*Sqrt[1 - c^2*x^2] - 2*c*x*ArcCos[c*
x]) + 9*Sqrt[b^(-1)]*Sqrt[2*Pi]*FresnelS[Sqrt[b^(-1)]*Sqrt[2/Pi]*Sqrt[a + b
*ArcCos[c*x]]]*(3*b*Cos[a/b] + 2*a*Sin[a/b]) - 9*Sqrt[b^(-1)]*Sqrt[2*Pi]*Fr
esnelC[Sqrt[b^(-1)]*Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]]]*(-2*a*Cos[a/b] + 3*
b*Sin[a/b]) + Sqrt[b^(-1)]*Sqrt[6*Pi]*FresnelS[Sqrt[b^(-1)]*Sqrt[6/Pi]*Sqrt
[a + b*ArcCos[c*x]]]*(b*Cos[(3*a)/b] + 2*a*Sin[(3*a)/b]) - Sqrt[b^(-1)]*Sqr
t[6*Pi]*FresnelC[Sqrt[b^(-1)]*Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]]]*(-2*a*Cos
[(3*a)/b] + b*Sin[(3*a)/b]) - 6*Sqrt[a + b*ArcCos[c*x]]*(-2*ArcCos[c*x]*Cos
[3*ArcCos[c*x]] + Sin[3*ArcCos[c*x]])))/(144*c^3)
```

Maple [B] time = 0.148, size = 541, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccos(c*x))^(3/2),x)
```

```
[Out] 1/144/c^3*(3^(1/2)*(1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*cos
(3*a/b)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/
2)/b)*b^2-3^(1/2)*(1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(
3*a/b)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2
)/b)*b^2+27*(1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(a/b)*F
resnelS(2^(1/2)/Pi^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b^2-27*(1/b
)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(a/b)*FresnelC(2^(1/2)/
Pi^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b^2+36*arccos(c*x)^2*cos((a
+b*arccos(c*x))/b-a/b)*b^2+12*arccos(c*x)^2*cos(3*(a+b*arccos(c*x))/b-3*a/b
)*b^2+72*arccos(c*x)*cos((a+b*arccos(c*x))/b-a/b)*a*b-54*arccos(c*x)*sin((a
+b*arccos(c*x))/b-a/b)*b^2+24*arccos(c*x)*cos(3*(a+b*arccos(c*x))/b-3*a/b)*
a*b-6*arccos(c*x)*sin(3*(a+b*arccos(c*x))/b-3*a/b)*b^2+36*cos((a+b*arccos(c
*x))/b-a/b)*a^2-54*sin((a+b*arccos(c*x))/b-a/b)*a*b+12*cos(3*(a+b*arccos(c*
x))/b-3*a/b)*a^2-6*sin(3*(a+b*arccos(c*x))/b-3*a/b)*a*b)/(a+b*arccos(c*x))^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \arccos (cx) + a)^{\frac{3}{2}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccos(c*x) + a)^(3/2)*x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{acos}(cx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acos(c*x))**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*acos(c*x))**(3/2), x)
```

Giac [B] time = 3.14784, size = 1800, normalized size = 5.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] -3/32*sqrt(2)*sqrt(pi)*b^4*i*erf(-1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/sqr
t(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(a*i/b)/(
```

```
(b^3*i/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c^3) - 3/32*sqrt(2)*sqrt(pi)*b^4*i*
erf(1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b
*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-a*i/b)/((b^3*i/sqrt(abs(b)) - b^2*sqr
t(abs(b)))*c^3) - 1/48*sqrt(pi)*b^(7/2)*i*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*
x) + a)*sqrt(b)*i/abs(b) - 1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b))*e^(
3*a*i/b)/((sqrt(6)*b^3*i/abs(b) + sqrt(6)*b^2)*c^3) - 1/16*sqrt(2)*sqrt(pi)
*a*b^3*erf(-1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2
)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(a*i/b)/((b^3*i/sqrt(abs(b)) +
b^2*sqrt(abs(b)))*c^3) + 1/16*sqrt(2)*sqrt(pi)*a*b^3*erf(1/2*sqrt(2)*sqrt(b
*arccos(c*x) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt
(abs(b))/b)*e^(-a*i/b)/((b^3*i/sqrt(abs(b)) - b^2*sqrt(abs(b)))*c^3) - 1/48
*sqrt(pi)*b^(7/2)*i*erf(1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b
) - 1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b))*e^(-3*a*i/b)/((sqrt(6)*b^3
*i/abs(b) - sqrt(6)*b^2)*c^3) - 1/24*sqrt(pi)*a*b^(5/2)*erf(-1/2*sqrt(6)*sq
rt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - 1/2*sqrt(6)*sqrt(b*arccos(c*x) + a
)/sqrt(b))*e^(3*a*i/b)/((sqrt(6)*b^3*i/abs(b) + sqrt(6)*b^2)*c^3) + 1/16*sq
rt(2)*sqrt(pi)*a*b^2*erf(-1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/sqrt(abs(b)
) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(a*i/b)/((b^2*i/s
qrt(abs(b)) + b*sqrt(abs(b)))*c^3) - 1/16*sqrt(2)*sqrt(pi)*a*b^2*erf(1/2*sq
rt(2)*sqrt(b*arccos(c*x) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*
x) + a)*sqrt(abs(b))/b)*e^(-a*i/b)/((b^2*i/sqrt(abs(b)) - b*sqrt(abs(b)))*c
^3) + 1/24*sqrt(pi)*a*b^(5/2)*erf(1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(
b)*i/abs(b) - 1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b))*e^(-3*a*i/b)/((s
qrt(6)*b^3*i/abs(b) - sqrt(6)*b^2)*c^3) + 1/24*sqrt(pi)*a*b^(3/2)*erf(-1/2*
sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - 1/2*sqrt(6)*sqrt(b*arcco
s(c*x) + a)/sqrt(b))*e^(3*a*i/b)/((sqrt(6)*b^2*i/abs(b) + sqrt(6)*b)*c^3) -
 1/24*sqrt(pi)*a*b^(3/2)*erf(1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)*i/
abs(b) - 1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b))*e^(-3*a*i/b)/((sqrt(6
)*b^2*i/abs(b) - sqrt(6)*b)*c^3) + 1/48*sqrt(b*arccos(c*x) + a)*b*i*e^(3*i*
arccos(c*x))/c^3 + 1/24*sqrt(b*arccos(c*x) + a)*b*arccos(c*x)*e^(3*i*arccos
(c*x))/c^3 + 3/16*sqrt(b*arccos(c*x) + a)*b*i*e^(i*arccos(c*x))/c^3 + 1/8*s
qrt(b*arccos(c*x) + a)*b*arccos(c*x)*e^(i*arccos(c*x))/c^3 - 3/16*sqrt(b*ar
ccos(c*x) + a)*b*i*e^(-i*arccos(c*x))/c^3 + 1/8*sqrt(b*arccos(c*x) + a)*b*a
rccos(c*x)*e^(-i*arccos(c*x))/c^3 - 1/48*sqrt(b*arccos(c*x) + a)*b*i*e^(-3*
i*arccos(c*x))/c^3 + 1/24*sqrt(b*arccos(c*x) + a)*b*arccos(c*x)*e^(-3*i*arc
cos(c*x))/c^3 + 1/24*sqrt(b*arccos(c*x) + a)*a*e^(3*i*arccos(c*x))/c^3 + 1/
8*sqrt(b*arccos(c*x) + a)*a*e^(i*arccos(c*x))/c^3 + 1/8*sqrt(b*arccos(c*x)
+ a)*a*e^(-i*arccos(c*x))/c^3 + 1/24*sqrt(b*arccos(c*x) + a)*a*e^(-3*i*arcc
os(c*x))/c^3
```

3.179 $\int x\left(a+b\cos^{-1}(cx)\right)^{3/2}\,dx$

Optimal. Leaf size=172

$$-\frac{3\sqrt{\pi}b^{3/2}\sin\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{\pi}\sqrt{b}}\right)}{32c^2}+\frac{3\sqrt{\pi}b^{3/2}\cos\left(\frac{2a}{b}\right)S\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{32c^2}-\frac{3bx\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{8c}-$$

```
[Out] (-3*b*x*Sqrt[1 - c^2*x^2]*Sqrt[a + b*ArcCos[c*x]])/(8*c) - (a + b*ArcCos[c*
x])^(3/2)/(4*c^2) + (x^2*(a + b*ArcCos[c*x])^(3/2))/2 + (3*b^(3/2)*Sqrt[Pi]
*Cos[(2*a)/b]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])])/(32
*c^2) - (3*b^(3/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*S
qrt[Pi])]*Sin[(2*a)/b])/(32*c^2)
```

Rubi [A] time = 0.461326, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {4630, 4708, 4642, 4636, 4406, 12, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\pi}b^{3/2}\sin\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{\pi}\sqrt{b}}\right)}{32c^2}+\frac{3\sqrt{\pi}b^{3/2}\cos\left(\frac{2a}{b}\right)S\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{32c^2}-\frac{3bx\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{8c}-$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcCos[c*x])^(3/2),x]
```

```
[Out] (-3*b*x*Sqrt[1 - c^2*x^2]*Sqrt[a + b*ArcCos[c*x]])/(8*c) - (a + b*ArcCos[c*
x])^(3/2)/(4*c^2) + (x^2*(a + b*ArcCos[c*x])^(3/2))/2 + (3*b^(3/2)*Sqrt[Pi]
*Cos[(2*a)/b]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])])/(32
*c^2) - (3*b^(3/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*S
qrt[Pi])]*Sin[(2*a)/b])/(32*c^2)
```

Rule 4630

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
x^(m + 1)*(a + b*ArcCos[c*x])^n)/(m + 1), x] + Dist[(b*c*n)/(m + 1), Int[(x
^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
```

```
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int x\left(a+b\cos^{-1}(cx)\right)^{3/2}\,dx=\frac{1}{2}x^2\left(a+b\cos^{-1}(cx)\right)^{3/2}+\frac{1}{4}(3bc)\int\frac{x^2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{1-c^2x^2}}\,dx$$

$$=-\frac{3bx\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{8c}+\frac{1}{2}x^2\left(a+b\cos^{-1}(cx)\right)^{3/2}-\frac{1}{16}\left(3b^2\right)\int\frac{x}{\sqrt{a+b\cos^{-1}(c}}$$

$$=-\frac{3bx\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{8c}-\frac{\left(a+b\cos^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\cos^{-1}(cx)\right)^{3/2}+\frac{\left(3b^2\right.}{}$$

$$=-\frac{3bx\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{8c}-\frac{\left(a+b\cos^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\cos^{-1}(cx)\right)^{3/2}+\frac{\left(3b^2\right.}{}$$

$$=-\frac{3bx\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{8c}-\frac{\left(a+b\cos^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\cos^{-1}(cx)\right)^{3/2}+\frac{\left(3b^2\right.}{}$$

$$=-\frac{3bx\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{8c}-\frac{\left(a+b\cos^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\cos^{-1}(cx)\right)^{3/2}+\frac{\left(3b^2\right.}{}$$

$$=-\frac{3bx\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{8c}-\frac{\left(a+b\cos^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\cos^{-1}(cx)\right)^{3/2}+\frac{\left(3b\right.}{}$$

$$=-\frac{3bx\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{8c}-\frac{\left(a+b\cos^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\cos^{-1}(cx)\right)^{3/2}+\frac{3b^{3/}}{}$$

Mathematica [A] time = 0.825214, size = 155, normalized size = 0.9

$$\frac{-3\sqrt{\pi}b\sin\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{\pi}}\right)+3\sqrt{\pi}b\cos\left(\frac{2a}{b}\right)S\left(\frac{2\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{\pi}}\right)+2\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(cx)}\left(4a\cos\left(2\cos\right.\right.}{32\sqrt{\frac{1}{b}}c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcCos[c*x])^(3/2),x]
```

```
[Out] (3*b*Sqrt[Pi]*Cos[(2*a)/b]*FresnelS[(2*Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[c*x]]
)/Sqrt[Pi]] - 3*b*Sqrt[Pi]*FresnelC[(2*Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[c*x]]
)/Sqrt[Pi]]*Sin[(2*a)/b] + 2*Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[c*x]]*(4*a*Cos[
2*ArcCos[c*x]] + 4*b*ArcCos[c*x]*Cos[2*ArcCos[c*x]] - 3*b*Sin[2*ArcCos[c*x]
]))/(32*Sqrt[b^(-1)]*c^2)
```

Maple [A] time = 0.114, size = 267, normalized size = 1.6

$$\frac{1}{32\,c^2}\left(3\,\sqrt{b^{-1}}\sqrt{\pi}\cos\left(2\,\frac{a}{b}\right)\text{FresnelS}\left(2\,\frac{\sqrt{a+b\arccos(cx)}}{\sqrt{b^{-1}}\sqrt{\pi}b}\right)\sqrt{a+b\arccos(cx)}b^2-3\,\sqrt{b^{-1}}\sqrt{\pi}\sin\left(2\,\frac{a}{b}\right)\text{FresnelC}\left(2\,\frac{\sqrt{a}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccos(c*x))^(3/2),x)
```

```
[Out] 1/32/c^2*(3*(1/b)^(1/2)*Pi^(1/2)*cos(2*a/b)*FresnelS(2/Pi^(1/2)/(1/b)^(1/2)
*(a+b*arccos(c*x))^(1/2)/b)*(a+b*arccos(c*x))^(1/2)*b^2-3*(1/b)^(1/2)*Pi^(1
/2)*sin(2*a/b)*FresnelC(2/Pi^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(
a+b*arccos(c*x))^(1/2)*b^2+8*arccos(c*x)^2*cos(2*(a+b*arccos(c*x))/b-2*a/b)
*b^2+16*arccos(c*x)*cos(2*(a+b*arccos(c*x))/b-2*a/b)*a*b-6*arccos(c*x)*sin(
2*(a+b*arccos(c*x))/b-2*a/b)*b^2+8*cos(2*(a+b*arccos(c*x))/b-2*a/b)*a^2-6*s
in(2*(a+b*arccos(c*x))/b-2*a/b)*a*b)/(a+b*arccos(c*x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\arccos(cx)+a)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccos(c*x) + a)^(3/2)*x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a+b\operatorname{acos}(cx)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acos(c*x))**(3/2),x)
```

```
[Out] Integral(x*(a + b*acos(c*x))**(3/2), x)
```

Giac [B] time = 1.97114, size = 791, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] -3/64*sqrt(pi)*b^(7/2)*i*erf(-sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - sq
rt(b*arccos(c*x) + a)/sqrt(b))*e^(2*a*i/b)/((b^3*i/abs(b) + b^2)*c^2) - 3/6
4*sqrt(pi)*b^(7/2)*i*erf(sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - sqrt(b*
arccos(c*x) + a)/sqrt(b))*e^(-2*a*i/b)/((b^3*i/abs(b) - b^2)*c^2) - 1/16*sq
```

```
rt(pi)*a*b^(5/2)*erf(-sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - sqrt(b*arc
cos(c*x) + a)/sqrt(b))*e^(2*a*i/b)/((b^3*i/abs(b) + b^2)*c^2) + 1/16*sqrt(p
i)*a*b^(5/2)*erf(sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - sqrt(b*arccos(c
*x) + a)/sqrt(b))*e^(-2*a*i/b)/((b^3*i/abs(b) - b^2)*c^2) + 1/16*sqrt(pi)*a
*b^(3/2)*erf(-sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - sqrt(b*arccos(c*x)
 + a)/sqrt(b))*e^(2*a*i/b)/((b^2*i/abs(b) + b)*c^2) - 1/16*sqrt(pi)*a*b^(3/
2)*erf(sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - sqrt(b*arccos(c*x) + a)/s
qrt(b))*e^(-2*a*i/b)/((b^2*i/abs(b) - b)*c^2) + 3/32*sqrt(b*arccos(c*x) + a
)*b*i*e^(2*i*arccos(c*x))/c^2 + 1/8*sqrt(b*arccos(c*x) + a)*b*arccos(c*x)*e
^(2*i*arccos(c*x))/c^2 - 3/32*sqrt(b*arccos(c*x) + a)*b*i*e^(-2*i*arccos(c*
x))/c^2 + 1/8*sqrt(b*arccos(c*x) + a)*b*arccos(c*x)*e^(-2*i*arccos(c*x))/c^
2 + 1/8*sqrt(b*arccos(c*x) + a)*a*e^(2*i*arccos(c*x))/c^2 + 1/8*sqrt(b*arcc
os(c*x) + a)*a*e^(-2*i*arccos(c*x))/c^2
```

3.180 $\int \left(a+b\cos^{-1}(cx)\right)^{3/2}\,dx$

Optimal. Leaf size=159

$$-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{2c}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{2c}-\frac{3b\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{2c}$$

```
[Out] (-3*b*Sqrt[1 - c^2*x^2]*Sqrt[a + b*ArcCos[c*x]])/(2*c) + x*(a + b*ArcCos[c*
x])^(3/2) + (3*b^(3/2)*Sqrt[Pi/2]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*
ArcCos[c*x]])/Sqrt[b]])/(2*c) - (3*b^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[2/Pi]*
Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(2*c)
```

Rubi [A] time = 0.237264, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4620, 4678, 4624, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{2c}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{2c}-\frac{3b\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[1 - c^2*x^2]*Sqrt[a + b*ArcCos[c*x]])/(2*c) + x*(a + b*ArcCos[c*
x])^(3/2) + (3*b^(3/2)*Sqrt[Pi/2]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*
ArcCos[c*x]])/Sqrt[b]])/(2*c) - (3*b^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[2/Pi]*
Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(2*c)
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
```

```
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4624

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Sub
st[Int[x^n*Sin[a/b - x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c,
 n}, x]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \left(a+b\cos^{-1}(cx)\right)^{3/2}\,dx = x\left(a+b\cos^{-1}(cx)\right)^{3/2}+\frac{1}{2}(3bc)\int \frac{x\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{1-c^2x^2}}\,dx$$

$$= -\frac{3b\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{2c}+x\left(a+b\cos^{-1}(cx)\right)^{3/2}-\frac{1}{4}\left(3b^2\right)\int \frac{1}{\sqrt{a+b\cos^{-1}(cx)}}\,dx$$

$$= -\frac{3b\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{2c}+x\left(a+b\cos^{-1}(cx)\right)^{3/2}-\frac{(3b)\,\mathrm{Subst}\left(\int \frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+\right.}{4c}$$

$$= -\frac{3b\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{2c}+x\left(a+b\cos^{-1}(cx)\right)^{3/2}+\frac{\left(3b\cos\left(\frac{a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx,\right.}{4c}$$

$$= -\frac{3b\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{2c}+x\left(a+b\cos^{-1}(cx)\right)^{3/2}+\frac{\left(3b\cos\left(\frac{a}{b}\right)\right)\mathrm{Subst}\left(\int \sin\left(\frac{x^2}{b}\right)\right.}{2c}$$

$$= -\frac{3b\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}{2c}+x\left(a+b\cos^{-1}(cx)\right)^{3/2}+\frac{3b^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}}}{\sqrt{b}}\right.}{2c}$$

Mathematica [C] time = 2.37094, size = 295, normalized size = 1.86

$$b\left(2\left(2cx\cos^{-1}(cx)-3\sqrt{1-c^2x^2}\right)\sqrt{a+b\cos^{-1}(cx)}-\sqrt{2\pi}\sqrt{\frac{1}{b}}\left(3b\sin\left(\frac{a}{b}\right)-2a\cos\left(\frac{a}{b}\right)\right)\mathrm{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[c*x])^(3/2),x]
```

```
[Out] ((-2*a*Sqrt[a + b*ArcCos[c*x]]*(-(Gamma[3/2, ((-I)*(a + b*ArcCos[c*x]))/b]/
Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]) - (E^(((2*I)*a)/b)*Gamma[3/2, (I*(a + b
*ArcCos[c*x]))/b])/Sqrt[(I*(a + b*ArcCos[c*x]))/b]))/E^((I*a)/b) + b*(2*Sqr
t[a + b*ArcCos[c*x]]*(-3*Sqrt[1 - c^2*x^2] + 2*c*x*ArcCos[c*x]) + Sqrt[b^(-
1)]*Sqrt[2*Pi]*FresnelS[Sqrt[b^(-1)]*Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]]]*(3
*b*Cos[a/b] + 2*a*Sin[a/b]) - Sqrt[b^(-1)]*Sqrt[2*Pi]*FresnelC[Sqrt[b^(-1)]
*Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]]]*(-2*a*Cos[a/b] + 3*b*Sin[a/b])))/(4*c)
```

Maple [B] time = 0.11, size = 270, normalized size = 1.7

$$\frac{1}{4c}\left(3\sqrt{b^{-1}}\sqrt{\pi}\sqrt{2}\sqrt{a+b\arccos(cx)}\cos\left(\frac{a}{b}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\sqrt{a+b\arccos(cx)}}{\sqrt{b^{-1}}\sqrt{\pi}b}\right)b^2-3\sqrt{b^{-1}}\sqrt{\pi}\sqrt{2}\sqrt{a+b\arccos(cx)}\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^(3/2),x)
```

```
[Out] 1/4/c*(3*(1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(a/b)*Fres
nelS(2^(1/2)/Pi^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b^2-3*(1/b)^(1
/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(a/b)*FresnelC(2^(1/2)/Pi^(
1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b^2+4*arccos(c*x)^2*cos((a+b*ar
ccos(c*x))/b-a/b)*b^2+8*arccos(c*x)*cos((a+b*arccos(c*x))/b-a/b)*a*b-6*arcc
os(c*x)*sin((a+b*arccos(c*x))/b-a/b)*b^2+4*cos((a+b*arccos(c*x))/b-a/b)*a^2
-6*sin((a+b*arccos(c*x))/b-a/b)*a*b)/(a+b*arccos(c*x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\arccos(cx)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccos(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\,\mathrm{acos}\,(cx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**(3/2),x)
```

```
[Out] Integral((a + b*acos(c*x))**(3/2), x)
```

Giac [B] time = 2.23727, size = 905, normalized size = 5.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] -3/8*sqrt(2)*sqrt(pi)*b^4*i*erf(-1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/sqrt
(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(a*i/b)/((
b^3*i/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c) - 3/8*sqrt(2)*sqrt(pi)*b^4*i*erf(
1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arc
cos(c*x) + a)*sqrt(abs(b))/b)*e^(-a*i/b)/((b^3*i/sqrt(abs(b)) - b^2*sqrt(ab
s(b)))*c) - 1/4*sqrt(2)*sqrt(pi)*a*b^3*erf(-1/2*sqrt(2)*sqrt(b*arccos(c*x)
+ a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e
^(a*i/b)/((b^3*i/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c) + 1/4*sqrt(2)*sqrt(pi)
*a*b^3*erf(1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2)
*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-a*i/b)/((b^3*i/sqrt(abs(b)) -
b^2*sqrt(abs(b)))*c) + 1/4*sqrt(2)*sqrt(pi)*a*b^2*erf(-1/2*sqrt(2)*sqrt(b*a
rccos(c*x) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(a
bs(b))/b)*e^(a*i/b)/((b^2*i/sqrt(abs(b)) + b*sqrt(abs(b)))*c) - 1/4*sqrt(2)
*sqrt(pi)*a*b^2*erf(1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/sqrt(abs(b)) - 1/
2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-a*i/b)/((b^2*i/sqrt(a
bs(b)) - b*sqrt(abs(b)))*c) + 3/4*sqrt(b*arccos(c*x) + a)*b*i*e^(i*arccos(c
*x))/c + 1/2*sqrt(b*arccos(c*x) + a)*b*arccos(c*x)*e^(i*arccos(c*x))/c - 3/
4*sqrt(b*arccos(c*x) + a)*b*i*e^(-i*arccos(c*x))/c + 1/2*sqrt(b*arccos(c*x)
 + a)*b*arccos(c*x)*e^(-i*arccos(c*x))/c + 1/2*sqrt(b*arccos(c*x) + a)*a*e^
(i*arccos(c*x))/c + 1/2*sqrt(b*arccos(c*x) + a)*a*e^(-i*arccos(c*x))/c
```

3.181 $\int \frac{\left(a+b\cos^{-1}(cx)\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\left(a+b\cos^{-1}(cx)\right)^{3/2}}{x},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCos[c*x])^(3/2)/x, x]
```

Rubi [A] time = 0.0391337, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^{3/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCos[c*x])^(3/2)/x,x]
```

```
[Out] Defer[Int][(a + b*ArcCos[c*x])^(3/2)/x, x]
```

Rubi steps

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^{3/2}}{x}\,dx=\int \frac{\left(a+b\cos^{-1}(cx)\right)^{3/2}}{x}\,dx$$

Mathematica [A] time = 3.19776, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^{3/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCos[c*x])^(3/2)/x,x]
```

```
[Out] Integrate[(a + b*ArcCos[c*x])^(3/2)/x, x]
```

Maple [A] time = 0.221, size = 0, normalized size = 0.

$$\int \frac{1}{x} (a + b \arccos (cx))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^(3/2)/x,x)
```

```
[Out] int((a+b*arccos(c*x))^(3/2)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arccos (cx) + a)^{\frac{3}{2}}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((b*arccos(c*x) + a)^(3/2)/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{acos}(cx))^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**(3/2)/x,x)
```

```
[Out] Integral((a + b*acos(c*x))**(3/2)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arccos(cx) + a)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)^(3/2)/x, x)
```

3.182 $\int \frac{\left(a+b\cos^{-1}(cx)\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\left(a+b\cos^{-1}(cx)\right)^{3/2}}{x^2},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCos[c*x])^(3/2)/x^2, x]
```

Rubi [A] time = 0.038918, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^{3/2}}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCos[c*x])^(3/2)/x^2,x]

[Out] Defer[Int][(a + b*ArcCos[c*x])^(3/2)/x^2, x]
```

Rubi steps

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^{3/2}}{x^2}\,dx=\int \frac{\left(a+b\cos^{-1}(cx)\right)^{3/2}}{x^2}\,dx$$

Mathematica [A] time = 9.75978, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^{3/2}}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCos[c*x])^(3/2)/x^2,x]
```

```
[Out] Integrate[(a + b*ArcCos[c*x])^(3/2)/x^2, x]
```

Maple [A] time = 0.286, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (a + b \arccos(cx))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^(3/2)/x^2,x)
```

```
[Out] int((a+b*arccos(c*x))^(3/2)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arccos(cx) + a)^{\frac{3}{2}}}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*arccos(c*x) + a)^(3/2)/x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{acos}\,(cx))^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**(3/2)/x**2,x)
```

```
[Out] Integral((a + b*acos(c*x))**(3/2)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arccos(cx) + a)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)^(3/2)/x^2, x)
```

3.183 $\int x^2 \left(a+b\cos^{-1}(cx)\right)^{5/2}\,dx$

Optimal. Leaf size=358

$$\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}+\frac{5\sqrt{\frac{\pi}{6}}b^{5/2}\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{144c^3}+\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}\sin\left(\frac{a}{b}\right)S\left(\sqrt{\right.}\right.}{16c^3}$$

```
[Out] (-5*b^2*x*Sqrt[a + b*ArcCos[c*x]])/(6*c^2) - (5*b^2*x^3*Sqrt[a + b*ArcCos[c
*x]])/36 - (5*b*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(3/2))/(9*c^3) - (5*b
*x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(3/2))/(18*c) + (x^3*(a + b*ArcC
os[c*x])^(5/2))/3 + (15*b^(5/2)*Sqrt[Pi/2]*Cos[a/b]*FresnelC[(Sqrt[2/Pi]*Sq
rt[a + b*ArcCos[c*x]])/Sqrt[b]])/(16*c^3) + (5*b^(5/2)*Sqrt[Pi/6]*Cos[(3*a)
/b]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(144*c^3) + (15
*b^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*
Sin[a/b])/(16*c^3) + (5*b^(5/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[6/Pi]*Sqrt[a + b*
ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/(144*c^3)
```

Rubi [A] time = 1.31637, antiderivative size = 358, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {4630, 4708, 4678, 4620, 4724, 3306, 3305, 3351, 3304, 3352, 3312}

$$\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}+\frac{5\sqrt{\frac{\pi}{6}}b^{5/2}\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{144c^3}+\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}\sin\left(\frac{a}{b}\right)S\left(\sqrt{\right.}\right.}{16c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcCos[c*x])^(5/2),x]
```


```
[Out] (-5*b^2*x*Sqrt[a + b*ArcCos[c*x]])/(6*c^2) - (5*b^2*x^3*Sqrt[a + b*ArcCos[c
*x]])/36 - (5*b*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(3/2))/(9*c^3) - (5*b
*x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(3/2))/(18*c) + (x^3*(a + b*ArcC
os[c*x])^(5/2))/3 + (15*b^(5/2)*Sqrt[Pi/2]*Cos[a/b]*FresnelC[(Sqrt[2/Pi]*Sq
rt[a + b*ArcCos[c*x]])/Sqrt[b]])/(16*c^3) + (5*b^(5/2)*Sqrt[Pi/6]*Cos[(3*a)
/b]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(144*c^3) + (15
*b^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*
Sin[a/b])/(16*c^3) + (5*b^(5/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[6/Pi]*Sqrt[a + b*
ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/(144*c^3)
```


Rule 4630

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
x^(m + 1)*(a + b*ArcCos[c*x])^n)/(m + 1), x] + Dist[(b*c*n)/(m + 1), Int[(x
^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int x^2 \left(a + b\cos^{-1}(cx)\right)^{5/2} dx = \frac{1}{3}x^3 \left(a + b\cos^{-1}(cx)\right)^{5/2} + \frac{1}{6}(5bc)\int \frac{x^3 \left(a + b\cos^{-1}(cx)\right)^{3/2}}{\sqrt{1 - c^2x^2}} dx$$

$$= -\frac{5bx^2\sqrt{1-c^2x^2}\left(a + b\cos^{-1}(cx)\right)^{3/2}}{18c} + \frac{1}{3}x^3 \left(a + b\cos^{-1}(cx)\right)^{5/2} - \frac{1}{12}\left(5b^2\right)\int x^2\sqrt{a + b}$$

$$= -\frac{5}{36}b^2x^3\sqrt{a + b\cos^{-1}(cx)} - \frac{5b\sqrt{1-c^2x^2}\left(a + b\cos^{-1}(cx)\right)^{3/2}}{9c^3} - \frac{5bx^2\sqrt{1-c^2x^2}\left(a + b\cos\right.}{18c}$$

$$= -\frac{5b^2x\sqrt{a + b\cos^{-1}(cx)}}{6c^2} - \frac{5}{36}b^2x^3\sqrt{a + b\cos^{-1}(cx)} - \frac{5b\sqrt{1-c^2x^2}\left(a + b\cos^{-1}(cx)\right)^{3/2}}{9c^3} -$$

$$= -\frac{5b^2x\sqrt{a + b\cos^{-1}(cx)}}{6c^2} - \frac{5}{36}b^2x^3\sqrt{a + b\cos^{-1}(cx)} - \frac{5b\sqrt{1-c^2x^2}\left(a + b\cos^{-1}(cx)\right)^{3/2}}{9c^3} -$$

$$= -\frac{5b^2x\sqrt{a + b\cos^{-1}(cx)}}{6c^2} - \frac{5}{36}b^2x^3\sqrt{a + b\cos^{-1}(cx)} - \frac{5b\sqrt{1-c^2x^2}\left(a + b\cos^{-1}(cx)\right)^{3/2}}{9c^3} -$$

$$= -\frac{5b^2x\sqrt{a + b\cos^{-1}(cx)}}{6c^2} - \frac{5}{36}b^2x^3\sqrt{a + b\cos^{-1}(cx)} - \frac{5b\sqrt{1-c^2x^2}\left(a + b\cos^{-1}(cx)\right)^{3/2}}{9c^3} -$$

$$= -\frac{5b^2x\sqrt{a + b\cos^{-1}(cx)}}{6c^2} - \frac{5}{36}b^2x^3\sqrt{a + b\cos^{-1}(cx)} - \frac{5b\sqrt{1-c^2x^2}\left(a + b\cos^{-1}(cx)\right)^{3/2}}{9c^3} -$$

$$= -\frac{5b^2x\sqrt{a + b\cos^{-1}(cx)}}{6c^2} - \frac{5}{36}b^2x^3\sqrt{a + b\cos^{-1}(cx)} - \frac{5b\sqrt{1-c^2x^2}\left(a + b\cos^{-1}(cx)\right)^{3/2}}{9c^3} -$$

Mathematica [C] time = 16.464, size = 1002, normalized size = 2.8

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcCos[c*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[a + b*ArcCos[c*x]]*(9*E^(((2*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x])
)/b]*Gamma[3/2, ((-I)*(a + b*ArcCos[c*x]))/b] + 9*E^(((4*I)*a)/b)*Sqrt[((-I
)*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, (I*(a + b*ArcCos[c*x]))/b] + Sqrt[3]*(
Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((-3*I)*(a + b*ArcCos[c*x]))/b]
```

```
+ E^(((6*I)*a)/b)*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((3*I)*(a +
 b*ArcCos[c*x]))/b])))/(72*c^3*E^(((3*I)*a)/b)*Sqrt[(a + b*ArcCos[c*x])^2/b
^2]) + (a*b*(-18*Sqrt[a + b*ArcCos[c*x]]*(3*Sqrt[1 - c^2*x^2] - 2*c*x*ArcCo
s[c*x]) + 9*Sqrt[b^(-1)]*Sqrt[2*Pi]*FresnelS[Sqrt[b^(-1)]*Sqrt[2/Pi]*Sqrt[a
 + b*ArcCos[c*x]]]*(3*b*Cos[a/b] + 2*a*Sin[a/b]) - 9*Sqrt[b^(-1)]*Sqrt[2*Pi
]*FresnelC[Sqrt[b^(-1)]*Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]]]*(-2*a*Cos[a/b]
+ 3*b*Sin[a/b]) + Sqrt[b^(-1)]*Sqrt[6*Pi]*FresnelS[Sqrt[b^(-1)]*Sqrt[6/Pi]*
Sqrt[a + b*ArcCos[c*x]]]*(b*Cos[(3*a)/b] + 2*a*Sin[(3*a)/b]) - Sqrt[b^(-1)]
*Sqrt[6*Pi]*FresnelC[Sqrt[b^(-1)]*Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]]]*(-2*a
*Cos[(3*a)/b] + b*Sin[(3*a)/b]) - 6*Sqrt[a + b*ArcCos[c*x]]*(-2*ArcCos[c*x]
*Cos[3*ArcCos[c*x]] + Sin[3*ArcCos[c*x]])))/(72*c^3) - ((-54*Sqrt[a + b*Arc
Cos[c*x]]*(2*Sqrt[1 - c^2*x^2]*(a - 5*b*ArcCos[c*x]) + b*c*x*(-15 + 4*ArcCo
s[c*x]^2)))/Sqrt[b^(-1)] + 27*Sqrt[2*Pi]*FresnelC[Sqrt[b^(-1)]*Sqrt[2/Pi]*S
qrt[a + b*ArcCos[c*x]]]*((4*a^2 - 15*b^2)*Cos[a/b] - 12*a*b*Sin[a/b]) - 27*
Sqrt[2*Pi]*FresnelS[Sqrt[b^(-1)]*Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]]]*(-12*a
*b*Cos[a/b] + (-4*a^2 + 15*b^2)*Sin[a/b]) + Sqrt[6*Pi]*FresnelC[Sqrt[b^(-1)
]*Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]]]*((12*a^2 - 5*b^2)*Cos[(3*a)/b] - 12*a
*b*Sin[(3*a)/b]) - Sqrt[6*Pi]*FresnelS[Sqrt[b^(-1)]*Sqrt[6/Pi]*Sqrt[a + b*A
rcCos[c*x]]]*(-12*a*b*Cos[(3*a)/b] + (-12*a^2 + 5*b^2)*Sin[(3*a)/b]) - (6*S
qrt[a + b*ArcCos[c*x]]*(b*(-5 + 12*ArcCos[c*x]^2)*Cos[3*ArcCos[c*x]] + 2*(a
 - 5*b*ArcCos[c*x])*Sin[3*ArcCos[c*x]]))/Sqrt[b^(-1)])/(864*Sqrt[b^(-1)]*c^
3)
```

Maple [B] time = 0.171, size = 792, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccos(c*x))^(5/2),x)
```

```
[Out] 1/864/c^3*(5*3^(1/2)*(1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*c
os(3*a/b)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(
1/2)/b)*b^3+5*3^(1/2)*(1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*
sin(3*a/b)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^
(1/2)/b)*b^3+405*(1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(a
/b)*FresnelC(2^(1/2)/Pi^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b^3+40
5*(1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(a/b)*FresnelS(2^
(1/2)/Pi^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b^3+216*arccos(c*x)^3
*cos((a+b*arccos(c*x))/b-a/b)*b^3+72*arccos(c*x)^3*cos(3*(a+b*arccos(c*x))/
b-3*a/b)*b^3+648*arccos(c*x)^2*cos((a+b*arccos(c*x))/b-a/b)*a*b^2-540*arcco
s(c*x)^2*sin((a+b*arccos(c*x))/b-a/b)*b^3+216*arccos(c*x)^2*cos(3*(a+b*arcc
```

```
os(c*x))/b-3*a/b)*a*b^2-60*arccos(c*x)^2*sin(3*(a+b*arccos(c*x))/b-3*a/b)*b
^3+648*arccos(c*x)*cos((a+b*arccos(c*x))/b-a/b)*a^2*b-810*arccos(c*x)*cos((
a+b*arccos(c*x))/b-a/b)*b^3-1080*arccos(c*x)*sin((a+b*arccos(c*x))/b-a/b)*a
*b^2+216*arccos(c*x)*cos(3*(a+b*arccos(c*x))/b-3*a/b)*a^2*b-30*arccos(c*x)*
cos(3*(a+b*arccos(c*x))/b-3*a/b)*b^3-120*arccos(c*x)*sin(3*(a+b*arccos(c*x)
)/b-3*a/b)*a*b^2+216*cos((a+b*arccos(c*x))/b-a/b)*a^3-810*cos((a+b*arccos(c
*x))/b-a/b)*a*b^2-540*sin((a+b*arccos(c*x))/b-a/b)*a^2*b+72*cos(3*(a+b*arcc
os(c*x))/b-3*a/b)*a^3-30*cos(3*(a+b*arccos(c*x))/b-3*a/b)*a*b^2-60*sin(3*(a
+b*arccos(c*x))/b-3*a/b)*a^2*b)/(a+b*arccos(c*x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \arccos(cx) + a)^{\frac{5}{2}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccos(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccos(c*x) + a)^(5/2)*x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccos(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acos(c*x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 4.66928, size = 3471, normalized size = 9.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccos(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] -3/16*sqrt(2)*sqrt(pi)*a*b^4*i*erf(-1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/s
qrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(a*i/b)
/((b^3*i/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c^3) - 3/16*sqrt(2)*sqrt(pi)*a*b^
4*i*erf(1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*sq
rt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-a*i/b)/((b^3*i/sqrt(abs(b)) - b^2
*sqrt(abs(b)))*c^3) - 1/24*sqrt(pi)*a*b^(7/2)*i*erf(-1/2*sqrt(6)*sqrt(b*arc
cos(c*x) + a)*sqrt(b)*i/abs(b) - 1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b
))*e^(3*a*i/b)/((sqrt(6)*b^3*i/abs(b) + sqrt(6)*b^2)*c^3) - 1/8*sqrt(2)*sqr
t(pi)*a^2*b^3*erf(-1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/sqrt(abs(b)) - 1/2
*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(a*i/b)/((b^3*i/sqrt(abs
(b)) + b^2*sqrt(abs(b)))*c^3) + 3/16*sqrt(2)*sqrt(pi)*a*b^3*i*erf(-1/2*sqrt
(2)*sqrt(b*arccos(c*x) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x)
 + a)*sqrt(abs(b))/b)*e^(a*i/b)/((b^2*i/sqrt(abs(b)) + b*sqrt(abs(b)))*c^3)
 + 1/8*sqrt(2)*sqrt(pi)*a^2*b^3*erf(1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/s
qrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-a*i/b
)/((b^3*i/sqrt(abs(b)) - b^2*sqrt(abs(b)))*c^3) + 3/16*sqrt(2)*sqrt(pi)*a*b
^3*i*erf(1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*s
qrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-a*i/b)/((b^2*i/sqrt(abs(b)) - b*
sqrt(abs(b)))*c^3) - 1/24*sqrt(pi)*a*b^(7/2)*i*erf(1/2*sqrt(6)*sqrt(b*arcco
s(c*x) + a)*sqrt(b)*i/abs(b) - 1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b))
*e^(-3*a*i/b)/((sqrt(6)*b^3*i/abs(b) - sqrt(6)*b^2)*c^3) - 1/12*sqrt(pi)*a^
2*b^(5/2)*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - 1/2*s
qrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b))*e^(3*a*i/b)/((sqrt(6)*b^3*i/abs(b)
+ sqrt(6)*b^2)*c^3) + 1/24*sqrt(pi)*a*b^(5/2)*i*erf(-1/2*sqrt(6)*sqrt(b*arc
cos(c*x) + a)*sqrt(b)*i/abs(b) - 1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b
))*e^(3*a*i/b)/((sqrt(6)*b^2*i/abs(b) + sqrt(6)*b)*c^3) + 1/8*sqrt(2)*sqrt(
pi)*a^2*b^2*erf(-1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/sqrt(abs(b)) - 1/2*s
qrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(a*i/b)/((b^2*i/sqrt(abs(b
)) + b*sqrt(abs(b)))*c^3) - 15/64*sqrt(2)*sqrt(pi)*b^4*erf(-1/2*sqrt(2)*sqr
t(b*arccos(c*x) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*s
qrt(abs(b))/b)*e^(a*i/b)/((b^2*i/sqrt(abs(b)) + b*sqrt(abs(b)))*c^3) - 1/8*
sqrt(2)*sqrt(pi)*a^2*b^2*erf(1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/sqrt(abs
```

(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-a*i/b)/((b^2*i/sqrt(abs(b)) - b*sqrt(abs(b)))*c^3) + 15/64*sqrt(2)*sqrt(pi)*b^4*erf(1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-a*i/b)/((b^2*i/sqrt(abs(b)) - b*sqrt(abs(b)))*c^3) + 1/12*sqrt(pi)*a^2*b^(5/2)*erf(1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - 1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b))*e^(-3*a*i/b)/((sqrt(6)*b^3*i/abs(b) - sqrt(6)*b^2)*c^3) + 1/24*sqrt(pi)*a*b^(5/2)*i*erf(1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - 1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b))*e^(-3*a*i/b)/((sqrt(6)*b^2*i/abs(b) - sqrt(6)*b)*c^3) + 5/144*sqrt(b*arccos(c*x) + a)*b^2*i*arccos(c*x)*e^(3*i*arccos(c*x))/c^3 + 1/24*sqrt(b*arccos(c*x) + a)*b^2*arccos(c*x)^2*e^(3*i*arccos(c*x))/c^3 + 5/16*sqrt(b*arccos(c*x) + a)*b^2*i*arccos(c*x)*e^(i*arccos(c*x))/c^3 + 1/8*sqrt(b*arccos(c*x) + a)*b^2*arccos(c*x)^2*e^(i*arccos(c*x))/c^3 - 5/16*sqrt(b*arccos(c*x) + a)*b^2*i*arccos(c*x)*e^(-i*arccos(c*x))/c^3 + 1/8*sqrt(b*arccos(c*x) + a)*b^2*arccos(c*x)^2*e^(-i*arccos(c*x))/c^3 - 5/144*sqrt(b*arccos(c*x) + a)*b^2*i*arccos(c*x)*e^(-3*i*arccos(c*x))/c^3 + 1/24*sqrt(b*arccos(c*x) + a)*b^2*arccos(c*x)^2*e^(-3*i*arccos(c*x))/c^3 + 1/24*sqrt(pi)*a^2*b^2*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - 1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b))*e^(3*a*i/b)/((sqrt(6)*b^(5/2)*i/abs(b) + sqrt(6)*b^(3/2))*c^3) - 1/24*sqrt(pi)*a^2*b^2*erf(1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - 1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b))*e^(-3*a*i/b)/((sqrt(6)*b^(5/2)*i/abs(b) - sqrt(6)*b^(3/2))*c^3) + 1/24*sqrt(pi)*a^2*b^(3/2)*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - 1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b))*e^(3*a*i/b)/((sqrt(6)*b^2*i/abs(b) + sqrt(6)*b)*c^3) - 5/288*sqrt(pi)*b^(7/2)*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - 1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b))*e^(3*a*i/b)/((sqrt(6)*b^2*i/abs(b) + sqrt(6)*b)*c^3) - 1/24*sqrt(pi)*a^2*b^(3/2)*erf(1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - 1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b))*e^(-3*a*i/b)/((sqrt(6)*b^2*i/abs(b) - sqrt(6)*b)*c^3) + 5/288*sqrt(pi)*b^(7/2)*erf(1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - 1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b))*e^(-3*a*i/b)/((sqrt(6)*b^2*i/abs(b) - sqrt(6)*b)*c^3) + 5/144*sqrt(b*arccos(c*x) + a)*a*b*i*e^(3*i*arccos(c*x))/c^3 + 1/12*sqrt(b*arccos(c*x) + a)*a*b*arccos(c*x)*e^(3*i*arccos(c*x))/c^3 + 5/16*sqrt(b*arccos(c*x) + a)*a*b*i*e^(i*arccos(c*x))/c^3 + 1/4*sqrt(b*arccos(c*x) + a)*a*b*arccos(c*x)*e^(i*arccos(c*x))/c^3 - 5/16*sqrt(b*arccos(c*x) + a)*a*b*i*e^(-i*arccos(c*x))/c^3 + 1/4*sqrt(b*arccos(c*x) + a)*a*b*arccos(c*x)*e^(-i*arccos(c*x))/c^3 - 5/144*sqrt(b*arccos(c*x) + a)*a*b*i*e^(-3*i*arccos(c*x))/c^3 + 1/12*sqrt(b*arccos(c*x) + a)*a*b*arccos(c*x)*e^(-3*i*arccos(c*x))/c^3 + 1/24*sqrt(b*arccos(c*x) + a)*a^2*e^(3*i*arccos(c*x))/c^3 - 5/288*sqrt(b*arccos(c*x) + a)*b^2*e^(3*i*arccos(c*x))/c^3 + 1/8*sqrt(b*arccos(c*x) + a)*a^2*e^(i*arccos(c*x))/c^3 - 15/32*sqrt(b*arccos(c*x) + a)*b^2*e^(i*arccos(c*x))/c^3 + 1/8*sqrt(b*arccos(c*x) + a)*a^2*e^(-i*arccos(c*x))/c^3 - 15/32*sqrt(b*arccos(c*x) + a)*b^2*e^(-i*arccos(c*x))/c^3 + 1/24*sqrt(b*arccos(c*x) + a)*a^2*e^(-3*i*arccos(c*x))/c^3 - 5/288*sqrt(b*arccos(c*x) + a)*b^2*e^(-3*i*arccos(c*x))/c^

3

3.184 $\int x\left(a+b\cos^{-1}(cx)\right)^{5/2}\,dx$

Optimal. Leaf size=216

$$\frac{15\sqrt{\pi}b^{5/2}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{\pi}\sqrt{b}}\right)}{128c^2}+\frac{15\sqrt{\pi}b^{5/2}\sin\left(\frac{2a}{b}\right)S\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{128c^2}+\frac{15b^2\sqrt{a+b\cos^{-1}(cx)}}{64c^2}-\frac{15}{32}b^2x^2$$

```
[Out] (15*b^2*Sqrt[a + b*ArcCos[c*x]])/(64*c^2) - (15*b^2*x^2*Sqrt[a + b*ArcCos[c
*x]])/32 - (5*b*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(3/2))/(8*c) - (a +
 b*ArcCos[c*x])^(5/2)/(4*c^2) + (x^2*(a + b*ArcCos[c*x])^(5/2))/2 + (15*b^(
5/2)*Sqrt[Pi]*Cos[(2*a)/b]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sq
rt[Pi])])/(128*c^2) + (15*b^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[a + b*ArcCos[c*
x]])/(Sqrt[b]*Sqrt[Pi])]*Sin[(2*a)/b])/(128*c^2)
```

Rubi [A] time = 0.702873, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {4630, 4708, 4642, 4724, 3312, 3306, 3305, 3351, 3304, 3352}

$$\frac{15\sqrt{\pi}b^{5/2}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{\pi}\sqrt{b}}\right)}{128c^2}+\frac{15\sqrt{\pi}b^{5/2}\sin\left(\frac{2a}{b}\right)S\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{128c^2}+\frac{15b^2\sqrt{a+b\cos^{-1}(cx)}}{64c^2}-\frac{15}{32}b^2x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcCos[c*x])^(5/2),x]
```

```
[Out] (15*b^2*Sqrt[a + b*ArcCos[c*x]])/(64*c^2) - (15*b^2*x^2*Sqrt[a + b*ArcCos[c
*x]])/32 - (5*b*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(3/2))/(8*c) - (a +
 b*ArcCos[c*x])^(5/2)/(4*c^2) + (x^2*(a + b*ArcCos[c*x])^(5/2))/2 + (15*b^(
5/2)*Sqrt[Pi]*Cos[(2*a)/b]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sq
rt[Pi])])/(128*c^2) + (15*b^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[a + b*ArcCos[c*
x]])/(Sqrt[b]*Sqrt[Pi])]*Sin[(2*a)/b])/(128*c^2)
```

Rule 4630

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
x^(m + 1)*(a + b*ArcCos[c*x])^n)/(m + 1), x] + Dist[(b*c*n)/(m + 1), Int[(x
^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4708

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcCos[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcCos[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int x\left(a+b\cos^{-1}(cx)\right)^{5/2}dx=\frac{1}{2}x^2\left(a+b\cos^{-1}(cx)\right)^{5/2}+\frac{1}{4}(5bc)\int\frac{x^2\left(a+b\cos^{-1}(cx)\right)^{3/2}}{\sqrt{1-c^2x^2}}dx$$

$$=-\frac{5bx\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^{3/2}}{8c}+\frac{1}{2}x^2\left(a+b\cos^{-1}(cx)\right)^{5/2}-\frac{1}{16}\left(15b^2\right)\int x\sqrt{a+b\,\mathrm{co}}$$

$$=-\frac{15}{32}b^2x^2\sqrt{a+b\cos^{-1}(cx)}-\frac{5bx\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^{3/2}}{8c}-\frac{\left(a+b\cos^{-1}(cx)\right)^{5/2}}{4c^2}+$$

$$=-\frac{15}{32}b^2x^2\sqrt{a+b\cos^{-1}(cx)}-\frac{5bx\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^{3/2}}{8c}-\frac{\left(a+b\cos^{-1}(cx)\right)^{5/2}}{4c^2}+$$

$$=-\frac{15}{32}b^2x^2\sqrt{a+b\cos^{-1}(cx)}-\frac{5bx\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^{3/2}}{8c}-\frac{\left(a+b\cos^{-1}(cx)\right)^{5/2}}{4c^2}+$$

$$=\frac{15b^2\sqrt{a+b\cos^{-1}(cx)}}{64c^2}-\frac{15}{32}b^2x^2\sqrt{a+b\cos^{-1}(cx)}-\frac{5bx\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^{3/2}}{8c}-$$

$$=\frac{15b^2\sqrt{a+b\cos^{-1}(cx)}}{64c^2}-\frac{15}{32}b^2x^2\sqrt{a+b\cos^{-1}(cx)}-\frac{5bx\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^{3/2}}{8c}-$$

$$=\frac{15b^2\sqrt{a+b\cos^{-1}(cx)}}{64c^2}-\frac{15}{32}b^2x^2\sqrt{a+b\cos^{-1}(cx)}-\frac{5bx\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^{3/2}}{8c}-$$

$$=\frac{15b^2\sqrt{a+b\cos^{-1}(cx)}}{64c^2}-\frac{15}{32}b^2x^2\sqrt{a+b\cos^{-1}(cx)}-\frac{5bx\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^{3/2}}{8c}-$$

Mathematica [A] time = 1.93229, size = 201, normalized size = 0.93

$$2\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(cx)}\left(\left(16a^2-15b^2\right)\cos\left(2\cos^{-1}(cx)\right)+4b\cos^{-1}(cx)\left(8a\cos\left(2\cos^{-1}(cx)\right)-5b\sin\left(2\cos^{-1}(cx)\right)\right)-20a$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcCos[c*x])^(5/2),x]
```

```
[Out] (15*b^2*Sqrt[Pi]*Cos[(2*a)/b]*FresnelC[(2*Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[c*
x]])/Sqrt[Pi]] + 15*b^2*Sqrt[Pi]*FresnelS[(2*Sqrt[b^(-1)]*Sqrt[a + b*ArcCos
[c*x]])/Sqrt[Pi]]*Sin[(2*a)/b] + 2*Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[c*x]]*((1
6*a^2 - 15*b^2)*Cos[2*ArcCos[c*x]] + 16*b^2*ArcCos[c*x]^2*Cos[2*ArcCos[c*x]
] - 20*a*b*Sin[2*ArcCos[c*x]] + 4*b*ArcCos[c*x]*(8*a*Cos[2*ArcCos[c*x]] - 5
*b*Sin[2*ArcCos[c*x]])))/(128*Sqrt[b^(-1)]*c^2)
```

Maple [B] time = 0.132, size = 366, normalized size = 1.7

$$\frac{b}{128\,c^2\sqrt{\pi}}\left(32\,(\arccos(cx))^2\sqrt{a+b\arccos(cx)}\cos\left(2\,\frac{a+b\arccos(cx)}{b}-2\,\frac{a}{b}\right)\sqrt{\pi}\sqrt{b^{-1}}b^2+64\,\arccos(cx)\sqrt{a+b\,\mathrm{arcc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccos(c*x))^(5/2),x)
```

```
[Out] 1/128/c^2*b*(32*arccos(c*x)^2*(a+b*arccos(c*x))^(1/2)*cos(2*(a+b*arccos(c*x
))/b-2*a/b)*Pi^(1/2)*(1/b)^(1/2)*b^2+64*arccos(c*x)*(a+b*arccos(c*x))^(1/2)
*cos(2*(a+b*arccos(c*x))/b-2*a/b)*Pi^(1/2)*(1/b)^(1/2)*a*b-40*arccos(c*x)*(
a+b*arccos(c*x))^(1/2)*sin(2*(a+b*arccos(c*x))/b-2*a/b)*Pi^(1/2)*(1/b)^(1/2
)*b^2+32*(a+b*arccos(c*x))^(1/2)*cos(2*(a+b*arccos(c*x))/b-2*a/b)*Pi^(1/2)*
(1/b)^(1/2)*a^2-30*(a+b*arccos(c*x))^(1/2)*cos(2*(a+b*arccos(c*x))/b-2*a/b)
*Pi^(1/2)*(1/b)^(1/2)*b^2-40*(a+b*arccos(c*x))^(1/2)*sin(2*(a+b*arccos(c*x)
)/b-2*a/b)*Pi^(1/2)*(1/b)^(1/2)*a*b+15*Pi*b^2*FresnelC(2/Pi^(1/2)/(1/b)^(1/
2)*(a+b*arccos(c*x))^(1/2)/b)*cos(2*a/b)+15*Pi*b^2*sin(2*a/b)*FresnelS(2/Pi
^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b))/Pi^(1/2)*(1/b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \arccos(cx) + a)^{\frac{5}{2}} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccos(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccos(c*x) + a)^(5/2)*x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccos(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acos(c*x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.40081, size = 1638, normalized size = 7.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccos(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] -3/32*sqrt(pi)*a*b^(7/2)*i*erf(-sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) -
sqrt(b*arccos(c*x) + a)/sqrt(b))*e^(2*a*i/b)/((b^3*i/abs(b) + b^2)*c^2) - 3
/32*sqrt(pi)*a*b^(7/2)*i*erf(sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - sqr
t(b*arccos(c*x) + a)/sqrt(b))*e^(-2*a*i/b)/((b^3*i/abs(b) - b^2)*c^2) - 1/8
*sqrt(pi)*a^2*b^(5/2)*erf(-sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - sqrt(
b*arccos(c*x) + a)/sqrt(b))*e^(2*a*i/b)/((b^3*i/abs(b) + b^2)*c^2) + 3/32*s
qrt(pi)*a*b^(5/2)*i*erf(-sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - sqrt(b*
arccos(c*x) + a)/sqrt(b))*e^(2*a*i/b)/((b^2*i/abs(b) + b)*c^2) + 1/8*sqrt(p
i)*a^2*b^(5/2)*erf(sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - sqrt(b*arccos
(c*x) + a)/sqrt(b))*e^(-2*a*i/b)/((b^3*i/abs(b) - b^2)*c^2) + 3/32*sqrt(pi)
*a*b^(5/2)*i*erf(sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - sqrt(b*arccos(c
*x) + a)/sqrt(b))*e^(-2*a*i/b)/((b^2*i/abs(b) - b)*c^2) + 5/32*sqrt(b*arcco
s(c*x) + a)*b^2*i*arccos(c*x)*e^(2*i*arccos(c*x))/c^2 + 1/8*sqrt(b*arccos(c
*x) + a)*b^2*arccos(c*x)^2*e^(2*i*arccos(c*x))/c^2 - 5/32*sqrt(b*arccos(c*x
) + a)*b^2*i*arccos(c*x)*e^(-2*i*arccos(c*x))/c^2 + 1/8*sqrt(b*arccos(c*x)
+ a)*b^2*arccos(c*x)^2*e^(-2*i*arccos(c*x))/c^2 + 1/16*sqrt(pi)*a^2*b^2*erf
(-sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - sqrt(b*arccos(c*x) + a)/sqrt(b
))*e^(2*a*i/b)/((b^(5/2)*i/abs(b) + b^(3/2))*c^2) - 1/16*sqrt(pi)*a^2*b^2*e
rf(sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - sqrt(b*arccos(c*x) + a)/sqrt(
b))*e^(-2*a*i/b)/((b^(5/2)*i/abs(b) - b^(3/2))*c^2) + 1/16*sqrt(pi)*a^2*b^(
3/2)*erf(-sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - sqrt(b*arccos(c*x) + a
)/sqrt(b))*e^(2*a*i/b)/((b^2*i/abs(b) + b)*c^2) - 15/256*sqrt(pi)*b^(7/2)*e
rf(-sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - sqrt(b*arccos(c*x) + a)/sqrt
(b))*e^(2*a*i/b)/((b^2*i/abs(b) + b)*c^2) - 1/16*sqrt(pi)*a^2*b^(3/2)*erf(s
qrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - sqrt(b*arccos(c*x) + a)/sqrt(b))*
e^(-2*a*i/b)/((b^2*i/abs(b) - b)*c^2) + 15/256*sqrt(pi)*b^(7/2)*erf(sqrt(b*
arccos(c*x) + a)*sqrt(b)*i/abs(b) - sqrt(b*arccos(c*x) + a)/sqrt(b))*e^(-2*
a*i/b)/((b^2*i/abs(b) - b)*c^2) + 5/32*sqrt(b*arccos(c*x) + a)*a*b*i*e^(2*i
*arccos(c*x))/c^2 + 1/4*sqrt(b*arccos(c*x) + a)*a*b*arccos(c*x)*e^(2*i*arcc
os(c*x))/c^2 - 5/32*sqrt(b*arccos(c*x) + a)*a*b*i*e^(-2*i*arccos(c*x))/c^2
+ 1/4*sqrt(b*arccos(c*x) + a)*a*b*arccos(c*x)*e^(-2*i*arccos(c*x))/c^2 + 1/
8*sqrt(b*arccos(c*x) + a)*a^2*e^(2*i*arccos(c*x))/c^2 - 15/128*sqrt(b*arcco
s(c*x) + a)*b^2*e^(2*i*arccos(c*x))/c^2 + 1/8*sqrt(b*arccos(c*x) + a)*a^2*e
^(-2*i*arccos(c*x))/c^2 - 15/128*sqrt(b*arccos(c*x) + a)*b^2*e^(-2*i*arccos
(c*x))/c^2
```

3.185 $\int \left(a+b\cos^{-1}(cx)\right)^{5/2}\,dx$

Optimal. Leaf size=179

$$\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{4c}+\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}\sin\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{15}{4}b^2x\sqrt{a+b\cos^{-1}(cx)}-\frac{5b\sqrt{\cdots}}{}$$

```
[Out] (-15*b^2*x*Sqrt[a + b*ArcCos[c*x]])/4 - (5*b*Sqrt[1 - c^2*x^2]*(a + b*ArcCo
s[c*x])^(3/2))/(2*c) + x*(a + b*ArcCos[c*x])^(5/2) + (15*b^(5/2)*Sqrt[Pi/2]
*Cos[a/b]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(4*c) + (
15*b^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]
]*Sin[a/b])/(4*c)
```

Rubi [A] time = 0.428301, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4620, 4678, 4724, 3306, 3305, 3351, 3304, 3352}

$$\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{4c}+\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}\sin\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{15}{4}b^2x\sqrt{a+b\cos^{-1}(cx)}-\frac{5b\sqrt{\cdots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])^(5/2),x]
```

```
[Out] (-15*b^2*x*Sqrt[a + b*ArcCos[c*x]])/4 - (5*b*Sqrt[1 - c^2*x^2]*(a + b*ArcCo
s[c*x])^(3/2))/(2*c) + x*(a + b*ArcCos[c*x])^(5/2) + (15*b^(5/2)*Sqrt[Pi/2]
*Cos[a/b]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(4*c) + (
15*b^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]
]*Sin[a/b])/(4*c)
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \left(a+b\cos^{-1}(cx)\right)^{5/2}\,dx = x\left(a+b\cos^{-1}(cx)\right)^{5/2}+\frac{1}{2}(5bc)\int\frac{x\left(a+b\cos^{-1}(cx)\right)^{3/2}}{\sqrt{1-c^2x^2}}\,dx$$

$$=-\frac{5b\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^{3/2}}{2c}+x\left(a+b\cos^{-1}(cx)\right)^{5/2}-\frac{1}{4}\left(15b^2\right)\int\sqrt{a+b\cos^{-1}(cx)}$$

$$=-\frac{15}{4}b^2x\sqrt{a+b\cos^{-1}(cx)}-\frac{5b\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^{3/2}}{2c}+x\left(a+b\cos^{-1}(cx)\right)^{5/2}-\frac{1}{8}\Big($$

$$=-\frac{15}{4}b^2x\sqrt{a+b\cos^{-1}(cx)}-\frac{5b\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^{3/2}}{2c}+x\left(a+b\cos^{-1}(cx)\right)^{5/2}+\frac{(15}{}$$

$$=-\frac{15}{4}b^2x\sqrt{a+b\cos^{-1}(cx)}-\frac{5b\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^{3/2}}{2c}+x\left(a+b\cos^{-1}(cx)\right)^{5/2}+\frac{(15}{}$$

$$=-\frac{15}{4}b^2x\sqrt{a+b\cos^{-1}(cx)}-\frac{5b\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^{3/2}}{2c}+x\left(a+b\cos^{-1}(cx)\right)^{5/2}+\frac{(15}{}$$

$$=-\frac{15}{4}b^2x\sqrt{a+b\cos^{-1}(cx)}-\frac{5b\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^{3/2}}{2c}+x\left(a+b\cos^{-1}(cx)\right)^{5/2}+\frac{15b}{}$$

Mathematica [C] time = 4.10564, size = 383, normalized size = 2.14

$$\frac{be^{-\frac{ia}{b}}\left(4ia^2\sqrt{-\frac{i\left(a+b\cos^{-1}(cx)\right)}{b}}\text{Gamma}\left(\frac{3}{2},-\frac{i\left(a+b\cos^{-1}(cx)\right)}{b}\right)-4ia^2e^{\frac{2ia}{b}}\sqrt{\frac{i\left(a+b\cos^{-1}(cx)\right)}{b}}\text{Gamma}\left(\frac{3}{2},\frac{i\left(a+b\cos^{-1}(cx)\right)}{b}\right)+\sqrt{\frac{\pi}{2}}\sqrt{\frac{1}{b}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[c*x])^(5/2),x]
```

```
[Out] (b*(-2*E^((I*a)/b)*(a + b*ArcCos[c*x])*(5*(3*b*c*x + 2*a*Sqrt[1 - c^2*x^2])
 + (-8*a*c*x + 10*b*Sqrt[1 - c^2*x^2])*ArcCos[c*x] - 4*b*c*x*ArcCos[c*x]^2)
 + Sqrt[b^(-1)]*(4*a^2 + 15*b^2)*(1 + E^(((2*I)*a)/b))*Sqrt[Pi/2]*Sqrt[a +
b*ArcCos[c*x]]*FresnelC[Sqrt[b^(-1)]*Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]]] -
I*Sqrt[b^(-1)]*(4*a^2 + 15*b^2)*(-1 + E^(((2*I)*a)/b))*Sqrt[Pi/2]*Sqrt[a +
b*ArcCos[c*x]]*FresnelS[Sqrt[b^(-1)]*Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]]] +
(4*I)*a^2*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((-I)*(a + b*ArcCos
[c*x]))/b] - (4*I)*a^2*E^(((2*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamm
```

```
a[3/2, (I*(a + b*ArcCos[c*x]))/b]))/(8*c*E^((I*a)/b)*Sqrt[a + b*ArcCos[c*x]
])
```

Maple [B] time = 0.123, size = 393, normalized size = 2.2

$$\frac{1}{8c}\left(15\sqrt{b^{-1}}\sqrt{\pi}\sqrt{2}\sqrt{a+b\arccos(cx)}\cos\left(\frac{a}{b}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\sqrt{a+b\arccos(cx)}}{\sqrt{b^{-1}}\sqrt{\pi}b}\right)b^3+15\sqrt{b^{-1}}\sqrt{\pi}\sqrt{2}\sqrt{a+b\arccos(cx)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^(5/2),x)
```

```
[Out] 1/8/c*(15*(1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(a/b)*Fre
snelC(2^(1/2)/Pi^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b^3+15*(1/b)^
(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(a/b)*FresnelS(2^(1/2)/Pi
^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b^3+8*arccos(c*x)^3*cos((a+b*
arccos(c*x))/b-a/b)*b^3+24*arccos(c*x)^2*cos((a+b*arccos(c*x))/b-a/b)*a*b^2
-20*arccos(c*x)^2*sin((a+b*arccos(c*x))/b-a/b)*b^3+24*arccos(c*x)*cos((a+b*
arccos(c*x))/b-a/b)*a^2*b-30*arccos(c*x)*cos((a+b*arccos(c*x))/b-a/b)*b^3-4
0*arccos(c*x)*sin((a+b*arccos(c*x))/b-a/b)*a*b^2+8*cos((a+b*arccos(c*x))/b-
a/b)*a^3-30*cos((a+b*arccos(c*x))/b-a/b)*a*b^2-20*sin((a+b*arccos(c*x))/b-a
/b)*a^2*b)/(a+b*arccos(c*x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\arccos(cx)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccos(c*x) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.90574, size = 1621, normalized size = 9.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] -3/4*sqrt(2)*sqrt(pi)*a*b^4*i*erf(-1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/sq
rt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(a*i/b)/
((b^3*i/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c) - 3/4*sqrt(2)*sqrt(pi)*a*b^4*i*
erf(1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b
*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-a*i/b)/((b^3*i/sqrt(abs(b)) - b^2*sqr
t(abs(b)))*c) - 1/2*sqrt(2)*sqrt(pi)*a^2*b^3*erf(-1/2*sqrt(2)*sqrt(b*arccos
(c*x) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b)
)/b)*e^(a*i/b)/((b^3*i/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c) + 3/4*sqrt(2)*sq
rt(pi)*a*b^3*i*erf(-1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/sqrt(abs(b)) - 1/
2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(a*i/b)/((b^2*i/sqrt(ab
s(b)) + b*sqrt(abs(b)))*c) + 1/2*sqrt(2)*sqrt(pi)*a^2*b^3*erf(1/2*sqrt(2)*s
qrt(b*arccos(c*x) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)
*sqrt(abs(b))/b)*e^(-a*i/b)/((b^3*i/sqrt(abs(b)) - b^2*sqrt(abs(b)))*c) + 3
/4*sqrt(2)*sqrt(pi)*a*b^3*i*erf(1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/sqrt(
abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-a*i/b)/((
```

```
b^2*i/sqrt(abs(b)) - b*sqrt(abs(b)))*c) + 1/2*sqrt(2)*sqrt(pi)*a^2*b^2*erf(
-1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*ar
ccos(c*x) + a)*sqrt(abs(b))/b)*e^(a*i/b)/((b^2*i/sqrt(abs(b)) + b*sqrt(abs(
b)))*c) - 15/16*sqrt(2)*sqrt(pi)*b^4*erf(-1/2*sqrt(2)*sqrt(b*arccos(c*x) +
a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(
a*i/b)/((b^2*i/sqrt(abs(b)) + b*sqrt(abs(b)))*c) - 1/2*sqrt(2)*sqrt(pi)*a^2
*b^2*erf(1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*s
qrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-a*i/b)/((b^2*i/sqrt(abs(b)) - b*
sqrt(abs(b)))*c) + 15/16*sqrt(2)*sqrt(pi)*b^4*erf(1/2*sqrt(2)*sqrt(b*arccos
(c*x) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b)
)/b)*e^(-a*i/b)/((b^2*i/sqrt(abs(b)) - b*sqrt(abs(b)))*c) + 5/4*sqrt(b*arcc
os(c*x) + a)*b^2*i*arccos(c*x)*e^(i*arccos(c*x))/c + 1/2*sqrt(b*arccos(c*x)
 + a)*b^2*arccos(c*x)^2*e^(i*arccos(c*x))/c - 5/4*sqrt(b*arccos(c*x) + a)*b
^2*i*arccos(c*x)*e^(-i*arccos(c*x))/c + 1/2*sqrt(b*arccos(c*x) + a)*b^2*arc
cos(c*x)^2*e^(-i*arccos(c*x))/c + 5/4*sqrt(b*arccos(c*x) + a)*a*b*i*e^(i*ar
ccos(c*x))/c + sqrt(b*arccos(c*x) + a)*a*b*arccos(c*x)*e^(i*arccos(c*x))/c
- 5/4*sqrt(b*arccos(c*x) + a)*a*b*i*e^(-i*arccos(c*x))/c + sqrt(b*arccos(c*
x) + a)*a*b*arccos(c*x)*e^(-i*arccos(c*x))/c + 1/2*sqrt(b*arccos(c*x) + a)*
a^2*e^(i*arccos(c*x))/c - 15/8*sqrt(b*arccos(c*x) + a)*b^2*e^(i*arccos(c*x)
)/c + 1/2*sqrt(b*arccos(c*x) + a)*a^2*e^(-i*arccos(c*x))/c - 15/8*sqrt(b*ar
ccos(c*x) + a)*b^2*e^(-i*arccos(c*x))/c
```

3.186 $\int \frac{(a+b\cos^{-1}(cx))^{5/2}}{x}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{(a+b\cos^{-1}(cx))^{5/2}}{x},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCos[c*x])^(5/2)/x, x]
```

Rubi [A] time = 0.0411428, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+b\cos^{-1}(cx))^{5/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCos[c*x])^(5/2)/x,x]

[Out] Defer[Int][(a + b*ArcCos[c*x])^(5/2)/x, x]
```

Rubi steps

$$\int \frac{(a+b\cos^{-1}(cx))^{5/2}}{x}\,dx = \int \frac{(a+b\cos^{-1}(cx))^{5/2}}{x}\,dx$$

Mathematica [A] time = 3.33561, size = 0, normalized size = 0.

$$\int \frac{(a+b\cos^{-1}(cx))^{5/2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCos[c*x])^(5/2)/x,x]
```

```
[Out] Integrate[(a + b*ArcCos[c*x])^(5/2)/x, x]
```

Maple [A] time = 0.217, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\arccos\left(cx\right)\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^(5/2)/x,x)
```

```
[Out] int((a+b*arccos(c*x))^(5/2)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\arccos\left(cx\right)+a\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((b*arccos(c*x) + a)^(5/2)/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**(5/2)/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arccos(cx) + a)^{\frac{5}{2}}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)^(5/2)/x, x)
```

3.187 $\int \frac{\left(a+b\cos^{-1}(cx)\right)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\left(a+b\cos^{-1}(cx)\right)^{5/2}}{x^2},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCos[c*x])^(5/2)/x^2, x]
```

Rubi [A] time = 0.040121, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^{5/2}}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCos[c*x])^(5/2)/x^2,x]
```

```
[Out] Defer[Int][(a + b*ArcCos[c*x])^(5/2)/x^2, x]
```

Rubi steps

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^{5/2}}{x^2}\,dx=\int \frac{\left(a+b\cos^{-1}(cx)\right)^{5/2}}{x^2}\,dx$$

Mathematica [A] time = 9.7634, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^{5/2}}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCos[c*x])^(5/2)/x^2,x]
```

```
[Out] Integrate[(a + b*ArcCos[c*x])^(5/2)/x^2, x]
```

Maple [A] time = 0.283, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (a + b \arccos(cx))^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^(5/2)/x^2,x)
```

```
[Out] int((a+b*arccos(c*x))^(5/2)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arccos(cx) + a)^{\frac{5}{2}}}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*arccos(c*x) + a)^(5/2)/x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**(5/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arccos{(cx)} + a)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)^(5/2)/x^2, x)
```

3.188 $\int \frac{x^2}{\sqrt{a+b\cos^{-1}(cx)}}\,dx$

Optimal. Leaf size=223

$$\frac{\sqrt{\frac{\pi}{2}}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{6}}\sin\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c^3}-\frac{\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c^3}$$

```
[Out] -(Sqrt[Pi/2]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]
])/(2*Sqrt[b]*c^3) - (Sqrt[Pi/6]*Cos[(3*a)/b]*FresnelS[(Sqrt[6/Pi]*Sqrt[a +
 b*ArcCos[c*x]])/Sqrt[b]])/(2*Sqrt[b]*c^3) + (Sqrt[Pi/2]*FresnelC[(Sqrt[2/P
i]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(2*Sqrt[b]*c^3) + (Sqrt[Pi/6
]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/(2*S
qrt[b]*c^3)
```

Rubi [A] time = 0.370696, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4636, 4406, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{6}}\sin\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c^3}-\frac{\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*ArcCos[c*x]],x]
```


```
[Out] -(Sqrt[Pi/2]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]
])/(2*Sqrt[b]*c^3) - (Sqrt[Pi/6]*Cos[(3*a)/b]*FresnelS[(Sqrt[6/Pi]*Sqrt[a +
 b*ArcCos[c*x]])/Sqrt[b]])/(2*Sqrt[b]*c^3) + (Sqrt[Pi/2]*FresnelC[(Sqrt[2/P
i]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(2*Sqrt[b]*c^3) + (Sqrt[Pi/6
]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/(2*S
qrt[b]*c^3)
```


Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+b\cos^{-1}(cx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{\cos^2(x)\sin(x)}{\sqrt{a+bx}}\,dx, x, \cos^{-1}(cx)\right)}{c^3}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{\sin(x)}{4\sqrt{a+bx}} + \frac{\sin(3x)}{4\sqrt{a+bx}}\right)dx, x, \cos^{-1}(cx)\right)}{c^3}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{a+bx}}\,dx, x, \cos^{-1}(cx)\right)}{4c^3} - \frac{\text{Subst}\left(\int \frac{\sin(3x)}{\sqrt{a+bx}}\,dx, x, \cos^{-1}(cx)\right)}{4c^3}$$
$$= -\frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}+x\right)}{\sqrt{a+bx}}\,dx, x, \cos^{-1}(cx)\right)}{4c^3} - \frac{\cos\left(\frac{3a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{3a}{b}+3x\right)}{\sqrt{a+bx}}\,dx, x, \cos^{-1}(cx)\right)}{4c^3}$$
$$= -\frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int \sin\left(\frac{x^2}{b}\right)dx, x, \sqrt{a+b\cos^{-1}(cx)}\right)}{2bc^3} - \frac{\cos\left(\frac{3a}{b}\right)\text{Subst}\left(\int \sin\left(\frac{3x^2}{b}\right)dx, x, \sqrt{a}\right.}{2bc^3}$$
$$= -\frac{\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c^3} - \frac{\sqrt{\frac{\pi}{6}}\cos\left(\frac{3a}{b}\right)S\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c^3} + \frac{\sqrt{\frac{\pi}{2}}C\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(c}}{\sqrt{b}}\right.}{2\sqrt{b}c^3}$$

Mathematica [C] time = 0.421384, size = 225, normalized size = 1.01

$$\frac{e^{-\frac{3ia}{b}}\left(3e^{\frac{2ia}{b}}\sqrt{-\frac{i(a+b\cos^{-1}(cx))}{b}}\text{Gamma}\left(\frac{1}{2}, -\frac{i(a+b\cos^{-1}(cx))}{b}\right) + 3e^{\frac{4ia}{b}}\sqrt{\frac{i(a+b\cos^{-1}(cx))}{b}}\text{Gamma}\left(\frac{1}{2}, \frac{i(a+b\cos^{-1}(cx))}{b}\right) + \sqrt{3}\left(\sqrt{-\frac{i}{}}\right.\right.}{24c^3\sqrt{a+b\cos^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/Sqrt[a + b*ArcCos[c*x]],x]
```

```
[Out] (3*E^(((2*I)*a)/b)*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-I)*(a +
 b*ArcCos[c*x]))/b] + 3*E^(((4*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gam
ma[1/2, (I*(a + b*ArcCos[c*x]))/b] + Sqrt[3]*(Sqrt[((-I)*(a + b*ArcCos[c*x]
))/b]*Gamma[1/2, ((-3*I)*(a + b*ArcCos[c*x]))/b] + E^(((6*I)*a)/b)*Sqrt[(I*
(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((3*I)*(a + b*ArcCos[c*x]))/b]))/(24*c^3
*E^(((3*I)*a)/b)*Sqrt[a + b*ArcCos[c*x]])
```

Maple [A] time = 0.085, size = 167, normalized size = 0.8

$$-\frac{\sqrt{2}\sqrt{\pi}}{12\,c^3}\sqrt{b^{-1}}\left(\sqrt{3}\cos\left(3\,\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{2}\sqrt{3}}{\sqrt{\pi}b}\sqrt{a+b\arccos(cx)}\frac{1}{\sqrt{b^{-1}}}\right) - \sqrt{3}\sin\left(3\,\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{2}\sqrt{3}}{\sqrt{\pi}b}\sqrt{a+b\,\text{arcco}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arccos(c*x))^(1/2),x)
```

```
[Out] -1/12/c^3*2^(1/2)*(1/b)^(1/2)*Pi^(1/2)*(3^(1/2)*cos(3*a/b)*FresnelS(2^(1/2)
/Pi^(1/2)*3^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)-3^(1/2)*sin(3*a/b)
*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)+3
*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)-
3*sin(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{b \arccos(cx) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(b*arccos(c*x) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a + b \operatorname{acos}(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*acos(c*x))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*acos(c*x)), x)
```

Giac [A] time = 2.086, size = 446, normalized size = 2.

$$\frac{\sqrt{\pi}i\operatorname{erf}\left(-\frac{\sqrt{6}\sqrt{b\arccos(cx)+a}\sqrt{b}i}{2|b|}-\frac{\sqrt{6}\sqrt{b\arccos(cx)+a}}{2\sqrt{b}}\right)e^{\left(\frac{3ai}{b}\right)}}{4\left(\frac{\sqrt{6}b^{\frac{3}{2}}i}{|b|}+\sqrt{6}\sqrt{b}\right)c^3}+\frac{\sqrt{\pi}i\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{b\arccos(cx)+a}i}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{b\arccos(cx)+a}\sqrt{|b|}}{2b}\right)e^{\left(\frac{ai}{b}\right)}}{4\left(\frac{\sqrt{2}bi}{\sqrt{|b|}}+\sqrt{2}\sqrt{|b|}\right)c^3}+\frac{\sqrt{\pi}i}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*i*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) -
1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b))*e^(3*a*i/b)/((sqrt(6)*b^(3/2)*
i/abs(b) + sqrt(6)*sqrt(b))*c^3) + 1/4*sqrt(pi)*i*erf(-1/2*sqrt(2)*sqrt(b*a
rccos(c*x) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(a
bs(b))/b)*e^(a*i/b)/((sqrt(2)*b*i/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b)))*c^3)
 + 1/4*sqrt(pi)*i*erf(1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/sqrt(abs(b)) -
1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-a*i/b)/((sqrt(2)*b*
i/sqrt(abs(b)) - sqrt(2)*sqrt(abs(b)))*c^3) + 1/4*sqrt(pi)*i*erf(1/2*sqrt(6
)*sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - 1/2*sqrt(6)*sqrt(b*arccos(c*x)
 + a)/sqrt(b))*e^(-3*a*i/b)/((sqrt(6)*b^(3/2)*i/abs(b) - sqrt(6)*sqrt(b))*c
^3)
```

3.189 $\int \frac{x}{\sqrt{a+b\cos^{-1}(cx)}}\,dx$

Optimal. Leaf size=99

$$\frac{\sqrt{\pi}\sin\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{\pi}\sqrt{b}}\right)}{2\sqrt{b}c^2}-\frac{\sqrt{\pi}\cos\left(\frac{2a}{b}\right)S\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{2\sqrt{b}c^2}$$

```
[Out] -(Sqrt[Pi]*Cos[(2*a)/b]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[
Pi])])/(2*Sqrt[b]*c^2) + (Sqrt[Pi]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sq
rt[b]*Sqrt[Pi])]*Sin[(2*a)/b])/(2*Sqrt[b]*c^2)
```

Rubi [A] time = 0.17077, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4636, 4406, 12, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\pi}\sin\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{\pi}\sqrt{b}}\right)}{2\sqrt{b}c^2}-\frac{\sqrt{\pi}\cos\left(\frac{2a}{b}\right)S\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{2\sqrt{b}c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*ArcCos[c*x]],x]
```


```
[Out] -(Sqrt[Pi]*Cos[(2*a)/b]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[
Pi])])/(2*Sqrt[b]*c^2) + (Sqrt[Pi]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sq
rt[b]*Sqrt[Pi])]*Sin[(2*a)/b])/(2*Sqrt[b]*c^2)
```


Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+b\cos^{-1}(cx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{\sqrt{a+bx}}\,dx, x, \cos^{-1}(cx)\right)}{c^2}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{a+bx}}\,dx, x, \cos^{-1}(cx)\right)}{c^2}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{a+bx}}\,dx, x, \cos^{-1}(cx)\right)}{2c^2}$$

$$= -\frac{\cos\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{2a}{b}+2x\right)}{\sqrt{a+bx}}\,dx, x, \cos^{-1}(cx)\right)}{2c^2} + \frac{\sin\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{2a}{b}+2x\right)}{\sqrt{a+bx}}\,dx, x, \cos^{-1}(cx)\right.}{2c^2}$$

$$= -\frac{\cos\left(\frac{2a}{b}\right)\text{Subst}\left(\int \sin\left(\frac{2x^2}{b}\right)dx, x, \sqrt{a+b\cos^{-1}(cx)}\right)}{bc^2} + \frac{\sin\left(\frac{2a}{b}\right)\text{Subst}\left(\int \cos\left(\frac{2x^2}{b}\right)dx, x, \sqrt{a}\right.}{bc^2}$$

$$= -\frac{\sqrt{\pi}\cos\left(\frac{2a}{b}\right)S\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{2\sqrt{b}c^2} + \frac{\sqrt{\pi}C\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)}{2\sqrt{b}c^2}$$

Mathematica [A] time = 0.159688, size = 91, normalized size = 0.92

$$-\frac{\sqrt{\pi}\sqrt{\frac{1}{b}}\left(\cos\left(\frac{2a}{b}\right)S\left(\frac{2\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{\pi}}\right)-\sin\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{\pi}}\right)\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*ArcCos[c*x]],x]
```

```
[Out] -(Sqrt[b^(-1)]*Sqrt[Pi]*(Cos[(2*a)/b]*FresnelS[(2*Sqrt[b^(-1)]*Sqrt[a + b*A
rcCos[c*x]])/Sqrt[Pi]] - FresnelC[(2*Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[c*x]])/
Sqrt[Pi]]*Sin[(2*a)/b]))/(2*c^2)
```

Maple [A] time = 0.06, size = 80, normalized size = 0.8

$$-\frac{\sqrt{\pi}}{2c^2}\sqrt{b^{-1}}\left(\cos\left(2\frac{a}{b}\right)\text{FresnelS}\left(2\frac{\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\sqrt{b^{-1}}b}\right)-\sin\left(2\frac{a}{b}\right)\text{FresnelC}\left(2\frac{\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\sqrt{b^{-1}}b}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arccos(c*x))^(1/2),x)
```

```
[Out] -1/2*Pi^(1/2)*(1/b)^(1/2)*(cos(2*a/b)*FresnelS(2/Pi^(1/2)/(1/b)^(1/2)*(a+b*
arccos(c*x))^(1/2)/b)-sin(2*a/b)*FresnelC(2/Pi^(1/2)/(1/b)^(1/2)*(a+b*arcco
s(c*x))^(1/2)/b))/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{b \arccos(cx) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*arccos(c*x) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a + b \operatorname{acos}(cx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*acos(c*x))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*acos(c*x)), x)
```

Giac [A] time = 1.82085, size = 188, normalized size = 1.9

$$\frac{\sqrt{\pi}i\,\mathrm{erf}\left(\frac{\sqrt{b\arccos(cx)+a}\sqrt{b}i}{|b|}-\frac{\sqrt{b\arccos(cx)+a}}{\sqrt{b}}\right)e^{\left(-\frac{2\,ai}{b}\right)}}{4\left(\frac{b^{\frac{3}{2}}i}{|b|}-\sqrt{b}\right)c^2}+\frac{\sqrt{\pi}i\,\mathrm{erf}\left(-\frac{\sqrt{b\arccos(cx)+a}\sqrt{b}i}{|b|}-\frac{\sqrt{b\arccos(cx)+a}}{\sqrt{b}}\right)e^{\left(\frac{2\,ai}{b}\right)}}{4\,\sqrt{b}c^2\left(\frac{bi}{|b|}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*i*erf(sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - sqrt(b*arccos
(c*x) + a)/sqrt(b))*e^(-2*a*i/b)/((b^(3/2)*i/abs(b) - sqrt(b))*c^2) + 1/4*s
qrt(pi)*i*erf(-sqrt(b*arccos(c*x) + a)*sqrt(b)*i/abs(b) - sqrt(b*arccos(c*x
) + a)/sqrt(b))*e^(2*a*i/b)/(sqrt(b)*c^2*(b*i/abs(b) + 1))
```

3.190 $\int \frac{1}{\sqrt{a+b\cos^{-1}(cx)}}\,dx$

Optimal. Leaf size=102

$$\frac{\sqrt{2\pi}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{\sqrt{b}c}-\frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{\sqrt{b}c}$$

```
[Out] -((Sqrt[2*Pi]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b
]])/(Sqrt[b]*c)) + (Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]]
)/Sqrt[b]]*Sin[a/b])/(Sqrt[b]*c)
```

Rubi [A] time = 0.0901623, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4624, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{2\pi}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{\sqrt{b}c}-\frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{\sqrt{b}c}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcCos[c*x]],x]
```

```
[Out] -((Sqrt[2*Pi]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b
]])/(Sqrt[b]*c)) + (Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]]
)/Sqrt[b]]*Sin[a/b])/(Sqrt[b]*c)
```

Rule 4624

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Sub
st[Int[x^n*Sin[a/b - x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c,
 n}, x]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
```

```
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cos^{-1}(cx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\cos^{-1}(cx)\right)}{bc}$$

$$= -\frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\cos^{-1}(cx)\right)}{bc} + \frac{\sin\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\cos^{-1}(c\cdots}{bc}$$

$$= -\frac{\left(2\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \sin\left(\frac{x^2}{b}\right)dx, x, \sqrt{a+b\cos^{-1}(cx)}\right)}{bc} + \frac{\left(2\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \cos\left(\frac{x^2}{b}\right)dx, x\cdots}{bc}$$

$$= -\frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{\sqrt{b}c} + \frac{\sqrt{2\pi}C\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{\sqrt{b}c}$$

Mathematica [C] time = 0.081981, size = 118, normalized size = 1.16

$$\frac{e^{-\frac{ia}{b}}\left(\sqrt{-\frac{i(a+b\cos^{-1}(cx))}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{i(a+b\cos^{-1}(cx))}{b}\right)+e^{\frac{2ia}{b}}\sqrt{\frac{i(a+b\cos^{-1}(cx))}{b}}\,\text{Gamma}\left(\frac{1}{2},\frac{i(a+b\cos^{-1}(cx))}{b}\right)\right)}{2c\sqrt{a+b\cos^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*ArcCos[c*x]],x]
```

```
[Out] (Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-I)*(a + b*ArcCos[c*x]))/b
] + E^(((2*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, (I*(a + b*Ar
cCos[c*x]))/b])/(2*c*E^((I*a)/b)*Sqrt[a + b*ArcCos[c*x]])
```

Maple [A] time = 0.056, size = 85, normalized size = 0.8

$$-\frac{\sqrt{2}\sqrt{\pi}}{c}\sqrt{b^{-1}}\left(\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{2}}{\sqrt{\pi}b}\sqrt{a+b\arccos(cx)}\frac{1}{\sqrt{b^{-1}}}\right)-\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}b}\sqrt{a+b\arccos(cx)}\frac{1}{\sqrt{b^{-1}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(c*x))^(1/2),x)
```

```
[Out] -2^(1/2)*Pi^(1/2)*(1/b)^(1/2)*(cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(1/b)^(1/
2)*(a+b*arccos(c*x))^(1/2)/b)-sin(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(1/b)^(1/2
)*(a+b*arccos(c*x))^(1/2)/b))/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\arccos(cx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arccos(c*x) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b \operatorname{acos}(cx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(c*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*acos(c*x)), x)
```

Giac [A] time = 1.69054, size = 220, normalized size = 2.16

$$\frac{\sqrt{\pi} i \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{b \arccos(cx)+a} i}{2\sqrt{|b|}} - \frac{\sqrt{2}\sqrt{b \arccos(cx)+a}\sqrt{|b|}}{2b}\right) e^{\left(\frac{ai}{b}\right)}}{\left(\frac{\sqrt{2} b i}{\sqrt{|b|}} + \sqrt{2}\sqrt{|b|}\right) c} + \frac{\sqrt{\pi} i \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b \arccos(cx)+a} i}{2\sqrt{|b|}} - \frac{\sqrt{2}\sqrt{b \arccos(cx)+a}\sqrt{|b|}}{2b}\right) e^{\left(-\frac{ai}{b}\right)}}{\left(\frac{\sqrt{2} b i}{\sqrt{|b|}} - \sqrt{2}\sqrt{|b|}\right) c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(pi)*i*erf(-1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*i/sqrt(abs(b)) - 1/2*sq
rt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(a*i/b)/((sqrt(2)*b*i/sqrt(
abs(b)) + sqrt(2)*sqrt(abs(b)))*c) + sqrt(pi)*i*erf(1/2*sqrt(2)*sqrt(b*arcc
os(c*x) + a)*i/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(
b))/b)*e^(-a*i/b)/((sqrt(2)*b*i/sqrt(abs(b)) - sqrt(2)*sqrt(abs(b)))*c)
```

3.191 $\int \frac{1}{x\sqrt{a+b\cos^{-1}(cx)}}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{1}{x\sqrt{a+b\cos^{-1}(cx)}},x\right)$$

```
[Out] Unintegrable[1/(x*Sqrt[a + b*ArcCos[c*x]]), x]
```

Rubi [A] time = 0.0355121, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{x\sqrt{a+b\cos^{-1}(cx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[a + b*ArcCos[c*x]]),x]

[Out] Defer[Int][1/(x*Sqrt[a + b*ArcCos[c*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+b\cos^{-1}(cx)}}\,dx = \int \frac{1}{x\sqrt{a+b\cos^{-1}(cx)}}\,dx$$

Mathematica [A] time = 3.2607, size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+b\cos^{-1}(cx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[a + b*ArcCos[c*x]]),x]

[Out] Integrate[1/(x*Sqrt[a + b*ArcCos[c*x]]), x]
```

Maple [A] time = 0.199, size = 0, normalized size = 0.

$$\int \frac{1}{x} \frac{1}{\sqrt{a + b \arccos(cx)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*arccos(c*x))^(1/2),x)
```

```
[Out] int(1/x/(a+b*arccos(c*x))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \arccos(cx) + a}x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*arccos(c*x) + a)*x), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a + b \operatorname{acos}(cx)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*acos(c*x))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*acos(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \arccos(cx) + a}x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*arccos(c*x) + a)*x), x)
```

3.192 $\int \frac{1}{x^2\sqrt{a+b\cos^{-1}(cx)}}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a+b\cos^{-1}(cx)}},x\right)$$

```
[Out] Unintegrable[1/(x^2*Sqrt[a + b*ArcCos[c*x]]), x]
```

Rubi [A] time = 0.0354743, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\sqrt{a+b\cos^{-1}(cx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[a + b*ArcCos[c*x]]),x]
```

```
[Out] Defer[Int][1/(x^2*Sqrt[a + b*ArcCos[c*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+b\cos^{-1}(cx)}}\,dx=\int \frac{1}{x^2\sqrt{a+b\cos^{-1}(cx)}}\,dx$$

Mathematica [A] time = 11.9607, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{a+b\cos^{-1}(cx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[a + b*ArcCos[c*x]]),x]
```

```
[Out] Integrate[1/(x^2*Sqrt[a + b*ArcCos[c*x]]), x]
```

Maple [A] time = 0.253, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\frac{1}{\sqrt{a+b\arccos(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arccos(c*x))^(1/2),x)
```

```
[Out] int(1/x^2/(a+b*arccos(c*x))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\arccos(cx)+a}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*arccos(c*x) + a)*x^2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{a+b\operatorname{acos}(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*acos(c*x))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*acos(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \arccos(cx) + a}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*arccos(c*x) + a)*x^2), x)
```

3.193 $\int \frac{x^2}{\left(a+b\cos^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=252

$$-\frac{\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3}-\frac{\sqrt{\frac{3\pi}{2}}\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3}-\frac{\sqrt{\frac{\pi}{2}}\sin\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3}$$

```
[Out] (2*x^2*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[a + b*ArcCos[c*x]]) - (Sqrt[Pi/2]*Cos[a
/b]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(b^(3/2)*c^3) -
 (Sqrt[(3*Pi)/2]*Cos[(3*a)/b]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])
/Sqrt[b]])/(b^(3/2)*c^3) - (Sqrt[Pi/2]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcC
os[c*x]])/Sqrt[b]]*Sin[a/b])/(b^(3/2)*c^3) - (Sqrt[(3*Pi)/2]*FresnelS[(Sqrt
[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/(b^(3/2)*c^3)
```

Rubi [A] time = 0.387983, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4632, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3}-\frac{\sqrt{\frac{3\pi}{2}}\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3}-\frac{\sqrt{\frac{\pi}{2}}\sin\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*ArcCos[c*x])^(3/2),x]
```


```
[Out] (2*x^2*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[a + b*ArcCos[c*x]]) - (Sqrt[Pi/2]*Cos[a
/b]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(b^(3/2)*c^3) -
 (Sqrt[(3*Pi)/2]*Cos[(3*a)/b]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])
/Sqrt[b]])/(b^(3/2)*c^3) - (Sqrt[Pi/2]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcC
os[c*x]])/Sqrt[b]]*Sin[a/b])/(b^(3/2)*c^3) - (Sqrt[(3*Pi)/2]*FresnelS[(Sqrt
[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/(b^(3/2)*c^3)
```


Rule 4632

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dis
t[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), Co
```

```
s[x]^(m - 1)*(m - (m + 1)*Cos[x]^2), x], x], x, ArcCos[c*x]], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x^2}{\left(a+b\cos^{-1}(cx)\right)^{3/2}}\,dx = \frac{2x^2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\cos^{-1}(cx)}} + \frac{2\,\text{Subst}\left(\int\left(-\frac{\cos(x)}{4\sqrt{a+bx}}-\frac{3\cos(3x)}{4\sqrt{a+bx}}\right)dx, x, \cos^{-1}(cx)\right)}{bc^3}$$

$$= \frac{2x^2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\cos^{-1}(cx)}} - \frac{\text{Subst}\left(\int\frac{\cos(x)}{\sqrt{a+bx}}dx, x, \cos^{-1}(cx)\right)}{2bc^3} - \frac{3\,\text{Subst}\left(\int\frac{\cos(3x)}{\sqrt{a+bx}}dx, x, \cos^{-1}(c\right.}{2bc^3}$$

$$= \frac{2x^2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\cos^{-1}(cx)}} - \frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int\frac{\cos\left(\frac{a}{b}+x\right)}{\sqrt{a+bx}}dx, x, \cos^{-1}(cx)\right)}{2bc^3} - \frac{\left(3\cos\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{2x^2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\cos^{-1}(cx)}} - \frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int\cos\left(\frac{x^2}{b}\right)dx, x, \sqrt{a+b\cos^{-1}(cx)}\right)}{b^2c^3} - \frac{\left(3\cos\left(\frac{3a}{b}\right)\right)\text{S}}{}$$

$$= \frac{2x^2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\cos^{-1}(cx)}} - \frac{\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)C\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3} - \frac{\sqrt{\frac{3\pi}{2}}\cos\left(\frac{3a}{b}\right)C\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right.}{b^{3/2}c^3}$$

Mathematica [C] time = 0.499206, size = 273, normalized size = 1.08

$$\frac{e^{-\frac{3ia}{b}}\left(ie^{\frac{2ia}{b}}\sqrt{-\frac{i\left(a+b\cos^{-1}(cx)\right)}{b}}\text{Gamma}\left(\frac{1}{2},-\frac{i\left(a+b\cos^{-1}(cx)\right)}{b}\right)-ie^{\frac{4ia}{b}}\sqrt{\frac{i\left(a+b\cos^{-1}(cx)\right)}{b}}\text{Gamma}\left(\frac{1}{2},\frac{i\left(a+b\cos^{-1}(cx)\right)}{b}\right)+i\sqrt{3}\sqrt{-\frac{i(a+}{}}\right.}{4bc^3\sqrt{a+b\,c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(a + b*ArcCos[c*x])^(3/2),x]
```

```
[Out] (8*c^2*E^(((3*I)*a)/b)*x^2*Sqrt[1 - c^2*x^2] + I*E^(((2*I)*a)/b)*Sqrt[((-I)
*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-I)*(a + b*ArcCos[c*x]))/b] - I*E^(((
4*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, (I*(a + b*ArcCos[c*x]
))/b] + I*Sqrt[3]*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-3*I)*(a
+ b*ArcCos[c*x]))/b] - I*Sqrt[3]*E^(((6*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]
))/b]*Gamma[1/2, ((3*I)*(a + b*ArcCos[c*x]))/b])/(4*b*c^3*E^(((3*I)*a)/b)*S
qrt[a + b*ArcCos[c*x]])
```

Maple [A] time = 0.118, size = 295, normalized size = 1.2

$$\frac{1}{2\,bc^3}\left(-\sqrt{3}\sqrt{\pi}\sqrt{2}\sqrt{a+b\arccos(cx)}\cos\left(3\,\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{2}\sqrt{3}}{\sqrt{\pi}b}\sqrt{a+b\arccos(cx)}\frac{1}{\sqrt{b^{-1}}}\right)\sqrt{b^{-1}}-\sqrt{3}\sqrt{\pi}\sqrt{2}\sqrt{a+b\,\text{ar}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arccos(c*x))^(3/2),x)
```

```
[Out] 1/2/c^3/b/(a+b*arccos(c*x))^(1/2)*(-3^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*
x))^(1/2)*cos(3*a/b)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(1/b)^(1/2)*(a+b*arc
cos(c*x))^(1/2)/b)*(1/b)^(1/2)-3^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(
1/2)*sin(3*a/b)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(1/b)^(1/2)*(a+b*arccos(c
*x))^(1/2)/b)*(1/b)^(1/2)-Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(a/b)
*FresnelC(2^(1/2)/Pi^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(1/b)^(1/
2)-Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(a/b)*FresnelS(2^(1/2)/Pi^(1
/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(1/b)^(1/2)+sin((a+b*arccos(c*x)
)/b-a/b)+sin(3*(a+b*arccos(c*x))/b-3*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(b \arccos(cx) + a)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*arccos(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a + b\operatorname{acos}(cx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*acos(c*x))**(3/2),x)
```

```
[Out] Integral(x**2/(a + b*acos(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(b\arccos(cx) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*arccos(c*x) + a)^(3/2), x)
```

3.194 $\int \frac{x}{\left(a+b\cos^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=130

$$-\frac{2\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{\pi}\sqrt{b}}\right)}{b^{3/2}c^2}-\frac{2\sqrt{\pi}\sin\left(\frac{2a}{b}\right)S\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{b^{3/2}c^2}+\frac{2x\sqrt{1-c^2x^2}}{bc\sqrt{a+b\cos^{-1}(cx)}}$$

```
[Out] (2*x*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[a + b*ArcCos[c*x]]) - (2*Sqrt[Pi]*Cos[(2*
a)/b]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])])/(b^(3/2)*c^
2) - (2*Sqrt[Pi]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])]*S
in[(2*a)/b])/(b^(3/2)*c^2)
```

Rubi [A] time = 0.153552, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4632, 3306, 3305, 3351, 3304, 3352}

$$-\frac{2\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{\pi}\sqrt{b}}\right)}{b^{3/2}c^2}-\frac{2\sqrt{\pi}\sin\left(\frac{2a}{b}\right)S\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{b^{3/2}c^2}+\frac{2x\sqrt{1-c^2x^2}}{bc\sqrt{a+b\cos^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*ArcCos[c*x])^(3/2),x]
```

```
[Out] (2*x*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[a + b*ArcCos[c*x]]) - (2*Sqrt[Pi]*Cos[(2*
a)/b]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])])/(b^(3/2)*c^
2) - (2*Sqrt[Pi]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])]*S
in[(2*a)/b])/(b^(3/2)*c^2)
```

Rule 4632

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dis
t[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), Co
s[x]^(m - 1)*(m - (m + 1)*Cos[x]^2), x], x], x, ArcCos[c*x]], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{x}{\left(a+b\cos^{-1}(cx)\right)^{3/2}}\,dx = \frac{2x\sqrt{1-c^2x^2}}{bc\sqrt{a+b\cos^{-1}(cx)}} - \frac{2\,\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{a+bx}}\,dx, x, \cos^{-1}(cx)\right)}{bc^2}$$

$$= \frac{2x\sqrt{1-c^2x^2}}{bc\sqrt{a+b\cos^{-1}(cx)}} - \frac{\left(2\cos\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{2a}{b}+2x\right)}{\sqrt{a+bx}}\,dx, x, \cos^{-1}(cx)\right)}{bc^2} - \frac{\left(2\sin\left(\frac{2a}{b}\right)\right)\text{Su}}{}$$

$$= \frac{2x\sqrt{1-c^2x^2}}{bc\sqrt{a+b\cos^{-1}(cx)}} - \frac{\left(4\cos\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \cos\left(\frac{2x^2}{b}\right)dx, x, \sqrt{a+b\cos^{-1}(cx)}\right)}{b^2c^2} - \frac{\left(4\sin\left(\right.\right.}{}$$

$$= \frac{2x\sqrt{1-c^2x^2}}{bc\sqrt{a+b\cos^{-1}(cx)}} - \frac{2\sqrt{\pi}\cos\left(\frac{2a}{b}\right)C\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{b^{3/2}c^2} - \frac{2\sqrt{\pi}S\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)}{b^{3/2}c^2}$$

Mathematica [A] time = 0.329857, size = 124, normalized size = 0.95

$$\frac{-2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{\pi}}\right)-2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2}\sin\left(\frac{2a}{b}\right)S\left(\frac{2\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{\pi}}\right)+\frac{\sin\left(2\cos^{-1}(cx)\right)}{b\sqrt{a+b\cos^{-1}(cx)}}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*ArcCos[c*x])^(3/2),x]
```

```
[Out] (-2*(b^(-1))^(3/2)*Sqrt[Pi]*Cos[(2*a)/b]*FresnelC[(2*Sqrt[b^(-1)]*Sqrt[a +
b*ArcCos[c*x]])/Sqrt[Pi]] - 2*(b^(-1))^(3/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b^(-
1)]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[Pi]]*Sin[(2*a)/b] + Sin[2*ArcCos[c*x]]/(b
*Sqrt[a + b*ArcCos[c*x]]))/c^2
```

Maple [A] time = 0.102, size = 142, normalized size = 1.1

$$\frac{1}{bc^2}\left(-2\sqrt{b^{-1}}\sqrt{\pi}\sqrt{a+b\arccos(cx)}\cos\left(2\frac{a}{b}\right)\text{FresnelC}\left(2\frac{\sqrt{a+b\arccos(cx)}}{\sqrt{b^{-1}}\sqrt{\pi}b}\right)-2\sqrt{b^{-1}}\sqrt{\pi}\sqrt{a+b\arccos(cx)}\sin\left(2\frac{a}{b}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arccos(c*x))^(3/2),x)
```

```
[Out] 1/c^2/b/(a+b*arccos(c*x))^(1/2)*(-2*(1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^
(1/2)*cos(2*a/b)*FresnelC(2/Pi^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)
-2*(1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(2*a/b)*FresnelS(2/Pi^(1
/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)+sin(2*(a+b*arccos(c*x))/b-2*a/b)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arccos(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a + b \operatorname{acos}{(cx)})^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*acos(c*x))**(3/2),x)
```

```
[Out] Integral(x/(a + b*acos(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(b \arccos{(cx)} + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*arccos(c*x) + a)^(3/2), x)
```

3.195 $\int \frac{1}{\left(a+b\cos^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{2\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}-\frac{2\sqrt{2\pi}\sin\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\cos^{-1}(cx)}}$$

```
[Out] (2*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[a + b*ArcCos[c*x]]) - (2*Sqrt[2*Pi]*Cos[a/b
]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(b^(3/2)*c) - (2*
Sqrt[2*Pi]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])
/(b^(3/2)*c)
```

Rubi [A] time = 0.265989, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {4622, 4724, 3306, 3305, 3351, 3304, 3352}

$$-\frac{2\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}-\frac{2\sqrt{2\pi}\sin\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\cos^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])^(-3/2),x]
```

```
[Out] (2*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[a + b*ArcCos[c*x]]) - (2*Sqrt[2*Pi]*Cos[a/b
]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(b^(3/2)*c) - (2*
Sqrt[2*Pi]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])
/(b^(3/2)*c)
```

Rule 4622

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(Sqrt[1 - c
^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1))
, Int[(x*(a + b*ArcCos[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && LtQ[n, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> -Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cos[x]^m*
Sin[x]^(2*p + 1), x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &
& EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Intege
rQ[p] || GtQ[d, 0])
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^{-1}(cx)\right)^{3/2}}\,dx = \frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\cos^{-1}(cx)}} + \frac{(2c)\int \frac{x}{\sqrt{1-c^2x^2}\sqrt{a+b\cos^{-1}(cx)}}\,dx}{b}$$

$$= \frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\cos^{-1}(cx)}} - \frac{2\,\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{a+bx}}\,dx, x, \cos^{-1}(cx)\right)}{bc}$$

$$= \frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\cos^{-1}(cx)}} - \frac{\left(2\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{a}{b}+x\right)}{\sqrt{a+bx}}\,dx, x, \cos^{-1}(cx)\right)}{bc} - \frac{\left(2\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\cos^{-1}(cx)}} - \frac{\left(4\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \cos\left(\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\cos^{-1}(cx)}\right)}{b^2c} - \frac{\left(4\sin\left(\frac{a}{b}\right)\right)\text{S}}{}$$

$$= \frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\cos^{-1}(cx)}} - \frac{2\sqrt{2\pi}\cos\left(\frac{a}{b}\right)C\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} - \frac{2\sqrt{2\pi}S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{b^{3/2}c}$$

Mathematica [C] time = 0.166727, size = 150, normalized size = 1.09

$$-\frac{ie^{-\frac{ia}{b}}\left(-\sqrt{-\frac{i\left(a+b\cos^{-1}(cx)\right)}{b}}\text{Gamma}\left(\frac{1}{2},-\frac{i\left(a+b\cos^{-1}(cx)\right)}{b}\right)+e^{\frac{2ia}{b}}\sqrt{\frac{i\left(a+b\cos^{-1}(cx)\right)}{b}}\text{Gamma}\left(\frac{1}{2},\frac{i\left(a+b\cos^{-1}(cx)\right)}{b}\right)+2ie^{\frac{ia}{b}}\sqrt{1-c^2x^2}\right)}{bc\sqrt{a+b\cos^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[c*x])^(-3/2),x]
```

```
[Out] ((-I)*((2*I)*E^((I*a)/b)*Sqrt[1 - c^2*x^2] - Sqrt[((-I)*(a + b*ArcCos[c*x])
)/b]*Gamma[1/2, ((-I)*(a + b*ArcCos[c*x]))/b] + E^(((2*I)*a)/b)*Sqrt[(I*(a
+ b*ArcCos[c*x]))/b]*Gamma[1/2, (I*(a + b*ArcCos[c*x]))/b]))/(b*c*E^((I*a)/
b)*Sqrt[a + b*ArcCos[c*x]])
```

Maple [A] time = 0.092, size = 150, normalized size = 1.1

$$2\frac{1}{bc\sqrt{a+b\arccos(cx)}}\left(-\sqrt{\pi}\sqrt{2}\sqrt{a+b\arccos(cx)}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{2}\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\sqrt{b^{-1}}b}\right)\sqrt{b^{-1}}-\sqrt{\pi}\sqrt{2}\sqrt{a+b\arccos(cx)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(c*x))^(3/2),x)
```

```
[Out] 2/c/b/(a+b*arccos(c*x))^(1/2)*(-Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*co
s(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(1/
b)^(1/2)-Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(a/b)*FresnelS(2^(1/2)
/Pi^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(1/b)^(1/2)+sin((a+b*arcco
s(c*x))/b-a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccos(cx) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccos(c*x) + a)^(-3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\,\mathrm{acos}(cx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(c*x))**(3/2),x)
```

```
[Out] Integral((a + b*acos(c*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\arccos(cx) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)^(-3/2), x)
```

3.196 $\int \frac{1}{x\left(a+b\cos^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{1}{x\left(a+b\cos^{-1}(cx)\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[1/(x*(a + b*ArcCos[c*x])^(3/2)), x]
```

Rubi [A] time = 0.0429365, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(a+b\cos^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + b*ArcCos[c*x])^(3/2)),x]
```

```
[Out] Defer[Int][1/(x*(a + b*ArcCos[c*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{x\left(a+b\cos^{-1}(cx)\right)^{3/2}}\,dx=\int \frac{1}{x\left(a+b\cos^{-1}(cx)\right)^{3/2}}\,dx$$

Mathematica [A] time = 4.11835, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\cos^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + b*ArcCos[c*x])^(3/2)),x]
```

```
[Out] Integrate[1/(x*(a + b*ArcCos[c*x])^(3/2)), x]
```

Maple [A] time = 0.159, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\arccos\left(cx\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*arccos(c*x))^(3/2),x)
```

```
[Out] int(1/x/(a+b*arccos(c*x))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\arccos\left(cx\right)+a\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arccos(c*x) + a)^(3/2)*x), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,(a + b\,\mathrm{acos}\,(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*acos(c*x))**(3/2),x)
```

```
[Out] Integral(1/(x*(a + b*acos(c*x))**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccos(cx) + a)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*arccos(c*x) + a)^(3/2)*x), x)
```

3.197 $\int \frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[1/(x^2*(a + b*ArcCos[c*x])^(3/2)), x]
```

Rubi [A] time = 0.0409574, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(a + b*ArcCos[c*x])^(3/2)),x]
```

```
[Out] Defer[Int][1/(x^2*(a + b*ArcCos[c*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)^{3/2}}\,dx$$

Mathematica [A] time = 12.2745, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(a + b*ArcCos[c*x])^(3/2)),x]
```

```
[Out] Integrate[1/(x^2*(a + b*ArcCos[c*x])^(3/2)), x]
```

Maple [A] time = 0.277, size = 0, normalized size = 0.

$$\int \frac{1}{x^{2}} \left(a+b \arccos \left(c x\right)\right)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arccos(c*x))^(3/2),x)
```

```
[Out] int(1/x^2/(a+b*arccos(c*x))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{{\left(b \arccos \left(c x\right) + a\right)}^{\frac{3}{2}} x^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arccos(c*x) + a)^(3/2)*x^2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a + b \operatorname{acos}(cx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*acos(c*x))**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*acos(c*x))**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccos(cx) + a)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*arccos(c*x) + a)^(3/2)*x^2), x)
```

3.198 $\int \frac{x^2}{\left(a+b\cos^{-1}(cx)\right)^{5/2}}\,dx$

Optimal. Leaf size=292

$$-\frac{\sqrt{2\pi}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^3}-\frac{\sqrt{6\pi}\sin\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{b^{5/2}c^3}+\frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(}\cdots}{\sqrt{b}}\right.}{3b^{5/2}c^3}\cdots$$

```
[Out] (2*x^2*Sqrt[1 - c^2*x^2])/(3*b*c*(a + b*ArcCos[c*x])^(3/2)) - (8*x)/(3*b^2*
c^2*Sqrt[a + b*ArcCos[c*x]]) + (4*x^3)/(b^2*Sqrt[a + b*ArcCos[c*x]]) + (Sqr
t[2*Pi]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(3
*b^(5/2)*c^3) + (Sqrt[6*Pi]*Cos[(3*a)/b]*FresnelS[(Sqrt[6/Pi]*Sqrt[a + b*Ar
cCos[c*x]])/Sqrt[b]])/(b^(5/2)*c^3) - (Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt
[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(3*b^(5/2)*c^3) - (Sqrt[6*Pi]*Fresn
elC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/(b^(5/2)*c^
3)
```

Rubi [A] time = 0.945343, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4634, 4720, 4636, 4406, 3306, 3305, 3351, 3304, 3352, 4624}

$$-\frac{\sqrt{2\pi}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^3}-\frac{\sqrt{6\pi}\sin\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{b^{5/2}c^3}+\frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(}\cdots}{\sqrt{b}}\right.}{3b^{5/2}c^3}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*ArcCos[c*x])^(5/2),x]
```

```
[Out] (2*x^2*Sqrt[1 - c^2*x^2])/(3*b*c*(a + b*ArcCos[c*x])^(3/2)) - (8*x)/(3*b^2*
c^2*Sqrt[a + b*ArcCos[c*x]]) + (4*x^3)/(b^2*Sqrt[a + b*ArcCos[c*x]]) + (Sqr
t[2*Pi]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(3
*b^(5/2)*c^3) + (Sqrt[6*Pi]*Cos[(3*a)/b]*FresnelS[(Sqrt[6/Pi]*Sqrt[a + b*Ar
cCos[c*x]])/Sqrt[b]])/(b^(5/2)*c^3) - (Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt
[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(3*b^(5/2)*c^3) - (Sqrt[6*Pi]*Fresn
elC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/(b^(5/2)*c^
3)
```

Rule 4634

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-D
ist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCos[c*x])^(n + 1))/Sq
rt[1 - c^2*x^2], x], x] + Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*ArcCo
s[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[
m, 0] && LtQ[n, -2]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4624

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Sub
st[Int[x^n*Sin[a/b - x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c,
 n}, x]
```

Rubi steps

$$\int \frac{x^2}{\left(a+b\cos^{-1}(cx)\right)^{5/2}}\,dx = \frac{2x^2\sqrt{1-c^2x^2}}{3bc\left(a+b\cos^{-1}(cx)\right)^{3/2}} - \frac{4\int \frac{x}{\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^{3/2}}\,dx}{3bc} + \frac{(2c)\int \frac{x^3}{\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^{3/2}}\,dx}{b}$$

$$= \frac{2x^2\sqrt{1-c^2x^2}}{3bc\left(a+b\cos^{-1}(cx)\right)^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{4x^3}{b^2\sqrt{a+b\cos^{-1}(cx)}} - \frac{12\int \frac{x^2}{\sqrt{a+b\cos^{-1}(cx)}}}{b^2}$$

$$= \frac{2x^2\sqrt{1-c^2x^2}}{3bc\left(a+b\cos^{-1}(cx)\right)^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{4x^3}{b^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{8\,\mathrm{Subst}\left(\int \frac{\sin\left(\frac{a}{b}\right.}{\sqrt{x}}\right.}{}$$

$$= \frac{2x^2\sqrt{1-c^2x^2}}{3bc\left(a+b\cos^{-1}(cx)\right)^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{4x^3}{b^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{12\,\mathrm{Subst}\left(\int\left(\frac{\sin}{4\sqrt{a}}\right.\right.}{}$$

$$= \frac{2x^2\sqrt{1-c^2x^2}}{3bc\left(a+b\cos^{-1}(cx)\right)^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{4x^3}{b^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{3\,\mathrm{Subst}\left(\int \frac{\sin(x)}{\sqrt{a+bx}}\right.}{b^2}$$

$$= \frac{2x^2\sqrt{1-c^2x^2}}{3bc\left(a+b\cos^{-1}(cx)\right)^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{4x^3}{b^2\sqrt{a+b\cos^{-1}(cx)}} - \frac{8\sqrt{2\pi}\cos\left(\frac{a}{b}\right)S\left(\right.}{3b^{5/}}$$

$$= \frac{2x^2\sqrt{1-c^2x^2}}{3bc\left(a+b\cos^{-1}(cx)\right)^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{4x^3}{b^2\sqrt{a+b\cos^{-1}(cx)}} - \frac{8\sqrt{2\pi}\cos\left(\frac{a}{b}\right)S\left(\right.}{3b^{5/}}$$

$$= \frac{2x^2\sqrt{1-c^2x^2}}{3bc\left(a+b\cos^{-1}(cx)\right)^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{4x^3}{b^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)S\left(\right.}{3b^{5/2}}$$

Mathematica [C] time = 2.57622, size = 322, normalized size = 1.1

$$-\frac{-\left(a+b\cos^{-1}(cx)\right)\left(-e^{-\frac{ia}{b}}\sqrt{-\frac{i\left(a+b\cos^{-1}(cx)\right)}{b}}\,\mathrm{Gamma}\left(\frac{1}{2},-\frac{i\left(a+b\cos^{-1}(cx)\right)}{b}\right)-e^{\frac{ia}{b}}\sqrt{\frac{i\left(a+b\cos^{-1}(cx)\right)}{b}}\,\mathrm{Gamma}\left(\frac{1}{2},\frac{i\left(a+b\cos^{-1}(cx)\right)}{b}\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(a + b*ArcCos[c*x])^(5/2),x]
```

```
[Out] -(-(b*Sqrt[1 - c^2*x^2]) - (a + b*ArcCos[c*x])*(E^((-I)*ArcCos[c*x]) + E^(I
*ArcCos[c*x]) - (Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-I)*(a + b
*ArcCos[c*x]))/b])/E^((I*a)/b) - E^((I*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b
]*Gamma[1/2, (I*(a + b*ArcCos[c*x]))/b]) - 3*(a + b*ArcCos[c*x])*(E^((-3*I)
*ArcCos[c*x]) + E^((3*I)*ArcCos[c*x]) - (Sqrt[3]*Sqrt[((-I)*(a + b*ArcCos[c
*x]))/b]*Gamma[1/2, ((-3*I)*(a + b*ArcCos[c*x]))/b])/E^(((3*I)*a)/b) - Sqrt
[3]*E^(((3*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((3*I)*(a +
b*ArcCos[c*x]))/b]) - b*Sin[3*ArcCos[c*x]])/(6*b^2*c^3*(a + b*ArcCos[c*x])^
(3/2))
```

Maple [B] time = 0.148, size = 659, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arccos(c*x))^(5/2),x)
```

```
[Out] 1/6/c^3/b^2*(6*arccos(c*x)*3^(1/2)*2^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)
*(1/b)^(1/2)*cos(3*a/b)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(1/b)^(1/2)*(a+b*
arccos(c*x))^(1/2)/b)*b-6*arccos(c*x)*3^(1/2)*2^(1/2)*Pi^(1/2)*(a+b*arccos(
c*x))^(1/2)*(1/b)^(1/2)*sin(3*a/b)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(1/b)^
(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b+2*arccos(c*x)*2^(1/2)*Pi^(1/2)*(a+b*arc
os(c*x))^(1/2)*(1/b)^(1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(1/b)^(1/2)*(
a+b*arccos(c*x))^(1/2)/b)*b-2*arccos(c*x)*2^(1/2)*Pi^(1/2)*(a+b*arccos(c*x)
)^(1/2)*(1/b)^(1/2)*sin(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(1/b)^(1/2)*(a+b*arc
cos(c*x))^(1/2)/b)*b+6*3^(1/2)*2^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*(1/
b)^(1/2)*cos(3*a/b)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(1/b)^(1/2)*(a+b*arc
os(c*x))^(1/2)/b)*a-6*3^(1/2)*2^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*(1/b
)^(1/2)*sin(3*a/b)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(1/b)^(1/2)*(a+b*arcco
s(c*x))^(1/2)/b)*a+2*2^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*(1/b)^(1/2)*c
os(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*a-
2*2^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*(1/b)^(1/2)*sin(a/b)*FresnelC(2^
(1/2)/Pi^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*a+2*arccos(c*x)*cos((
a+b*arccos(c*x))/b-a/b)*b+6*arccos(c*x)*cos(3*(a+b*arccos(c*x))/b-3*a/b)*b+
sin((a+b*arccos(c*x))/b-a/b)*b+2*cos((a+b*arccos(c*x))/b-a/b)*a+sin(3*(a+b*
arccos(c*x))/b-3*a/b)*b+6*cos(3*(a+b*arccos(c*x))/b-3*a/b)*a)/(a+b*arccos(c
*x))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(b \arccos(cx) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccos(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*arccos(c*x) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccos(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a + b \operatorname{acos}(cx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*acos(c*x))**(5/2),x)
```

```
[Out] Integral(x**2/(a + b*acos(c*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(b \arccos(cx) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccos(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*arccos(c*x) + a)^(5/2), x)
```

3.199 $\int \frac{x}{\left(a+b\cos^{-1}(cx)\right)^{5/2}}\,dx$

Optimal. Leaf size=180

$$-\frac{8\sqrt{\pi}\sin\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{\pi}\sqrt{b}}\right)}{3b^{5/2}c^2}+\frac{8\sqrt{\pi}\cos\left(\frac{2a}{b}\right)S\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{3b^{5/2}c^2}-\frac{4}{3b^2c^2\sqrt{a+b\cos^{-1}(cx)}}+\frac{8x^2}{3b^2\sqrt{a+b\cos^{-1}}}$$

```
[Out] (2*x*Sqrt[1 - c^2*x^2])/(3*b*c*(a + b*ArcCos[c*x])^(3/2)) - 4/(3*b^2*c^2*Sq
rt[a + b*ArcCos[c*x]]) + (8*x^2)/(3*b^2*Sqrt[a + b*ArcCos[c*x]]) + (8*Sqrt[
Pi]*Cos[(2*a)/b]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])])/
(3*b^(5/2)*c^2) - (8*Sqrt[Pi]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]
*Sqrt[Pi])]*Sin[(2*a)/b])/(3*b^(5/2)*c^2)
```

Rubi [A] time = 0.48727, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {4634, 4720, 4636, 4406, 12, 3306, 3305, 3351, 3304, 3352, 4642}

$$-\frac{8\sqrt{\pi}\sin\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{\pi}\sqrt{b}}\right)}{3b^{5/2}c^2}+\frac{8\sqrt{\pi}\cos\left(\frac{2a}{b}\right)S\left(\frac{2\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{3b^{5/2}c^2}-\frac{4}{3b^2c^2\sqrt{a+b\cos^{-1}(cx)}}+\frac{8x^2}{3b^2\sqrt{a+b\cos^{-1}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*ArcCos[c*x])^(5/2),x]
```

```
[Out] (2*x*Sqrt[1 - c^2*x^2])/(3*b*c*(a + b*ArcCos[c*x])^(3/2)) - 4/(3*b^2*c^2*Sq
rt[a + b*ArcCos[c*x]]) + (8*x^2)/(3*b^2*Sqrt[a + b*ArcCos[c*x]]) + (8*Sqrt[
Pi]*Cos[(2*a)/b]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])])/
(3*b^(5/2)*c^2) - (8*Sqrt[Pi]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]
*Sqrt[Pi])]*Sin[(2*a)/b])/(3*b^(5/2)*c^2)
```

Rule 4634

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Simp[
(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-D
ist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCos[c*x])^(n + 1))/Sq
rt[1 - c^2*x^2], x], x] + Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*ArcCo
s[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[
m, 0] && LtQ[n, -2]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4636

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Cos[x]^m*Sin[x], x], x, ArcCos[c*x]
], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+b\cos^{-1}(cx)\right)^{5/2}}\,dx = \frac{2x\sqrt{1-c^2x^2}}{3bc\left(a+b\cos^{-1}(cx)\right)^{3/2}} - \frac{2\int \frac{1}{\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^{3/2}}\,dx}{3bc} + \frac{(4c)\int \frac{x^2}{\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^{3/2}}\,dx}{3b}$$

$$= \frac{2x\sqrt{1-c^2x^2}}{3bc\left(a+b\cos^{-1}(cx)\right)^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{8x^2}{3b^2\sqrt{a+b\cos^{-1}(cx)}} - \frac{16\int \frac{x}{\sqrt{a+b\cos^{-1}}}}{3b^2}$$

$$= \frac{2x\sqrt{1-c^2x^2}}{3bc\left(a+b\cos^{-1}(cx)\right)^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{8x^2}{3b^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{16\,\mathrm{Subst}\left(\int \frac{\mathrm{c}}{}\right.}{}$$

$$= \frac{2x\sqrt{1-c^2x^2}}{3bc\left(a+b\cos^{-1}(cx)\right)^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{8x^2}{3b^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{16\,\mathrm{Subst}\left(\int \frac{\mathrm{s}}{2}\right.}{}$$

$$= \frac{2x\sqrt{1-c^2x^2}}{3bc\left(a+b\cos^{-1}(cx)\right)^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{8x^2}{3b^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{8\,\mathrm{Subst}\left(\int \frac{\sin}{\sqrt{a}}\right.}{}$$

$$= \frac{2x\sqrt{1-c^2x^2}}{3bc\left(a+b\cos^{-1}(cx)\right)^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{8x^2}{3b^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{\left(8\cos\left(\frac{2a}{b}\right)\right)\mathrm{S}}{}$$

$$= \frac{2x\sqrt{1-c^2x^2}}{3bc\left(a+b\cos^{-1}(cx)\right)^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{8x^2}{3b^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{\left(16\cos\left(\frac{2a}{b}\right)\right)\mathrm{S}}{}$$

$$= \frac{2x\sqrt{1-c^2x^2}}{3bc\left(a+b\cos^{-1}(cx)\right)^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{8x^2}{3b^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{8\sqrt{\pi}\cos\left(\frac{2a}{b}\right)}{3b}$$

Mathematica [A] time = 0.667768, size = 176, normalized size = 0.98

$$\frac{-8\sqrt{\pi}\sqrt{\frac{1}{b}}\sin\left(\frac{2a}{b}\right)\left(a+b\cos^{-1}(cx)\right)^{3/2}\text{FresnelC}\left(\frac{2\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{\pi}}\right)+8\sqrt{\pi}\sqrt{\frac{1}{b}}\cos\left(\frac{2a}{b}\right)\left(a+b\cos^{-1}(cx)\right)^{3/2}S\left(\frac{2\sqrt{\frac{1}{b}}\sqrt{a+b}}{\sqrt{}}\right.}{3b^2c^2\left(a+b\cos^{-1}(cx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*ArcCos[c*x])^(5/2),x]
```

```
[Out] (4*a*Cos[2*ArcCos[c*x]] + 4*b*ArcCos[c*x]*Cos[2*ArcCos[c*x]] + 8*Sqrt[b^(-1
)]*Sqrt[Pi]*(a + b*ArcCos[c*x])^(3/2)*Cos[(2*a)/b]*FresnelS[(2*Sqrt[b^(-1)]
```

```
*Sqrt[a + b*ArcCos[c*x]])/Sqrt[Pi]] - 8*Sqrt[b^(-1)]*Sqrt[Pi]*(a + b*ArcCos
[c*x])^(3/2)*FresnelC[(2*Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[Pi]]*Si
n[(2*a)/b] + b*Sin[2*ArcCos[c*x]])/(3*b^2*c^2*(a + b*ArcCos[c*x])^(3/2))
```

Maple [B] time = 0.114, size = 311, normalized size = 1.7

$$\frac{1}{3\,b^2c^2}\left(8\,\arccos(cx)\,\sqrt{\pi}\sqrt{b^{-1}}\cos\left(2\,\frac{a}{b}\right)\mathrm{FresnelS}\left(2\,\frac{\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\sqrt{b^{-1}}b}\right)\sqrt{a+b\arccos(cx)}b-8\,\arccos(cx)\,\sqrt{\pi}\sqrt{b^{-1}}\,\mathrm{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arccos(c*x))^(5/2),x)
```

```
[Out] 1/3/c^2/b^2*(8*arccos(c*x)*Pi^(1/2)*(1/b)^(1/2)*cos(2*a/b)*FresnelS(2/Pi^(1
/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(a+b*arccos(c*x))^(1/2)*b-8*arcc
os(c*x)*Pi^(1/2)*(1/b)^(1/2)*sin(2*a/b)*FresnelC(2/Pi^(1/2)/(1/b)^(1/2)*(a+
b*arccos(c*x))^(1/2)/b)*(a+b*arccos(c*x))^(1/2)*b+8*Pi^(1/2)*(1/b)^(1/2)*co
s(2*a/b)*FresnelS(2/Pi^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(a+b*ar
ccos(c*x))^(1/2)*a-8*Pi^(1/2)*(1/b)^(1/2)*sin(2*a/b)*FresnelC(2/Pi^(1/2)/(1
/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(a+b*arccos(c*x))^(1/2)*a+4*arccos(c*x
)*cos(2*(a+b*arccos(c*x))/b-2*a/b)*b+sin(2*(a+b*arccos(c*x))/b-2*a/b)*b+4*c
os(2*(a+b*arccos(c*x))/b-2*a/b)*a)/(a+b*arccos(c*x))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(b\arccos(cx)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccos(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arccos(c*x) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccos(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a + b \operatorname{acos}(cx))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*acos(c*x))**(5/2),x)
```

```
[Out] Integral(x/(a + b*acos(c*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(b \arccos(cx) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccos(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*arccos(c*x) + a)^(5/2), x)
```

3.200 $\int \frac{1}{\left(a+b\cos^{-1}(cx)\right)^{5/2}}\,dx$

Optimal. Leaf size=163

$$-\frac{4\sqrt{2\pi}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c}+\frac{4\sqrt{2\pi}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c}+\frac{4x}{3b^2\sqrt{a+b\cos^{-1}(cx)}}+\frac{2\sqrt{1-\cdots}}{3bc\left(a+b\cos\cdots\right.}$$

```
[Out] (2*Sqrt[1 - c^2*x^2])/(3*b*c*(a + b*ArcCos[c*x])^(3/2)) + (4*x)/(3*b^2*Sqrt
[a + b*ArcCos[c*x]]) + (4*Sqrt[2*Pi]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a +
 b*ArcCos[c*x]])/Sqrt[b]])/(3*b^(5/2)*c) - (4*Sqrt[2*Pi]*FresnelC[(Sqrt[2/P
i]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(3*b^(5/2)*c)
```

Rubi [A] time = 0.258632, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4622, 4720, 4624, 3306, 3305, 3351, 3304, 3352}

$$-\frac{4\sqrt{2\pi}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c}+\frac{4\sqrt{2\pi}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c}+\frac{4x}{3b^2\sqrt{a+b\cos^{-1}(cx)}}+\frac{2\sqrt{1-\cdots}}{3bc\left(a+b\cos\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])^(-5/2),x]
```

```
[Out] (2*Sqrt[1 - c^2*x^2])/(3*b*c*(a + b*ArcCos[c*x])^(3/2)) + (4*x)/(3*b^2*Sqrt
[a + b*ArcCos[c*x]]) + (4*Sqrt[2*Pi]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a +
 b*ArcCos[c*x]])/Sqrt[b]])/(3*b^(5/2)*c) - (4*Sqrt[2*Pi]*FresnelC[(Sqrt[2/P
i]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(3*b^(5/2)*c)
```

Rule 4622

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(Sqrt[1 - c
^2*x^2]*(a + b*ArcCos[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1))
, Int[(x*(a + b*ArcCos[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && LtQ[n, -1]
```

Rule 4720

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> -Simp[((f*x)^m*(a + b*ArcCos[c*x])^(n + 1))/(
b*c*Sqrt[d]*(n + 1)), x] + Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
 1)*(a + b*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 4624

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Sub
st[Int[x^n*Sin[a/b - x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c,
 n}, x]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^{-1}(cx)\right)^{5/2}}\,dx = \frac{2\sqrt{1-c^2x^2}}{3bc\left(a+b\cos^{-1}(cx)\right)^{3/2}} + \frac{(2c)\int \frac{x}{\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^{3/2}}\,dx}{3b}$$

$$= \frac{2\sqrt{1-c^2x^2}}{3bc\left(a+b\cos^{-1}(cx)\right)^{3/2}} + \frac{4x}{3b^2\sqrt{a+b\cos^{-1}(cx)}} - \frac{4\int \frac{1}{\sqrt{a+b\cos^{-1}(cx)}}\,dx}{3b^2}$$

$$= \frac{2\sqrt{1-c^2x^2}}{3bc\left(a+b\cos^{-1}(cx)\right)^{3/2}} + \frac{4x}{3b^2\sqrt{a+b\cos^{-1}(cx)}} - \frac{4\,\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\cos^{-1}(cx)\right)}{3b^3c}$$

$$= \frac{2\sqrt{1-c^2x^2}}{3bc\left(a+b\cos^{-1}(cx)\right)^{3/2}} + \frac{4x}{3b^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{\left(4\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\cos\ldots\right.}{3b^3c}$$

$$= \frac{2\sqrt{1-c^2x^2}}{3bc\left(a+b\cos^{-1}(cx)\right)^{3/2}} + \frac{4x}{3b^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{\left(8\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \sin\left(\frac{x^2}{b}\right)dx, x, \sqrt{a+\ldots}\right.}{3b^3c}$$

$$= \frac{2\sqrt{1-c^2x^2}}{3bc\left(a+b\cos^{-1}(cx)\right)^{3/2}} + \frac{4x}{3b^2\sqrt{a+b\cos^{-1}(cx)}} + \frac{4\sqrt{2\pi}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c} - \frac{4\sqrt{2}\ldots}{}$$

<u>Mathematica</u> [C] time = 1.62112, size = 194, normalized size = 1.19

$$\frac{2\left(e^{-\frac{ia}{b}}\left(a+b\cos^{-1}(cx)\right)\left(e^{\frac{i\left(a+b\cos^{-1}(cx)\right)}{b}}-\sqrt{-\frac{i\left(a+b\cos^{-1}(cx)\right)}{b}}\text{Gamma}\left(\frac{1}{2},-\frac{i\left(a+b\cos^{-1}(cx)\right)}{b}\right)\right)-e^{-i\cos^{-1}(cx)}\left(a+b\cos^{-1}(cx)\right)\left(-\ldots\right.\right.}{3b^2c\left(a+b\cos^{-1}(cx)\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[c*x])^(-5/2),x]
```

```
[Out] (2*(b*Sqrt[1 - c^2*x^2] + ((a + b*ArcCos[c*x])*(E^((I*(a + b*ArcCos[c*x]))/
b) - Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-I)*(a + b*ArcCos[c*x]
))/b]))/E^((I*a)/b) - ((a + b*ArcCos[c*x])*(-1 + E^((I*(a + b*ArcCos[c*x]))
/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, (I*(a + b*ArcCos[c*x]))/b]))
/E^(I*ArcCos[c*x])))/(3*b^2*c*(a + b*ArcCos[c*x])^(3/2))
```

Maple [B] time = 0.111, size = 324, normalized size = 2.

$$\frac{2\left(2\arccos\left(cx\right)\sqrt{2}\,\sqrt{\pi}\,\sqrt{a+b\arccos\left(cx\right)}\,\sqrt{b^{-1}}\cos\left(\frac{a}{b}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos\left(cx\right)}}{\sqrt{\pi}\,\sqrt{b^{-1}}\,b}\right)b-2\arccos\left(cx\right)\sqrt{2}\,\sqrt{\pi}\right.}{3\,b^{2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(c*x))^(5/2),x)
```

```
[Out] 2/3/c/b^2*(2*arccos(c*x)*2^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*(1/b)^(1/
2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b
)*b-2*arccos(c*x)*2^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*(1/b)^(1/2)*sin(
a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b+2*2
^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*(1/b)^(1/2)*cos(a/b)*FresnelS(2^(1/
2)/Pi^(1/2)/(1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*a-2*2^(1/2)*Pi^(1/2)*(a+
b*arccos(c*x))^(1/2)*(1/b)^(1/2)*sin(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(1/b)^(
1/2)*(a+b*arccos(c*x))^(1/2)/b)*a+2*arccos(c*x)*cos((a+b*arccos(c*x))/b-a/b
)*b+sin((a+b*arccos(c*x))/b-a/b)*b+2*cos((a+b*arccos(c*x))/b-a/b)*a)/(a+b*a
rccos(c*x))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\arccos\left(cx\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccos(c*x) + a)^(-5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\operatorname{acos}(cx))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(c*x))**(5/2),x)
```

```
[Out] Integral((a + b*acos(c*x))**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\arccos(cx) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)^(-5/2), x)
```

3.201 $\int \frac{1}{x\left(a+b\cos^{-1}(cx)\right)^{5/2}}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{1}{x\left(a+b\cos^{-1}(cx)\right)^{5/2}},x\right)$$

```
[Out] Unintegrable[1/(x*(a + b*ArcCos[c*x])^(5/2)), x]
```

Rubi [A] time = 0.0406992, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(a+b\cos^{-1}(cx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + b*ArcCos[c*x])^(5/2)),x]
```

```
[Out] Defer[Int][1/(x*(a + b*ArcCos[c*x])^(5/2)), x]
```

Rubi steps

$$\int \frac{1}{x\left(a+b\cos^{-1}(cx)\right)^{5/2}}\,dx = \int \frac{1}{x\left(a+b\cos^{-1}(cx)\right)^{5/2}}\,dx$$

Mathematica [A] time = 4.21234, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\cos^{-1}(cx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + b*ArcCos[c*x])^(5/2)),x]
```

```
[Out] Integrate[1/(x*(a + b*ArcCos[c*x])^(5/2)), x]
```

Maple [A] time = 0.16, size = 0, normalized size = 0.

$$\int \frac{1}{x} (a + b \arccos (cx))^{-\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*arccos(c*x))^(5/2),x)
```

```
[Out] int(1/x/(a+b*arccos(c*x))^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccos (cx) + a)^{\frac{5}{2}} x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccos(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arccos(c*x) + a)^(5/2)*x), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccos(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\,\mathrm{acos}\left(cx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*acos(c*x))**(5/2),x)
```

```
[Out] Integral(1/(x*(a + b*acos(c*x))**(5/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\arccos\left(cx\right)+a\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccos(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*arccos(c*x) + a)^(5/2)*x), x)
```

3.202 $\int \frac{1}{x^2(a+b\cos^{-1}(cx))^{5/2}}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)^{5/2}},x\right)$$

[Out] Unintegrable[1/(x^2*(a + b*ArcCos[c*x])^(5/2)), x]

Rubi [A] time = 0.0405848, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2*(a + b*ArcCos[c*x])^(5/2)),x]

[Out] Defer[Int][1/(x^2*(a + b*ArcCos[c*x])^(5/2)), x]

Rubi steps

$$\int \frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)^{5/2}}\,dx=\int \frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)^{5/2}}\,dx$$

Mathematica [A] time = 12.702, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\cos^{-1}(cx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^2*(a + b*ArcCos[c*x])^(5/2)),x]

```
[Out] Integrate[1/(x^2*(a + b*ArcCos[c*x])^(5/2)), x]
```

Maple [A] time = 0.272, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (a + b \arccos (cx))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arccos(c*x))^(5/2),x)
```

```
[Out] int(1/x^2/(a+b*arccos(c*x))^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccos (cx) + a)^{\frac{5}{2}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arccos(c*x) + a)^(5/2)*x^2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*acos(c*x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccos(cx) + a)^{\frac{5}{2}} x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*arccos(c*x) + a)^(5/2)*x^2), x)
```

3.203 $\int (dx)^{5/2} \left(a + b\cos^{-1}(cx)\right) dx$

Optimal. Leaf size=120

$$\frac{20bd^{5/2}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right), -1\right)}{147c^{7/2}} + \frac{2(dx)^{7/2}\left(a + b\cos^{-1}(cx)\right)}{7d} - \frac{20bd^2\sqrt{1-c^2x^2}\sqrt{dx}}{147c^3} - \frac{4b\sqrt{1-c^2x^2}(dx)^{5/2}}{49c}$$

```
[Out] (-20*b*d^2*Sqrt[d*x]*Sqrt[1 - c^2*x^2])/(147*c^3) - (4*b*(d*x)^(5/2)*Sqrt[1
 - c^2*x^2])/(49*c) + (2*(d*x)^(7/2)*(a + b*ArcCos[c*x]))/(7*d) + (20*b*d^(
5/2)*EllipticF[ArcSin[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]], -1])/(147*c^(7/2))
```

Rubi [A] time = 0.0669195, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4628, 321, 329, 221}

$$\frac{2(dx)^{7/2}\left(a + b\cos^{-1}(cx)\right)}{7d} - \frac{20bd^2\sqrt{1-c^2x^2}\sqrt{dx}}{147c^3} + \frac{20bd^{5/2}F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{147c^{7/2}} - \frac{4b\sqrt{1-c^2x^2}(dx)^{5/2}}{49c}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (-20*b*d^2*Sqrt[d*x]*Sqrt[1 - c^2*x^2])/(147*c^3) - (4*b*(d*x)^(5/2)*Sqrt[1
 - c^2*x^2])/(49*c) + (2*(d*x)^(7/2)*(a + b*ArcCos[c*x]))/(7*d) + (20*b*d^(
5/2)*EllipticF[ArcSin[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]], -1])/(147*c^(7/2))
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int (dx)^{5/2}\left(a+b\cos^{-1}(cx)\right)dx = \frac{2(dx)^{7/2}\left(a+b\cos^{-1}(cx)\right)}{7d}+\frac{(2bc)\int\frac{(dx)^{7/2}}{\sqrt{1-c^2x^2}}\,dx}{7d}$$

$$= -\frac{4b(dx)^{5/2}\sqrt{1-c^2x^2}}{49c}+\frac{2(dx)^{7/2}\left(a+b\cos^{-1}(cx)\right)}{7d}+\frac{(10bd)\int\frac{(dx)^{3/2}}{\sqrt{1-c^2x^2}}\,dx}{49c}$$

$$= -\frac{20bd^2\sqrt{dx}\sqrt{1-c^2x^2}}{147c^3}-\frac{4b(dx)^{5/2}\sqrt{1-c^2x^2}}{49c}+\frac{2(dx)^{7/2}\left(a+b\cos^{-1}(cx)\right)}{7d}+\frac{\left(10bd^3\right)\int \cdots}{147 \cdots}$$

$$= -\frac{20bd^2\sqrt{dx}\sqrt{1-c^2x^2}}{147c^3}-\frac{4b(dx)^{5/2}\sqrt{1-c^2x^2}}{49c}+\frac{2(dx)^{7/2}\left(a+b\cos^{-1}(cx)\right)}{7d}+\frac{\left(20bd^2\right)\text{Sub}\cdots}{\cdots}$$

$$= -\frac{20bd^2\sqrt{dx}\sqrt{1-c^2x^2}}{147c^3}-\frac{4b(dx)^{5/2}\sqrt{1-c^2x^2}}{49c}+\frac{2(dx)^{7/2}\left(a+b\cos^{-1}(cx)\right)}{7d}+\frac{20bd^{5/2}F\left(\text{si}\cdots\right.}{1\cdots}$$

Mathematica [C] time = 0.254003, size = 158, normalized size = 1.32

$$\frac{2d^2\sqrt{dx}\left(\frac{10ib\sqrt{x}\sqrt{1-\frac{1}{c^2x^2}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{1}{c}}}{\sqrt{x}}\right),-1\right)}{\sqrt{-\frac{1}{c}}}+21ac^3x^3\sqrt{1-c^2x^2}+6bc^4x^4+4bc^2x^2+21bc^3x^3\sqrt{1-c^2x^2}\cos^{-1}(cx)-10\cdots\right.}{147c^3\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (2*d^2*Sqrt[d*x]*(-10*b + 4*b*c^2*x^2 + 6*b*c^4*x^4 + 21*a*c^3*x^3*Sqrt[1 -
 c^2*x^2] + 21*b*c^3*x^3*Sqrt[1 - c^2*x^2]*ArcCos[c*x] + ((10*I)*b*Sqrt[1 -
 1/(c^2*x^2)]*Sqrt[x]*EllipticF[I*ArcSinh[Sqrt[-c^(-1)]/Sqrt[x]], -1])/Sqrt
[-c^(-1)]))/(147*c^3*Sqrt[1 - c^2*x^2])
```

Maple [A] time = 0.024, size = 144, normalized size = 1.2

$$2\,\frac{1}{d}\left(1/7\,(dx)^{7/2}\,a+b\left(1/7\,(dx)^{7/2}\arccos(cx)+2/7\,\frac{c}{d}\left(-1/7\,\frac{d^2\,(dx)^{5/2}\,\sqrt{-c^2x^2+1}}{c^2}-\frac{5\,d^4\sqrt{dx}\sqrt{-c^2x^2+1}}{21\,c^4}+\frac{5\,d^4\sqrt{-cx+ \cdots}}{21\,c^4\sqrt{-c^{\cdots}}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(a+b*arccos(c*x)),x)
```

```
[Out] 2/d*(1/7*(d*x)^(7/2)*a+b*(1/7*(d*x)^(7/2)*arccos(c*x)+2/7*c/d*(-1/7/c^2*d^2
*(d*x)^(5/2)*(-c^2*x^2+1)^(1/2)-5/21/c^4*d^4*(d*x)^(1/2)*(-c^2*x^2+1)^(1/2)
+5/21/c^4*d^4/(c/d)^(1/2)*(-c*x+1)^(1/2)*(c*x+1)^(1/2)/(-c^2*x^2+1)^(1/2)*E
llipticF((d*x)^(1/2)*(c/d)^(1/2),I))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bd^2x^2\arccos(cx)+ad^2x^2\right)\sqrt{dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*d^2*x^2*arccos(c*x) + a*d^2*x^2)*sqrt(d*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(a+b*acos(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{5}{2}}\,(b\arccos(cx)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(5/2)*(b*arccos(c*x) + a), x)
```

3.204 $\int (dx)^{3/2} \left(a + b\cos^{-1}(cx)\right) dx$

Optimal. Leaf size=124

$$-\frac{12bd^{3/2}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{25c^{5/2}}+\frac{2(dx)^{5/2}\left(a+b\cos^{-1}(cx)\right)}{5d}+\frac{12bd^{3/2}E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{25c^{5/2}}-\frac{4b\sqrt{1-c^2x^2}(dx}{25c}$$

```
[Out] (-4*b*(d*x)^(3/2)*Sqrt[1 - c^2*x^2])/(25*c) + (2*(d*x)^(5/2)*(a + b*ArcCos[
c*x]))/(5*d) + (12*b*d^(3/2)*EllipticE[ArcSin[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]],
 -1])/(25*c^(5/2)) - (12*b*d^(3/2)*EllipticF[ArcSin[(Sqrt[c]*Sqrt[d*x])/Sqr
t[d]], -1])/(25*c^(5/2))
```

Rubi [A] time = 0.0912919, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4628, 321, 329, 307, 221, 1199, 424}

$$\frac{2(dx)^{5/2}\left(a+b\cos^{-1}(cx)\right)}{5d}-\frac{12bd^{3/2}F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{25c^{5/2}}+\frac{12bd^{3/2}E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{25c^{5/2}}-\frac{4b\sqrt{1-c^2x^2}(dx)^{3/2}}{25c}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (-4*b*(d*x)^(3/2)*Sqrt[1 - c^2*x^2])/(25*c) + (2*(d*x)^(5/2)*(a + b*ArcCos[
c*x]))/(5*d) + (12*b*d^(3/2)*EllipticE[ArcSin[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]],
 -1])/(25*c^(5/2)) - (12*b*d^(3/2)*EllipticF[ArcSin[(Sqrt[c]*Sqrt[d*x])/Sqr
t[d]], -1])/(25*c^(5/2))
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int (dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)dx = \frac{2(dx)^{5/2}\left(a+b\cos^{-1}(cx)\right)}{5d} + \frac{(2bc)\int \frac{(dx)^{5/2}}{\sqrt{1-c^2x^2}}dx}{5d}$$

$$= -\frac{4b(dx)^{3/2}\sqrt{1-c^2x^2}}{25c} + \frac{2(dx)^{5/2}\left(a+b\cos^{-1}(cx)\right)}{5d} + \frac{(6bd)\int \frac{\sqrt{dx}}{\sqrt{1-c^2x^2}}dx}{25c}$$

$$= -\frac{4b(dx)^{3/2}\sqrt{1-c^2x^2}}{25c} + \frac{2(dx)^{5/2}\left(a+b\cos^{-1}(cx)\right)}{5d} + \frac{(12b)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{c^2x^4}{d^2}}}dx, x, \sqrt{dx}\right.}{25c}$$

$$= -\frac{4b(dx)^{3/2}\sqrt{1-c^2x^2}}{25c} + \frac{2(dx)^{5/2}\left(a+b\cos^{-1}(cx)\right)}{5d} - \frac{(12bd)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{c^2x^4}{d^2}}}dx, x, \sqrt{d\right.}{25c^2}$$

$$= -\frac{4b(dx)^{3/2}\sqrt{1-c^2x^2}}{25c} + \frac{2(dx)^{5/2}\left(a+b\cos^{-1}(cx)\right)}{5d} - \frac{12bd^{3/2}F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{25c^{5/2}} + $$

$$= -\frac{4b(dx)^{3/2}\sqrt{1-c^2x^2}}{25c} + \frac{2(dx)^{5/2}\left(a+b\cos^{-1}(cx)\right)}{5d} + \frac{12bd^{3/2}E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{25c^{5/2}} - $$

Mathematica [C] time = 0.0672701, size = 66, normalized size = 0.53

$$\frac{2(dx)^{3/2}\left(2b\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},c^2x^2\right)+5acx-2b\sqrt{1-c^2x^2}+5bcx\cos^{-1}(cx)\right)}{25c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (2*(d*x)^(3/2)*(5*a*c*x - 2*b*Sqrt[1 - c^2*x^2] + 5*b*c*x*ArcCos[c*x] + 2*b
*Hypergeometric2F1[1/2, 3/4, 7/4, c^2*x^2]))/(25*c)
```

Maple [A] time = 0.007, size = 138, normalized size = 1.1

$$2\frac{1}{d}\left(1/5\,(dx)^{5/2}a+b\left(1/5\,(dx)^{5/2}\arccos(cx)+2/5\,\frac{c}{d}\left(-1/5\,\frac{d^2(dx)^{3/2}\sqrt{-c^2x^2+1}}{c^2}-3/5\,\frac{d^3\sqrt{-cx+1}\sqrt{cx+1}}{c^3\sqrt{-c^2x^2+1}}\left(\text{EllipticF}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a+b*arccos(c*x)),x)
```

```
[Out] 2/d*(1/5*(d*x)^(5/2)*a+b*(1/5*(d*x)^(5/2)*arccos(c*x)+2/5*c/d*(-1/5/c^2*d^2
*(d*x)^(3/2)*(-c^2*x^2+1)^(1/2)-3/5/c^3*d^3/(c/d)^(1/2)*(-c*x+1)^(1/2)*(c*x
+1)^(1/2)/(-c^2*x^2+1)^(1/2)*(EllipticF((d*x)^(1/2)*(c/d)^(1/2),I)-Elliptic
E((d*x)^(1/2)*(c/d)^(1/2),I)))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bdx \arccos(cx) + adx)\sqrt{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*d*x*arccos(c*x) + a*d*x)*sqrt(d*x), x)
```

Sympy [A] time = 117.363, size = 82, normalized size = 0.66

$$a\left(\begin{cases} 0 & \text{for}\ d = 0 \\ \frac{2(dx)^{\frac{5}{2}}}{5d} & \text{otherwise} \end{cases}\right) + bc\left(\begin{cases} 0 & \text{for}\ d = 0 \\ \frac{d^{\frac{3}{2}} x^{\frac{7}{2}} \Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\middle| c^2x^2e^{2i\pi}\right)}{5\Gamma\left(\frac{11}{4}\right)} & \text{otherwise} \end{cases}\right) + b\left(\begin{cases} 0 & \text{for}\ d = 0 \\ \frac{2(dx)^{\frac{5}{2}}}{5d} & \text{otherwise} \end{cases}\right)\operatorname{acos}(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(a+b*acos(c*x)),x)
```

```
[Out] a*Piecewise((0, Eq(d, 0)), (2*(d*x)**(5/2)/(5*d), True)) + b*c*Piecewise((0
, Eq(d, 0)), (d**(3/2)*x**(7/2)*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c**2*
x**2*exp_polar(2*I*pi))/(5*gamma(11/4)), True)) + b*Piecewise((0, Eq(d, 0))
, (2*(d*x)**(5/2)/(5*d), True))*acos(c*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} \left(b \arccos\left(cx\right) + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*(b*arccos(c*x) + a), x)
```

3.205 $\int \sqrt{dx} \left(a + b\cos^{-1}(cx)\right) dx$

Optimal. Leaf size=88

$$\frac{4b\sqrt{d}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right), -1\right)}{9c^{3/2}} + \frac{2(dx)^{3/2}\left(a + b\cos^{-1}(cx)\right)}{3d} - \frac{4b\sqrt{1 - c^2x^2}\sqrt{dx}}{9c}$$

```
[Out] (-4*b*Sqrt[d*x]*Sqrt[1 - c^2*x^2])/(9*c) + (2*(d*x)^(3/2)*(a + b*ArcCos[c*x
]))/(3*d) + (4*b*Sqrt[d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]], -1]
)/(9*c^(3/2))
```

Rubi [A] time = 0.0456004, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4628, 321, 329, 221}

$$\frac{2(dx)^{3/2}\left(a + b\cos^{-1}(cx)\right)}{3d} - \frac{4b\sqrt{1 - c^2x^2}\sqrt{dx}}{9c} + \frac{4b\sqrt{d}F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle| -1\right)}{9c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a + b*ArcCos[c*x]),x]
```

```
[Out] (-4*b*Sqrt[d*x]*Sqrt[1 - c^2*x^2])/(9*c) + (2*(d*x)^(3/2)*(a + b*ArcCos[c*x
]))/(3*d) + (4*b*Sqrt[d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]], -1]
)/(9*c^(3/2))
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{dx}\left(a+b\cos^{-1}(cx)\right)dx &= \frac{2(dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)}{3d}+\frac{(2bc)\int \frac{(dx)^{3/2}}{\sqrt{1-c^2x^2}}\,dx}{3d}\\
&= -\frac{4b\sqrt{dx}\sqrt{1-c^2x^2}}{9c}+\frac{2(dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)}{3d}+\frac{(2bd)\int \frac{1}{\sqrt{dx}\sqrt{1-c^2x^2}}\,dx}{9c}\\
&= -\frac{4b\sqrt{dx}\sqrt{1-c^2x^2}}{9c}+\frac{2(dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)}{3d}+\frac{(4b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{c^2x^4}{d^2}}}\,dx,x,\sqrt{dx}\right)}{9c}\\
&= -\frac{4b\sqrt{dx}\sqrt{1-c^2x^2}}{9c}+\frac{2(dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)}{3d}+\frac{4b\sqrt{d}F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{9c^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.188331, size = 113, normalized size = 1.28

$$\frac{2}{9}\sqrt{dx}\left(-\frac{2ib\sqrt{-\frac{1}{c}}\sqrt{x}\sqrt{1-\frac{1}{c^2x^2}}\,\mathrm{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{1}{c}}}{\sqrt{x}}\right),-1\right)}{\sqrt{1-c^2x^2}}+3ax-\frac{2b\sqrt{1-c^2x^2}}{c}+3bx\cos^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a + b*ArcCos[c*x]),x]
```

```
[Out] (2*Sqrt[d*x]*(3*a*x - (2*b*Sqrt[1 - c^2*x^2])/c + 3*b*x*ArcCos[c*x] - ((2*I
)*b*Sqrt[-c^(-1)]*Sqrt[1 - 1/(c^2*x^2)]*Sqrt[x]*EllipticF[I*ArcSinh[Sqrt[-c
```

```
^(-1)]/Sqrt[x]], -1])/Sqrt[1 - c^2*x^2]))/9
```

Maple [A] time = 0.006, size = 119, normalized size = 1.4

$$2\frac{1}{d}\left(1/3\ (dx)^{3/2}a+b\left(1/3\ (dx)^{3/2}\arccos(cx)+2/3\,\frac{c}{d}\left(-1/3\,\frac{d^2\sqrt{dx}\sqrt{-c^2x^2+1}}{c^2}+1/3\,\frac{d^2\sqrt{-cx+1}\sqrt{cx+1}}{c^2\sqrt{-c^2x^2+1}}\mathrm{EllipticF}\left(\sqrt{dx}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))*(d*x)^(1/2),x)
```

```
[Out] 2/d*(1/3*(d*x)^(3/2)*a+b*(1/3*(d*x)^(3/2)*arccos(c*x)+2/3*c/d*(-1/3/c^2*d^2
*(d*x)^(1/2)*(-c^2*x^2+1)^(1/2)+1/3/c^2*d^2/(c/d)^(1/2)*(-c*x+1)^(1/2)*(c*x
+1)^(1/2)/(-c^2*x^2+1)^(1/2)*EllipticF((d*x)^(1/2)*(c/d)^(1/2),I))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{dx}(b\arccos(cx)+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(b*arccos(c*x) + a), x)
```

Sympy [A] time = 3.34889, size = 76, normalized size = 0.86

$$\frac{2a\,(dx)^{\frac{3}{2}}}{3d}+\frac{bc\,(dx)^{\frac{5}{2}}\,\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|c^{2}x^{2}e^{2i\pi}\right)}{3d^{2}\Gamma\left(\frac{9}{4}\right)}+\frac{2b\,(dx)^{\frac{3}{2}}\,\mathrm{acos}\,(cx)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))*(d*x)**(1/2),x)
```

```
[Out] 2*a*(d*x)**(3/2)/(3*d) + b*c*(d*x)**(5/2)*gamma(5/4)*hyper((1/2, 5/4), (9/4
,), c**2*x**2*exp_polar(2*I*pi))/(3*d**2*gamma(9/4)) + 2*b*(d*x)**(3/2)*aco
s(c*x)/(3*d)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{dx}(b\arccos\left(cx\right)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*(b*arccos(c*x) + a), x)
```

3.206 $\int \frac{a+b\cos^{-1}(cx)}{\sqrt{dx}}\,dx$

Optimal. Leaf size=89

$$-\frac{4b\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{\sqrt{c}\sqrt{d}}+\frac{2\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)}{d}+\frac{4bE\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{c}\sqrt{d}}$$

```
[Out] (2*Sqrt[d*x]*(a + b*ArcCos[c*x]))/d + (4*b*EllipticE[ArcSin[(Sqrt[c]*Sqrt[d
*x])/Sqrt[d]], -1])/(Sqrt[c]*Sqrt[d]) - (4*b*EllipticF[ArcSin[(Sqrt[c]*Sqrt
[d*x])/Sqrt[d]], -1])/(Sqrt[c]*Sqrt[d])
```

Rubi [A] time = 0.0739902, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4628, 329, 307, 221, 1199, 424}

$$\frac{2\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)}{d}-\frac{4bF\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{c}\sqrt{d}}+\frac{4bE\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{c}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])/Sqrt[d*x],x]
```

```
[Out] (2*Sqrt[d*x]*(a + b*ArcCos[c*x]))/d + (4*b*EllipticE[ArcSin[(Sqrt[c]*Sqrt[d
*x])/Sqrt[d]], -1])/(Sqrt[c]*Sqrt[d]) - (4*b*EllipticF[ArcSin[(Sqrt[c]*Sqrt
[d*x])/Sqrt[d]], -1])/(Sqrt[c]*Sqrt[d])
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\cos^{-1}(cx)}{\sqrt{dx}}\,dx &= \frac{2\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)}{d}+\frac{(2bc)\int \frac{\sqrt{dx}}{\sqrt{1-c^2x^2}}\,dx}{d} \\
&= \frac{2\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)}{d}+\frac{(4bc)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{c^2x^4}{d^2}}}\,dx,x,\sqrt{dx}\right)}{d^2} \\
&= \frac{2\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)}{d}-\frac{(4b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{c^2x^4}{d^2}}}\,dx,x,\sqrt{dx}\right)}{d}+\frac{(4b)\,\mathrm{Subst}\left(\int \frac{1+\frac{cx^2}{d}}{\sqrt{1-\frac{c^2x^4}{d^2}}}\,dx,x,\sqrt{dx}\right)}{d} \\
&= \frac{2\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)}{d}-\frac{4bF\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{c}\sqrt{d}}+\frac{(4b)\,\mathrm{Subst}\left(\int \frac{\sqrt{1+\frac{cx^2}{d}}}{\sqrt{1-\frac{cx^2}{d}}}\,dx,x,\sqrt{dx}\right)}{d} \\
&= \frac{2\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)}{d}+\frac{4bE\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{c}\sqrt{d}}-\frac{4bF\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{c}\sqrt{d}}
\end{aligned}
$$

Mathematica [C] time = 0.034045, size = 45, normalized size = 0.51

$$
\frac{2x\left(2bcx\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},c^2x^2\right)+3\left(a+b\cos^{-1}(cx)\right)\right)}{3\sqrt{dx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[c*x])/Sqrt[d*x],x]
```

```
[Out] (2*x*(3*(a + b*ArcCos[c*x]) + 2*b*c*x*Hypergeometric2F1[1/2, 3/4, 7/4, c^2*
x^2]))/(3*Sqrt[d*x])
```

Maple [A] time = 0.008, size = 98, normalized size = 1.1

$$
2\,\frac{1}{d}\left(a\sqrt{dx}+b\left(\sqrt{dx}\arccos(cx)-2\,\frac{\sqrt{-cx+1}\sqrt{cx+1}}{\sqrt{-c^2x^2+1}}\left(\mathrm{EllipticF}\left(\sqrt{dx}\sqrt{\frac{c}{d}},i\right)-\mathrm{EllipticE}\left(\sqrt{dx}\sqrt{\frac{c}{d}},i\right)\right)\frac{1}{\sqrt{\frac{c}{d}}}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))/(d*x)^(1/2),x)
```

```
[Out] 2/d*(a*(d*x)^(1/2)+b*((d*x)^(1/2)*arccos(c*x)-2/(c/d)^(1/2)*(-c*x+1)^(1/2)*
(c*x+1)^(1/2)/(-c^2*x^2+1)^(1/2)*(EllipticF((d*x)^(1/2)*(c/d)^(1/2),I)-Elli
pticE((d*x)^(1/2)*(c/d)^(1/2),I))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}(b\arccos(cx)+a)}{dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(b*arccos(c*x) + a)/(d*x), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))/(d*x)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\arccos(cx) + a}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)/sqrt(d*x), x)
```

3.207 $\int \frac{a+b\cos^{-1}(cx)}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=55

$$-\frac{4b\sqrt{c}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{d^{3/2}}-\frac{2\left(a+b\cos^{-1}(cx)\right)}{d\sqrt{dx}}$$

```
[Out] (-2*(a + b*ArcCos[c*x]))/(d*Sqrt[d*x]) - (4*b*Sqrt[c]*EllipticF[ArcSin[(Sqr
t[c]*Sqrt[d*x])/Sqrt[d]], -1])/d^(3/2)
```

Rubi [A] time = 0.0340409, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4628, 329, 221}

$$-\frac{2\left(a+b\cos^{-1}(cx)\right)}{d\sqrt{dx}}-\frac{4b\sqrt{c}F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])/(d*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcCos[c*x]))/(d*Sqrt[d*x]) - (4*b*Sqrt[c]*EllipticF[ArcSin[(Sqr
t[c]*Sqrt[d*x])/Sqrt[d]], -1])/d^(3/2)
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\cos^{-1}(cx)}{(dx)^{3/2}}\,dx &= -\frac{2\left(a+b\cos^{-1}(cx)\right)}{d\sqrt{dx}}-\frac{(2bc)\int \frac{1}{\sqrt{dx}\sqrt{1-c^2x^2}}\,dx}{d}\\ &= -\frac{2\left(a+b\cos^{-1}(cx)\right)}{d\sqrt{dx}}-\frac{(4bc)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{c^2x^4}{d^2}}}\,dx,x,\sqrt{dx}\right)}{d^2}\\ &= -\frac{2\left(a+b\cos^{-1}(cx)\right)}{d\sqrt{dx}}-\frac{4b\sqrt{c}F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{d^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.150651, size = 93, normalized size = 1.69

$$\frac{2x\left(\frac{2ib\sqrt{-\frac{1}{c}}c^2x^{3/2}\sqrt{1-\frac{1}{c^2x^2}}\,\mathrm{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{1}{c}}}{\sqrt{x}}\right),-1\right)}{\sqrt{1-c^2x^2}}-a-b\cos^{-1}(cx)\right)}{(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[c*x])/(d*x)^(3/2),x]
```

```
[Out] (2*x*(-a - b*ArcCos[c*x] + ((2*I)*b*Sqrt[-c^(-1)]*c^2*Sqrt[1 - 1/(c^2*x^2)]
*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c^(-1)]/Sqrt[x]], -1])/Sqrt[1 - c^2*x^2]
))/(d*x)^(3/2)
```

Maple [A] time = 0.006, size = 85, normalized size = 1.6

$$2\,\frac{1}{d}\left(-\frac{a}{\sqrt{dx}}+b\left(-\frac{\arccos(cx)}{\sqrt{dx}}-2\,\frac{c\sqrt{-cx+1}\sqrt{cx+1}}{d\sqrt{-c^2x^2+1}}\mathrm{EllipticF}\left(\sqrt{dx}\sqrt{\frac{c}{d}},i\right)\frac{1}{\sqrt{\frac{c}{d}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))/(d*x)^(3/2),x)
```

```
[Out] 2/d*(-1/(d*x)^(1/2)*a+b*(-1/(d*x)^(1/2)*arccos(c*x)-2*c/d/(c/d)^(1/2)*(-c*x
+1)^(1/2)*(c*x+1)^(1/2)/(-c^2*x^2+1)^(1/2)*EllipticF((d*x)^(1/2)*(c/d)^(1/2
),I)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}(b\arccos(cx)+a)}{d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(b*arccos(c*x) + a)/(d^2*x^2), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))/(d*x)**(3/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arccos(cx) + a}{(dx)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)/(d*x)^(3/2), x)
```

3.208 $\int \frac{a+b\cos^{-1}(cx)}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{4bc^{3/2}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{3d^{5/2}}-\frac{2\left(a+b\cos^{-1}(cx)\right)}{3d(dx)^{3/2}}+\frac{4bc\sqrt{1-c^2x^2}}{3d^2\sqrt{dx}}+\frac{4bc^{3/2}E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{3d^{5/2}}$$

```
[Out] (4*b*c*Sqrt[1 - c^2*x^2])/(3*d^2*Sqrt[d*x]) - (2*(a + b*ArcCos[c*x]))/(3*d*
(d*x)^(3/2)) + (4*b*c^(3/2)*EllipticE[ArcSin[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]],
-1])/(3*d^(5/2)) - (4*b*c^(3/2)*EllipticF[ArcSin[(Sqrt[c]*Sqrt[d*x])/Sqrt[d
]], -1])/(3*d^(5/2))
```

Rubi [A] time = 0.0915139, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4628, 325, 329, 307, 221, 1199, 424}

$$-\frac{2\left(a+b\cos^{-1}(cx)\right)}{3d(dx)^{3/2}}+\frac{4bc\sqrt{1-c^2x^2}}{3d^2\sqrt{dx}}-\frac{4bc^{3/2}F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{3d^{5/2}}+\frac{4bc^{3/2}E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{3d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])/(d*x)^(5/2),x]
```


```
[Out] (4*b*c*Sqrt[1 - c^2*x^2])/(3*d^2*Sqrt[d*x]) - (2*(a + b*ArcCos[c*x]))/(3*d*
(d*x)^(3/2)) + (4*b*c^(3/2)*EllipticE[ArcSin[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]],
-1])/(3*d^(5/2)) - (4*b*c^(3/2)*EllipticF[ArcSin[(Sqrt[c]*Sqrt[d*x])/Sqrt[d
]], -1])/(3*d^(5/2))
```


Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{a+b\cos^{-1}(cx)}{(dx)^{5/2}}\,dx = -\frac{2\left(a+b\cos^{-1}(cx)\right)}{3d(dx)^{3/2}} - \frac{(2bc)\int \frac{1}{(dx)^{3/2}\sqrt{1-c^2x^2}}\,dx}{3d}$$

$$= \frac{4bc\sqrt{1-c^2x^2}}{3d^2\sqrt{dx}} - \frac{2\left(a+b\cos^{-1}(cx)\right)}{3d(dx)^{3/2}} + \frac{\left(2bc^3\right)\int \frac{\sqrt{dx}}{\sqrt{1-c^2x^2}}\,dx}{3d^3}$$

$$= \frac{4bc\sqrt{1-c^2x^2}}{3d^2\sqrt{dx}} - \frac{2\left(a+b\cos^{-1}(cx)\right)}{3d(dx)^{3/2}} + \frac{\left(4bc^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{c^2x^4}{d^2}}}\,dx, x, \sqrt{dx}\right)}{3d^4}$$

$$= \frac{4bc\sqrt{1-c^2x^2}}{3d^2\sqrt{dx}} - \frac{2\left(a+b\cos^{-1}(cx)\right)}{3d(dx)^{3/2}} - \frac{\left(4bc^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{c^2x^4}{d^2}}}\,dx, x, \sqrt{dx}\right)}{3d^3} + \frac{\left(4bc^2\right)\text{Subst}\left(\int \right.}{}$$

$$= \frac{4bc\sqrt{1-c^2x^2}}{3d^2\sqrt{dx}} - \frac{2\left(a+b\cos^{-1}(cx)\right)}{3d(dx)^{3/2}} - \frac{4bc^{3/2}F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{3d^{5/2}} + \frac{\left(4bc^2\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{cx^2}{d}}}{\sqrt{1-\frac{cx^2}{d}}}\right.}{3d^3}$$

$$= \frac{4bc\sqrt{1-c^2x^2}}{3d^2\sqrt{dx}} - \frac{2\left(a+b\cos^{-1}(cx)\right)}{3d(dx)^{3/2}} + \frac{4bc^{3/2}E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{3d^{5/2}} - \frac{4bc^{3/2}F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\right|}{3d^{5/2}}$$

Mathematica [C] time = 0.0673355, size = 68, normalized size = 0.54

$$\frac{2x\left(2bc^3x^3\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},c^2x^2\right)-3\left(a-2bcx\sqrt{1-c^2x^2}+b\cos^{-1}(cx)\right)\right)}{9(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[c*x])/(d*x)^(5/2),x]
```

```
[Out] (2*x*(-3*(a - 2*b*c*x*Sqrt[1 - c^2*x^2] + b*ArcCos[c*x]) + 2*b*c^3*x^3*Hype
rgeometric2F1[1/2, 3/4, 7/4, c^2*x^2]))/(9*(d*x)^(5/2))
```

Maple [A] time = 0.007, size = 129, normalized size = 1.

$$2\frac{1}{d}\left(-1/3\frac{a}{(dx)^{3/2}}+b\left(-1/3\frac{\arccos(cx)}{(dx)^{3/2}}-2/3\frac{c}{d}\left(-\frac{\sqrt{-c^2x^2+1}}{\sqrt{dx}}+\frac{c\sqrt{-cx+1}\sqrt{cx+1}}{d\sqrt{-c^2x^2+1}}\left(\text{EllipticF}\left(\sqrt{dx}\sqrt{\frac{c}{d}},i\right)-\text{Elliptic}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))/(d*x)^(5/2),x)
```

```
[Out] 2/d*(-1/3*a/(d*x)^(3/2)+b*(-1/3/(d*x)^(3/2)*arccos(c*x)-2/3*c/d*(-(-c^2*x^2
+1)^(1/2)/(d*x)^(1/2)+c/d/(c/d)^(1/2)*(-c*x+1)^(1/2)*(c*x+1)^(1/2)/(-c^2*x^
2+1)^(1/2)*(EllipticF((d*x)^(1/2)*(c/d)^(1/2),I)-EllipticE((d*x)^(1/2)*(c/d
)^(1/2),I)))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}(b\arccos(cx)+a)}{d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(b*arccos(c*x) + a)/(d^3*x^3), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))/(d*x)**(5/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arccos(cx) + a}{(dx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)/(d*x)^(5/2), x)
```

3.209 $\int (dx)^{5/2} \left(a+b\cos^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=109

$$\frac{16b^2c^2(dx)^{11/2}\text{HypergeometricPFQ}\left(\left\{1,\frac{11}{4},\frac{11}{4}\right\},\left\{\frac{13}{4},\frac{15}{4}\right\},c^2x^2\right)}{693d^3}+\frac{8bc(dx)^{9/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{9}{4},\frac{13}{4},c^2x^2\right)\left(a\right.}{63d^2}$$

```
[Out] (2*(d*x)^(7/2)*(a + b*ArcCos[c*x])^2)/(7*d) + (8*b*c*(d*x)^(9/2)*(a + b*Arc
Cos[c*x])*Hypergeometric2F1[1/2, 9/4, 13/4, c^2*x^2])/(63*d^2) + (16*b^2*c^
2*(d*x)^(11/2)*HypergeometricPFQ[{1, 11/4, 11/4}, {13/4, 15/4}, c^2*x^2])/(
693*d^3)
```

Rubi [A] time = 0.13904, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4628, 4712}

$$\frac{16b^2c^2(dx)^{11/2}\,{}_3F_2\left(1,\frac{11}{4},\frac{11}{4};\frac{13}{4},\frac{15}{4};c^2x^2\right)}{693d^3}+\frac{8bc(dx)^{9/2}\,{}_2F_1\left(\frac{1}{2},\frac{9}{4};\frac{13}{4};c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{63d^2}+\frac{2(dx)^{7/2}\left(a+b\cos^{-1}(cx)\right)^2}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a + b*ArcCos[c*x])^2,x]
```

```
[Out] (2*(d*x)^(7/2)*(a + b*ArcCos[c*x])^2)/(7*d) + (8*b*c*(d*x)^(9/2)*(a + b*Arc
Cos[c*x])*Hypergeometric2F1[1/2, 9/4, 13/4, c^2*x^2])/(63*d^2) + (16*b^2*c^
2*(d*x)^(11/2)*HypergeometricPFQ[{1, 11/4, 11/4}, {13/4, 15/4}, c^2*x^2])/(
693*d^3)
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4712

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcCos[c*x])*Hypergeomet
ric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(Sqrt[d]*f*(m + 1)), x] + Simp[
```

```
(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 +
 m/2}, c^2*x^2])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (dx)^{5/2}\left(a+b\cos^{-1}(cx)\right)^2\,dx = \frac{2(dx)^{7/2}\left(a+b\cos^{-1}(cx)\right)^2}{7d}+\frac{(4bc)\int\frac{(dx)^{7/2}\left(a+b\cos^{-1}(cx)\right)}{\sqrt{1-c^2x^2}}\,dx}{7d}$$
$$=\frac{2(dx)^{7/2}\left(a+b\cos^{-1}(cx)\right)^2}{7d}+\frac{8bc(dx)^{9/2}\left(a+b\cos^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{9}{4};\frac{13}{4};c^2x^2\right)}{63d^2}+\frac{16b^2c}{}$$

Mathematica [B] time = 1.26082, size = 234, normalized size = 2.15

$$\frac{(dx)^{5/2}\left(\frac{b^2\left(\frac{105\sqrt{2}\pi cx\text{HypergeometricPFQ}\left(\left\{\frac{3}{4},\frac{3}{4},1\right\},\left\{\frac{5}{4},\frac{7}{4}\right\},c^2x^2\right)}{\text{Gamma}\left(\frac{5}{4}\right)\text{Gamma}\left(\frac{7}{4}\right)}+840\sqrt{1-c^2x^2}\cos^{-1}(cx)\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{5}{4},c^2x^2\right)-16cx\left(9c^2x^2+35\right)+882c^3x^3\cos^{-1}(cx)\right.}{c^3}\right.}{3087x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^(5/2)*(a + b*ArcCos[c*x])^2,x]
```

```
[Out] ((d*x)^(5/2)*(882*a^2*x^3 + (84*a*b*(-2*Sqrt[1 - c^2*x^2]*(5 + 3*c^2*x^2) +
 21*c^3*x^3*ArcCos[c*x] + 10*Hypergeometric2F1[1/4, 1/2, 5/4, c^2*x^2]))/c^
3 + (b^2*(-16*c*x*(35 + 9*c^2*x^2) - 168*Sqrt[1 - c^2*x^2]*(5 + 3*c^2*x^2)*
ArcCos[c*x] + 882*c^3*x^3*ArcCos[c*x]^2 + 840*Sqrt[1 - c^2*x^2]*ArcCos[c*x]
*Hypergeometric2F1[3/4, 1, 5/4, c^2*x^2] + (105*Sqrt[2]*c*Pi*x*Hypergeometr
icPFQ[{3/4, 3/4, 1}, {5/4, 7/4}, c^2*x^2])/(Gamma[5/4]*Gamma[7/4])))/c^3))/
(3087*x^2)
```

Maple [F] time = 0.358, size = 0, normalized size = 0.

$$\int (dx)^{\frac{5}{2}}\left(a+b\arccos(cx)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(a+b*arccos(c*x))^2,x)
```

```
[Out] int((d*x)^(5/2)*(a+b*arccos(c*x))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2d^2x^2\arccos{(cx)}^2+2\,abd^2x^2\arccos{(cx)}+a^2d^2x^2\right)\sqrt{dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*d^2*x^2*arccos(c*x)^2 + 2*a*b*d^2*x^2*arccos(c*x) + a^2*d^2*x
^2)*sqrt(d*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(a+b*acos(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{5}{2}} (b \arccos (cx) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^(5/2)*(b*arccos(c*x) + a)^2, x)
```

3.210 $\int (dx)^{3/2} \left(a + b \cos^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=109

$$\frac{16b^2c^2(dx)^{9/2}\text{HypergeometricPFQ}\left(\left\{1,\frac{9}{4},\frac{9}{4}\right\},\left\{\frac{11}{4},\frac{13}{4}\right\},c^2x^2\right)}{315d^3}+\frac{8bc(dx)^{7/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{4},\frac{11}{4},c^2x^2\right)\left(a+b\right.}{35d^2}$$

```
[Out] (2*(d*x)^(5/2)*(a + b*ArcCos[c*x])^2)/(5*d) + (8*b*c*(d*x)^(7/2)*(a + b*Arc
Cos[c*x])*Hypergeometric2F1[1/2, 7/4, 11/4, c^2*x^2])/(35*d^2) + (16*b^2*c^
2*(d*x)^(9/2)*HypergeometricPFQ[{1, 9/4, 9/4}, {11/4, 13/4}, c^2*x^2])/(315
*d^3)
```

Rubi [A] time = 0.140138, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4628, 4712}

$$\frac{16b^2c^2(dx)^{9/2}\,{}_3F_2\left(1,\frac{9}{4},\frac{9}{4};\frac{11}{4},\frac{13}{4};c^2x^2\right)}{315d^3}+\frac{8bc(dx)^{7/2}\,{}_2F_1\left(\frac{1}{2},\frac{7}{4};\frac{11}{4};c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{35d^2}+\frac{2(dx)^{5/2}\left(a+b\cos^{-1}(cx)\right)^2}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```


```
[Out] (2*(d*x)^(5/2)*(a + b*ArcCos[c*x])^2)/(5*d) + (8*b*c*(d*x)^(7/2)*(a + b*Arc
Cos[c*x])*Hypergeometric2F1[1/2, 7/4, 11/4, c^2*x^2])/(35*d^2) + (16*b^2*c^
2*(d*x)^(9/2)*HypergeometricPFQ[{1, 9/4, 9/4}, {11/4, 13/4}, c^2*x^2])/(315
*d^3)
```


Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4712

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcCos[c*x])*Hypergeomet
ric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(Sqrt[d]*f*(m + 1)), x] + Simp[
```

```
(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 +
 m/2}, c^2*x^2])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)^2\,dx=\frac{2(dx)^{5/2}\left(a+b\cos^{-1}(cx)\right)^2}{5d}+\frac{(4bc)\int\frac{(dx)^{5/2}\left(a+b\cos^{-1}(cx)\right)}{\sqrt{1-c^2x^2}}\,dx}{5d}$$
$$=\frac{2(dx)^{5/2}\left(a+b\cos^{-1}(cx)\right)^2}{5d}+\frac{8bc(dx)^{7/2}\left(a+b\cos^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{7}{4};\frac{11}{4};c^2x^2\right)}{35d^2}+\frac{16b^2c}{}$$

Mathematica [A] time = 7.41986, size = 176, normalized size = 1.61

$$\frac{(dx)^{3/2}\left(\frac{525\sqrt{2}\pi b^2c^2x^3\text{HypergeometricPFQ}\left(\left\{1,\frac{9}{4},\frac{9}{4}\right\},\left\{\frac{11}{4},\frac{13}{4}\right\},c^2x^2\right)}{\text{Gamma}\left(\frac{11}{4}\right)\text{Gamma}\left(\frac{13}{4}\right)}+\frac{128b\left(28a\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},c^2x^2\right)+20bc^2x^2\sqrt{1-c^2x^2}\cos^{-1}(cx)\text{Hyperg}\right.}{c}\right.}{11200}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

```
[Out] ((d*x)^(3/2)*(4480*a^2*x + (128*b*(-28*a*Sqrt[1 - c^2*x^2] + 70*a*c*x*ArcCo
s[c*x] + 35*b*c*x*ArcCos[c*x]^2 + 28*a*Hypergeometric2F1[1/2, 3/4, 7/4, c^2
*x^2] + 20*b*c^2*x^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Hypergeometric2F1[1, 9/4
, 11/4, c^2*x^2]))/c + (525*Sqrt[2]*b^2*c^2*Pi*x^3*HypergeometricPFQ[{1, 9/
4, 9/4}, {11/4, 13/4}, c^2*x^2])/(Gamma[11/4]*Gamma[13/4])))/11200
```

Maple [F] time = 0.335, size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}}\left(a+b\arccos(cx)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a+b*arccos(c*x))^2,x)
```

```
[Out] int((d*x)^(3/2)*(a+b*arccos(c*x))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2dx\arccos\left(cx\right)^2 + 2\,abdx\arccos\left(cx\right) + a^2dx\right)\sqrt{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*d*x*arccos(c*x)^2 + 2*a*b*d*x*arccos(c*x) + a^2*d*x)*sqrt(d*x
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(a+b*acos(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}}\left(b\arccos\left(cx\right) + a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*(b*arccos(c*x) + a)^2, x)
```

3.211 $\int \sqrt{dx}\left(a+b\cos^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=109

$$\frac{16b^2c^2(dx)^{7/2}\text{HypergeometricPFQ}\left(\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},c^2x^2\right)}{105d^3}+\frac{8bc(dx)^{5/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{15d^2}$$

```
[Out] (2*(d*x)^(3/2)*(a + b*ArcCos[c*x])^2)/(3*d) + (8*b*c*(d*x)^(5/2)*(a + b*Arc
Cos[c*x])*Hypergeometric2F1[1/2, 5/4, 9/4, c^2*x^2])/(15*d^2) + (16*b^2*c^2
*(d*x)^(7/2)*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, c^2*x^2])/(105*d
^3)
```

Rubi [A] time = 0.134042, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4628, 4712}

$$\frac{16b^2c^2(dx)^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{105d^3}+\frac{8bc(dx)^{5/2}\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{15d^2}+\frac{2(dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a + b*ArcCos[c*x])^2,x]
```


```
[Out] (2*(d*x)^(3/2)*(a + b*ArcCos[c*x])^2)/(3*d) + (8*b*c*(d*x)^(5/2)*(a + b*Arc
Cos[c*x])*Hypergeometric2F1[1/2, 5/4, 9/4, c^2*x^2])/(15*d^2) + (16*b^2*c^2
*(d*x)^(7/2)*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, c^2*x^2])/(105*d
^3)
```


Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4712

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcCos[c*x])*Hypergeomet
ric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(Sqrt[d]*f*(m + 1)), x] + Simp[
```

```
(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 +
 m/2}, c^2*x^2])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \sqrt{dx}\left(a+b\cos^{-1}(cx)\right)^2\,dx = \frac{2(dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)^2}{3d} + \frac{(4bc)\int \frac{(dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)}{\sqrt{1-c^2x^2}}\,dx}{3d}$$

$$= \frac{2(dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)^2}{3d} + \frac{8bc(dx)^{5/2}\left(a+b\cos^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};c^2x^2\right)}{15d^2} + \frac{16b^2c^2(a}{}$$

Mathematica [A] time = 0.531985, size = 202, normalized size = 1.85

$$\frac{1}{27}\sqrt{dx}\left(\frac{3\sqrt{2}\pi b^2x\text{HypergeometricPFQ}\left(\left\{\frac{3}{4},\frac{3}{4},1\right\},\left\{\frac{5}{4},\frac{7}{4}\right\},c^2x^2\right)}{\text{Gamma}\left(\frac{5}{4}\right)\text{Gamma}\left(\frac{7}{4}\right)} + \frac{2\left(12ab\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},c^2x^2\right)+12\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*x]*(a + b*ArcCos[c*x])^2,x]
```

```
[Out] (Sqrt[d*x]*((2*(9*a^2*c*x - 8*b^2*c*x - 12*a*b*Sqrt[1 - c^2*x^2] + 18*a*b*c
*x*ArcCos[c*x] - 12*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x] + 9*b^2*c*x*ArcCos[c*
x]^2 + 12*a*b*Hypergeometric2F1[1/4, 1/2, 5/4, c^2*x^2] + 12*b^2*Sqrt[1 - c
^2*x^2]*ArcCos[c*x]*Hypergeometric2F1[3/4, 1, 5/4, c^2*x^2]))/c + (3*Sqrt[2
]*b^2*Pi*x*HypergeometricPFQ[{3/4, 3/4, 1}, {5/4, 7/4}, c^2*x^2])/(Gamma[5/
4]*Gamma[7/4])))/27
```

Maple [F] time = 0.352, size = 0, normalized size = 0.

$$\int (a+b\arccos(cx))^2\sqrt{dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^2*(d*x)^(1/2),x)
```

```
[Out] int((a+b*arccos(c*x))^2*(d*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2 \arccos(cx)^2 + 2\,ab \arccos(cx) + a^2\right)\sqrt{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}\,(a + b\,\text{acos}\,(cx))^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**2*(d*x)**(1/2),x)
```

```
[Out] Integral(sqrt(d*x)*(a + b*acos(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}(b \arccos(cx) + a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*(b*arccos(c*x) + a)^2, x)
```

3.212 $\int \frac{\left(a+b\cos^{-1}(cx)\right)^2}{\sqrt{dx}}\,dx$

Optimal. Leaf size=107

$$\frac{16b^2c^2(dx)^{5/2}\text{HypergeometricPFQ}\left(\left\{1,\frac{5}{4},\frac{5}{4}\right\},\left\{\frac{7}{4},\frac{9}{4}\right\},c^2x^2\right)}{15d^3}+\frac{8bc(dx)^{3/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},c^2x^2\right)\left(a+b\cos\right.}{3d^2}$$

```
[Out] (2*Sqrt[d*x]*(a + b*ArcCos[c*x])^2)/d + (8*b*c*(d*x)^(3/2)*(a + b*ArcCos[c*
x])*Hypergeometric2F1[1/2, 3/4, 7/4, c^2*x^2])/(3*d^2) + (16*b^2*c^2*(d*x)^
(5/2)*HypergeometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, c^2*x^2])/(15*d^3)
```

Rubi [A] time = 0.124588, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4628, 4712}

$$\frac{16b^2c^2(dx)^{5/2}\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};c^2x^2\right)}{15d^3}+\frac{8bc(dx)^{3/2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{3d^2}+\frac{2\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])^2/Sqrt[d*x],x]
```

```
[Out] (2*Sqrt[d*x]*(a + b*ArcCos[c*x])^2)/d + (8*b*c*(d*x)^(3/2)*(a + b*ArcCos[c*
x])*Hypergeometric2F1[1/2, 3/4, 7/4, c^2*x^2])/(3*d^2) + (16*b^2*c^2*(d*x)^
(5/2)*HypergeometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, c^2*x^2])/(15*d^3)
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4712

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcCos[c*x])*Hypergeomet
ric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(Sqrt[d]*f*(m + 1)), x] + Simp[
(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 +
```

```
m/2}, c^2*x^2])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d, e,
f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^2}{\sqrt{dx}}\,dx = \frac{2\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)^2}{d} + \frac{(4bc)\int \frac{\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)}{\sqrt{1-c^2x^2}}\,dx}{d}$$

$$= \frac{2\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)^2}{d} + \frac{8bc(dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};c^2x^2\right)}{3d^2} + \frac{16b^2c^2(dx)^{5/2}\,{}_3\ldots}{\ldots}$$

Mathematica [A] time = 1.27198, size = 142, normalized size = 1.33

$$\frac{3\sqrt{2}\pi b^2c^2x^3\text{HypergeometricPFQ}\left(\left\{1,\frac{5}{4},\frac{5}{4}\right\},\left\{\frac{7}{4},\frac{9}{4}\right\},c^2x^2\right)+8x\text{Gamma}\left(\frac{7}{4}\right)\text{Gamma}\left(\frac{9}{4}\right)\left(4abcx\text{Hypergeometric2}\ldots\right.}{12\text{Gamma}\left(\frac{7}{4}\right)\text{Gam}\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[c*x])^2/Sqrt[d*x],x]
```

```
[Out] (3*Sqrt[2]*b^2*c^2*Pi*x^3*HypergeometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, c^2*
x^2] + 8*x*Gamma[7/4]*Gamma[9/4]*(3*(a + b*ArcCos[c*x])^2 + 4*a*b*c*x*Hyper
geometric2F1[1/2, 3/4, 7/4, c^2*x^2] + 2*b^2*ArcCos[c*x]*Hypergeometric2F1[
1, 5/4, 7/4, c^2*x^2]*Sin[2*ArcCos[c*x]]))/(12*Sqrt[d*x]*Gamma[7/4]*Gamma[9
/4])
```

Maple [F] time = 0.518, size = 0, normalized size = 0.

$$\int (a+b\arccos(cx))^2\frac{1}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^2/(d*x)^(1/2),x)
```

```
[Out] int((a+b*arccos(c*x))^2/(d*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2 \arccos\left(cx\right)^2 + 2\,ab \arccos\left(cx\right) + a^2\right)\sqrt{dx}}{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(d*x)/(d*x), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**2/(d*x)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \arccos\left(cx\right) + a\right)^2}{\sqrt{dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)^2/sqrt(d*x), x)
```

3.213 $\int \frac{\left(a+b\cos^{-1}(cx)\right)^2}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{16b^2c^2(dx)^{3/2}\text{HypergeometricPFQ}\left(\left\{\frac{3}{4},\frac{3}{4},1\right\},\left\{\frac{5}{4},\frac{7}{4}\right\},c^2x^2\right)}{3d^3}-\frac{8bc\sqrt{dx}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},c^2x^2\right)\left(a+b\cos\right.}{d^2}$$

```
[Out] (-2*(a + b*ArcCos[c*x])^2)/(d*Sqrt[d*x]) - (8*b*c*Sqrt[d*x]*(a + b*ArcCos[c
*x])*Hypergeometric2F1[1/4, 1/2, 5/4, c^2*x^2])/d^2 - (16*b^2*c^2*(d*x)^(3/
2)*HypergeometricPFQ[{3/4, 3/4, 1}, {5/4, 7/4}, c^2*x^2])/(3*d^3)
```

Rubi [A] time = 0.130147, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4628, 4712}

$$-\frac{16b^2c^2(dx)^{3/2}\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};c^2x^2\right)}{3d^3}-\frac{8bc\sqrt{dx}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{d^2}-\frac{2\left(a+b\cos^{-1}(cx)\right)^2}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])^2/(d*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcCos[c*x])^2)/(d*Sqrt[d*x]) - (8*b*c*Sqrt[d*x]*(a + b*ArcCos[c
*x])*Hypergeometric2F1[1/4, 1/2, 5/4, c^2*x^2])/d^2 - (16*b^2*c^2*(d*x)^(3/
2)*HypergeometricPFQ[{3/4, 3/4, 1}, {5/4, 7/4}, c^2*x^2])/(3*d^3)
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4712

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcCos[c*x])*Hypergeomet
ric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(Sqrt[d]*f*(m + 1)), x] + Simp[
(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 +
```

```
m/2}, c^2*x^2])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d, e,
f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && !IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^2}{(dx)^{3/2}}\,dx = -\frac{2\left(a+b\cos^{-1}(cx)\right)^2}{d\sqrt{dx}} - \frac{(4bc)\int \frac{a+b\cos^{-1}(cx)}{\sqrt{dx}\sqrt{1-c^2x^2}}\,dx}{d}$$

$$= -\frac{2\left(a+b\cos^{-1}(cx)\right)^2}{d\sqrt{dx}} - \frac{8bc\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};c^2x^2\right)}{d^2} - \frac{16b^2c^2(dx)^{3/2}\,{}_3F_2\left(\frac{3}{4},\right.}{3d^3}$$

Mathematica [A] time = 0.432541, size = 129, normalized size = 1.23

$$\frac{x\left(-\frac{\sqrt{2}\pi b^2c^2x^2\text{HypergeometricPFQ}\left(\left\{\frac{3}{4},\frac{3}{4},1\right\},\left\{\frac{5}{4},\frac{7}{4}\right\},c^2x^2\right)}{\text{Gamma}\left(\frac{5}{4}\right)\text{Gamma}\left(\frac{7}{4}\right)}-2\left(4abcx\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},c^2x^2\right)+2b^2\cos^{-1}(cx)\text{Hyperg}\right.\right.}{(dx)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[c*x])^2/(d*x)^(3/2),x]
```

```
[Out] (x*(-((Sqrt[2]*b^2*c^2*Pi*x^2*HypergeometricPFQ[{3/4, 3/4, 1}, {5/4, 7/4},
c^2*x^2])/(Gamma[5/4]*Gamma[7/4])) - 2*((a + b*ArcCos[c*x])^2 + 4*a*b*c*x*H
ypergeometric2F1[1/4, 1/2, 5/4, c^2*x^2] + 2*b^2*ArcCos[c*x]*Hypergeometric
2F1[3/4, 1, 5/4, c^2*x^2]*Sin[2*ArcCos[c*x]])))/(d*x)^(3/2)
```

Maple [F] time = 0.339, size = 0, normalized size = 0.

$$\int \left(a+b\arccos(cx)\right)^2(dx)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^2/(d*x)^(3/2),x)
```

```
[Out] int((a+b*arccos(c*x))^2/(d*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2\arccos(cx)^2+2\,ab\arccos(cx)+a^2\right)\sqrt{dx}}{d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(d*x)/(d^2*x^2),
 x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**2/(d*x)**(3/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\arccos(cx)+a)^2}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)^2/(d*x)^(3/2), x)
```

3.214 $\int \frac{\left(a+b\cos^{-1}(cx)\right)^2}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=109

$$\frac{16b^2c^2\sqrt{dx}\text{HypergeometricPFQ}\left(\left\{\frac{1}{4},\frac{1}{4},1\right\},\left\{\frac{3}{4},\frac{5}{4}\right\},c^2x^2\right)}{3d^3}+\frac{8bc\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{3d^2\sqrt{dx}}$$

```
[Out] (-2*(a + b*ArcCos[c*x])^2)/(3*d*(d*x)^(3/2)) + (8*b*c*(a + b*ArcCos[c*x])*H
ypergeometric2F1[-1/4, 1/2, 3/4, c^2*x^2])/(3*d^2*Sqrt[d*x]) + (16*b^2*c^2*
Sqrt[d*x]*HypergeometricPFQ[{1/4, 1/4, 1}, {3/4, 5/4}, c^2*x^2])/(3*d^3)
```

Rubi [A] time = 0.141738, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4628, 4712}

$$\frac{16b^2c^2\sqrt{dx}\,{}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};c^2x^2\right)}{3d^3}+\frac{8bc\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};c^2x^2\right)\left(a+b\cos^{-1}(cx)\right)}{3d^2\sqrt{dx}}-\frac{2\left(a+b\cos^{-1}(cx)\right)^2}{3d(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])^2/(d*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcCos[c*x])^2)/(3*d*(d*x)^(3/2)) + (8*b*c*(a + b*ArcCos[c*x])*H
ypergeometric2F1[-1/4, 1/2, 3/4, c^2*x^2])/(3*d^2*Sqrt[d*x]) + (16*b^2*c^2*
Sqrt[d*x]*HypergeometricPFQ[{1/4, 1/4, 1}, {3/4, 5/4}, c^2*x^2])/(3*d^3)
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4712

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcCos[c*x])*Hypergeomet
ric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(Sqrt[d]*f*(m + 1)), x] + Simp[
(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 +
```

```
m/2}, c^2*x^2])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d, e,
f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^2}{(dx)^{5/2}}\,dx = -\frac{2\left(a+b\cos^{-1}(cx)\right)^2}{3d(dx)^{3/2}} - \frac{(4bc)\int \frac{a+b\cos^{-1}(cx)}{(dx)^{3/2}\sqrt{1-c^2x^2}}\,dx}{3d}$$
$$= -\frac{2\left(a+b\cos^{-1}(cx)\right)^2}{3d(dx)^{3/2}} + \frac{8bc\left(a+b\cos^{-1}(cx)\right)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};c^2x^2\right)}{3d^2\sqrt{dx}} + \frac{16b^2c^2\sqrt{dx}\,{}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\right.}{3d^3}$$

Mathematica [A] time = 0.786154, size = 198, normalized size = 1.82

$$\frac{x\left(3\sqrt{2}\pi b^2c^4x^4\text{HypergeometricPFQ}\left(\left\{1,\frac{5}{4},\frac{5}{4}\right\},\left\{\frac{7}{4},\frac{9}{4}\right\},c^2x^2\right)-8\text{Gamma}\left(\frac{7}{4}\right)\text{Gamma}\left(\frac{9}{4}\right)\left(-12abcx\text{Hypergeomet}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[c*x])^2/(d*x)^(5/2),x]
```

```
[Out] (x*(-8*Gamma[7/4]*Gamma[9/4]*(3*(a^2 - 8*b^2*c^2*x^2 + 2*b*(a - 2*b*c*x*Sqr
t[1 - c^2*x^2])*ArcCos[c*x] + b^2*ArcCos[c*x]^2) - 12*a*b*c*x*Hypergeometri
c2F1[-1/4, 1/2, 3/4, c^2*x^2] - 4*b^2*c^3*x^3*Sqrt[1 - c^2*x^2]*ArcCos[c*x]
*Hypergeometric2F1[1, 5/4, 7/4, c^2*x^2]) + 3*Sqrt[2]*b^2*c^4*Pi*x^4*Hyperg
eometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, c^2*x^2]))/(36*(d*x)^(5/2)*Gamma[7/4
]*Gamma[9/4])
```

Maple [F] time = 0.342, size = 0, normalized size = 0.

$$\int (a+b\arccos(cx))^2\,(dx)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^2/(d*x)^(5/2),x)
```

```
[Out] int((a+b*arccos(c*x))^2/(d*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2\arccos\left(cx\right)^2+2\,ab\arccos\left(cx\right)+a^2\right)\sqrt{dx}}{d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(d*x)/(d^3*x^3),
 x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**2/(d*x)**(5/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arccos(cx) + a)^2}{(dx)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)^2/(d*x)^(5/2), x)
```

3.215 $\int (dx)^{3/2} \left(a+b\cos^{-1}(cx)\right)^3 dx$

Optimal. Leaf size=68

$$\frac{6bc\text{Unintegrable}\left(\frac{(dx)^{5/2}\left(a+b\cos^{-1}(cx)\right)^2}{\sqrt{1-c^2x^2}},x\right)}{5d}+\frac{2(dx)^{5/2}\left(a+b\cos^{-1}(cx)\right)^3}{5d}$$

```
[Out] (2*(d*x)^(5/2)*(a + b*ArcCos[c*x])^3)/(5*d) + (6*b*c*Unintegrable[((d*x)^(5
/2)*(a + b*ArcCos[c*x])^2)/Sqrt[1 - c^2*x^2], x])/(5*d)
```

Rubi [A] time = 0.164384, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (dx)^{3/2} \left(a+b\cos^{-1}(cx)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^(3/2)*(a + b*ArcCos[c*x])^3,x]
```

```
[Out] (2*(d*x)^(5/2)*(a + b*ArcCos[c*x])^3)/(5*d) + (6*b*c*Defer[Int][((d*x)^(5/2
)*(a + b*ArcCos[c*x])^2)/Sqrt[1 - c^2*x^2], x])/(5*d)
```

Rubi steps

$$\int (dx)^{3/2} \left(a+b\cos^{-1}(cx)\right)^3 dx=\frac{2(dx)^{5/2}\left(a+b\cos^{-1}(cx)\right)^3}{5d}+\frac{(6bc)\int \frac{(dx)^{5/2}\left(a+b\cos^{-1}(cx)\right)^2}{\sqrt{1-c^2x^2}}\,dx}{5d}$$

Mathematica [A] time = 30.752, size = 0, normalized size = 0.

$$\int (dx)^{3/2} \left(a+b\cos^{-1}(cx)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^(3/2)*(a + b*ArcCos[c*x])^3,x]
```

```
[Out] Integrate[(d*x)^(3/2)*(a + b*ArcCos[c*x])^3, x]
```

Maple [A] time = 0.335, size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} (a + b \arccos(cx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a+b*arccos(c*x))^3,x)
```

```
[Out] int((d*x)^(3/2)*(a+b*arccos(c*x))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arccos(c*x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3 dx \arccos(cx)^3 + 3\,ab^2 dx \arccos(cx)^2 + 3\,a^2 bdx \arccos(cx) + a^3 dx\right)\sqrt{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arccos(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*d*x*arccos(c*x)^3 + 3*a*b^2*d*x*arccos(c*x)^2 + 3*a^2*b*d*x*a
rccos(c*x) + a^3*d*x)*sqrt(d*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(a+b*acos(c*x))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} (b \arccos (cx) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arccos(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*(b*arccos(c*x) + a)^3, x)
```

3.216 $\int \sqrt{dx}\left(a+b\cos^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=66

$$\frac{2bc\text{Unintegrable}\left(\frac{(dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)^2}{\sqrt{1-c^2x^2}},x\right)}{d}+\frac{2(dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)^3}{3d}$$

```
[Out] (2*(d*x)^(3/2)*(a + b*ArcCos[c*x])^3)/(3*d) + (2*b*c*Unintegrable[((d*x)^(3
/2)*(a + b*ArcCos[c*x])^2)/Sqrt[1 - c^2*x^2], x])/d
```

Rubi [A] time = 0.166553, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{dx}\left(a+b\cos^{-1}(cx)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[d*x]*(a + b*ArcCos[c*x])^3,x]
```

```
[Out] (2*(d*x)^(3/2)*(a + b*ArcCos[c*x])^3)/(3*d) + (2*b*c*Defer[Int][((d*x)^(3/2
)*(a + b*ArcCos[c*x])^2)/Sqrt[1 - c^2*x^2], x])/d
```

Rubi steps

$$\int \sqrt{dx}\left(a+b\cos^{-1}(cx)\right)^3\,dx=\frac{2(dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)^3}{3d}+\frac{(2bc)\int \frac{(dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)^2}{\sqrt{1-c^2x^2}}\,dx}{d}$$

Mathematica [A] time = 135.147, size = 0, normalized size = 0.

$$\int \sqrt{dx}\left(a+b\cos^{-1}(cx)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[d*x]*(a + b*ArcCos[c*x])^3,x]
```

```
[Out] Integrate[Sqrt[d*x]*(a + b*ArcCos[c*x])^3, x]
```

Maple [A] time = 0.349, size = 0, normalized size = 0.

$$\int (a + b \arccos(cx))^3 \sqrt{dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^3*(d*x)^(1/2),x)
```

```
[Out] int((a+b*arccos(c*x))^3*(d*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^3*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^3 \arccos(cx)^3 + 3\,ab^2 \arccos(cx)^2 + 3\,a^2 b \arccos(cx) + a^3\right)\sqrt{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^3*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x) +
 a^3)*sqrt(d*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}\,(a + b\,\text{acos}\,(cx))^3\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**3*(d*x)**(1/2),x)
```

```
[Out] Integral(sqrt(d*x)*(a + b*acos(c*x))**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}(b\arccos{(cx)} + a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^3*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*(b*arccos(c*x) + a)^3, x)
```

3.217 $\int \frac{\left(a+b\cos^{-1}(cx)\right)^3}{\sqrt{dx}}\,dx$

Optimal. Leaf size=64

$$\frac{6bc\text{Unintegrable}\left(\frac{\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)^2}{\sqrt{1-c^2x^2}},x\right)}{d}+\frac{2\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)^3}{d}$$

```
[Out] (2*Sqrt[d*x]*(a + b*ArcCos[c*x])^3)/d + (6*b*c*Unintegrable[(Sqrt[d*x]*(a +
 b*ArcCos[c*x])^2)/Sqrt[1 - c^2*x^2], x])/d
```

Rubi [A] time = 0.146574, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^3}{\sqrt{dx}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCos[c*x])^3/Sqrt[d*x],x]
```

```
[Out] (2*Sqrt[d*x]*(a + b*ArcCos[c*x])^3)/d + (6*b*c*Defer[Int][(Sqrt[d*x]*(a + b
*ArcCos[c*x])^2)/Sqrt[1 - c^2*x^2], x])/d
```

Rubi steps

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^3}{\sqrt{dx}}\,dx=\frac{2\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)^3}{d}+\frac{(6bc)\int \frac{\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)^2}{\sqrt{1-c^2x^2}}\,dx}{d}$$

Mathematica [A] time = 10.4271, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^3}{\sqrt{dx}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCos[c*x])^3/Sqrt[d*x],x]
```

```
[Out] Integrate[(a + b*ArcCos[c*x])^3/Sqrt[d*x], x]
```

Maple [A] time = 0.356, size = 0, normalized size = 0.

$$\int (a + b \arccos(cx))^3 \frac{1}{\sqrt{dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^3/(d*x)^(1/2),x)
```

```
[Out] int((a+b*arccos(c*x))^3/(d*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^3/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^3 \arccos(cx)^3 + 3\,ab^2 \arccos(cx)^2 + 3\,a^2 b \arccos(cx) + a^3\right)\sqrt{dx}}{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^3/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x) +
 a^3)*sqrt(d*x)/(d*x), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**3/(d*x)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arccos(cx) + a)^3}{\sqrt{dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^3/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)^3/sqrt(d*x), x)
```

3.218 $\int \frac{\left(a+b\cos^{-1}(cx)\right)^3}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=64

$$-\frac{6bc\text{Unintegrable}\left(\frac{\left(a+b\cos^{-1}(cx)\right)^2}{\sqrt{1-c^2x^2}\sqrt{dx}},x\right)}{d}-\frac{2\left(a+b\cos^{-1}(cx)\right)^3}{d\sqrt{dx}}$$

```
[Out] (-2*(a + b*ArcCos[c*x])^3)/(d*Sqrt[d*x]) - (6*b*c*Unintegrable[(a + b*ArcCo
s[c*x])^2/(Sqrt[d*x]*Sqrt[1 - c^2*x^2]), x])/d
```

Rubi [A] time = 0.157522, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^3}{(dx)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCos[c*x])^3/(d*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcCos[c*x])^3)/(d*Sqrt[d*x]) - (6*b*c*Defer[Int][(a + b*ArcCos[
c*x])^2/(Sqrt[d*x]*Sqrt[1 - c^2*x^2]), x])/d
```

Rubi steps

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^3}{(dx)^{3/2}}\,dx=-\frac{2\left(a+b\cos^{-1}(cx)\right)^3}{d\sqrt{dx}}-\frac{(6bc)\int \frac{\left(a+b\cos^{-1}(cx)\right)^2}{\sqrt{dx}\sqrt{1-c^2x^2}}\,dx}{d}$$

Mathematica [A] time = 7.31337, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^3}{(dx)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCos[c*x])^3/(d*x)^(3/2),x]
```

```
[Out] Integrate[(a + b*ArcCos[c*x])^3/(d*x)^(3/2), x]
```

Maple [A] time = 0.334, size = 0, normalized size = 0.

$$\int (a+b\arccos(cx))^3 (dx)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^3/(d*x)^(3/2),x)
```

```
[Out] int((a+b*arccos(c*x))^3/(d*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^3/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^3\arccos(cx)^3+3\,ab^2\arccos(cx)^2+3\,a^2b\arccos(cx)+a^3\right)\sqrt{dx}}{d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^3/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x) +
 a^3)*sqrt(d*x)/(d^2*x^2), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**3/(d*x)**(3/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arccos(cx) + a)^3}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^3/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)^3/(d*x)^(3/2), x)
```

3.219 $\int \frac{(a+b\cos^{-1}(cx))^3}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{2bc\text{Unintegrable}\left(\frac{(a+b\cos^{-1}(cx))^2}{\sqrt{1-c^2x^2}(dx)^{3/2}},x\right)}{d}-\frac{2\left(a+b\cos^{-1}(cx)\right)^3}{3d(dx)^{3/2}}$$

```
[Out] (-2*(a + b*ArcCos[c*x])^3)/(3*d*(d*x)^(3/2)) - (2*b*c*Unintegrable[(a + b*A
rcCos[c*x])^2/((d*x)^(3/2)*Sqrt[1 - c^2*x^2]), x])/d
```

Rubi [A] time = 0.167126, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^3}{(dx)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCos[c*x])^3/(d*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcCos[c*x])^3)/(3*d*(d*x)^(3/2)) - (2*b*c*Defer[Int][(a + b*Arc
Cos[c*x])^2/((d*x)^(3/2)*Sqrt[1 - c^2*x^2]), x])/d
```

Rubi steps

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^3}{(dx)^{5/2}}\,dx=-\frac{2\left(a+b\cos^{-1}(cx)\right)^3}{3d(dx)^{3/2}}-\frac{(2bc)\int \frac{(a+b\cos^{-1}(cx))^2}{(dx)^{3/2}\sqrt{1-c^2x^2}}\,dx}{d}$$

Mathematica [A] time = 11.4726, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^3}{(dx)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCos[c*x])^3/(d*x)^(5/2),x]
```

```
[Out] Integrate[(a + b*ArcCos[c*x])^3/(d*x)^(5/2), x]
```

Maple [A] time = 0.341, size = 0, normalized size = 0.

$$\int \left(a+b\arccos \left(cx\right)\right)^{3} \left(dx\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^3/(d*x)^(5/2),x)
```

```
[Out] int((a+b*arccos(c*x))^3/(d*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^3/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^{3} \arccos \left(cx\right)^{3}+3\,ab^{2} \arccos \left(cx\right)^{2}+3\,a^{2}b \arccos \left(cx\right)+a^{3}\right)\sqrt{dx}}{d^{3}x^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^3/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x) +
 a^3)*sqrt(d*x)/(d^3*x^3), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**3/(d*x)**(5/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arccos(cx)+a)^3}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^3/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)^3/(d*x)^(5/2), x)
```

3.220 $\int \frac{(dx)^{3/2}}{a+b\cos^{-1}(cx)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{(dx)^{3/2}}{a+b\cos^{-1}(cx)},x\right)$$

```
[Out] Unintegrable[(d*x)^(3/2)/(a + b*ArcCos[c*x]), x]
```

Rubi [A] time = 0.0282532, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(dx)^{3/2}}{a+b\cos^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^(3/2)/(a + b*ArcCos[c*x]),x]
```

```
[Out] Defer[Int][(d*x)^(3/2)/(a + b*ArcCos[c*x]), x]
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{a+b\cos^{-1}(cx)}\,dx = \int \frac{(dx)^{3/2}}{a+b\cos^{-1}(cx)}\,dx$$

Mathematica [A] time = 3.47324, size = 0, normalized size = 0.

$$\int \frac{(dx)^{3/2}}{a+b\cos^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^(3/2)/(a + b*ArcCos[c*x]),x]
```

```
[Out] Integrate[(d*x)^(3/2)/(a + b*ArcCos[c*x]), x]
```

Maple [A] time = 0.238, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\arccos\left(cx\right)}\left(dx\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(a+b*arccos(c*x)),x)
```

```
[Out] int((d*x)^(3/2)/(a+b*arccos(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx\right)^{\frac{3}{2}}}{b\arccos\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^(3/2)/(b*arccos(c*x) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dx}dx}{b\arccos\left(cx\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*d*x/(b*arccos(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{3}{2}}}{a + b \operatorname{acos}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(a+b*acos(c*x)),x)
```

```
[Out] Integral((d*x)**(3/2)/(a + b*acos(c*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{3}{2}}}{b \arccos(cx) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)/(b*arccos(c*x) + a), x)
```

3.221 $\int \frac{\sqrt{dx}}{a+b\cos^{-1}(cx)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{\sqrt{dx}}{a+b\cos^{-1}(cx)},x\right)$$

```
[Out] Unintegrable[Sqrt[d*x]/(a + b*ArcCos[c*x]), x]
```

Rubi [A] time = 0.0250688, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{dx}}{a+b\cos^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[d*x]/(a + b*ArcCos[c*x]),x]
```

```
[Out] Defer[Int][Sqrt[d*x]/(a + b*ArcCos[c*x]), x]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{a+b\cos^{-1}(cx)}\,dx=\int \frac{\sqrt{dx}}{a+b\cos^{-1}(cx)}\,dx$$

Mathematica [A] time = 2.27137, size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx}}{a+b\cos^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[d*x]/(a + b*ArcCos[c*x]),x]
```

```
[Out] Integrate[Sqrt[d*x]/(a + b*ArcCos[c*x]), x]
```

Maple [A] time = 0.245, size = 0, normalized size = 0.

$$\int \frac{1}{a + b \arccos\left(cx\right)}\sqrt{dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(a+b*arccos(c*x)),x)
```

```
[Out] int((d*x)^(1/2)/(a+b*arccos(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx}}{b \arccos\left(cx\right) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x)/(b*arccos(c*x) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}}{b \arccos\left(cx\right) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)/(b*arccos(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx}}{a + b\operatorname{acos}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(a+b*acos(c*x)),x)
```

```
[Out] Integral(sqrt(d*x)/(a + b*acos(c*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx}}{b\arccos(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)/(b*arccos(c*x) + a), x)
```

3.222 $\int \frac{1}{\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/(Sqrt[d*x]*(a + b*ArcCos[c*x])), x]
```

Rubi [A] time = 0.02601, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sqrt[d*x]*(a + b*ArcCos[c*x])),x]
```

```
[Out] Defer[Int][1/(Sqrt[d*x]*(a + b*ArcCos[c*x])), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)}\,dx=\int \frac{1}{\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.853932, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[d*x]*(a + b*ArcCos[c*x])),x]
```

```
[Out] Integrate[1/(Sqrt[d*x]*(a + b*ArcCos[c*x])), x]
```

Maple [A] time = 0.24, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\arccos\left(cx\right)}\frac{1}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(c*x))/(d*x)^(1/2),x)
```

```
[Out] int(1/(a+b*arccos(c*x))/(d*x)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx}\left(b\arccos\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(c*x))/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x)*(b*arccos(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dx}}{bdx\arccos\left(cx\right)+adx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(c*x))/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)/(b*d*x*arccos(c*x) + a*d*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx}\,(a + b\,\mathrm{acos}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(c*x))/(d*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*(a + b*acos(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx}(b\arccos{(cx)} + a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(c*x))/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x)*(b*arccos(c*x) + a)), x)
```

3.223 $\int \frac{1}{(dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{(dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/((d*x)^(3/2)*(a + b*ArcCos[c*x])), x]
```

Rubi [A] time = 0.0290772, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d*x)^(3/2)*(a + b*ArcCos[c*x])),x]
```

```
[Out] Defer[Int][1/((d*x)^(3/2)*(a + b*ArcCos[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)}\,dx=\int \frac{1}{(dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 2.9465, size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d*x)^(3/2)*(a + b*ArcCos[c*x])),x]
```

```
[Out] Integrate[1/((d*x)^(3/2)*(a + b*ArcCos[c*x])), x]
```

Maple [A] time = 0.229, size = 0, normalized size = 0.

$$\int \frac{1}{a + b \arccos(cx)} (dx)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(a+b*arccos(c*x)),x)
```

```
[Out] int(1/(d*x)^(3/2)/(a+b*arccos(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{3}{2}} (b \arccos(cx) + a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x)^(3/2)*(b*arccos(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}}{bd^2x^2 \arccos(cx) + ad^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)/(b*d^2*x^2*arccos(c*x) + a*d^2*x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{3}{2}}\,(a + b\,\mathrm{acos}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(a+b*acos(c*x)),x)
```

```
[Out] Integral(1/((d*x)**(3/2)*(a + b*acos(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{3}{2}}\,(b \arccos{(cx)} + a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x)^(3/2)*(b*arccos(c*x) + a)), x)
```

3.224 $\int \frac{(dx)^{3/2}}{\left(a+b\cos^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{(dx)^{3/2}}{\left(a+b\cos^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[(d*x)^(3/2)/(a + b*ArcCos[c*x])^2, x]
```

Rubi [A] time = 0.0269893, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(dx)^{3/2}}{\left(a+b\cos^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^(3/2)/(a + b*ArcCos[c*x])^2,x]
```

```
[Out] Defer[Int][(d*x)^(3/2)/(a + b*ArcCos[c*x])^2, x]
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{\left(a+b\cos^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^{3/2}}{\left(a+b\cos^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 6.46136, size = 0, normalized size = 0.

$$\int \frac{(dx)^{3/2}}{\left(a+b\cos^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^(3/2)/(a + b*ArcCos[c*x])^2,x]
```

```
[Out] Integrate[(d*x)^(3/2)/(a + b*ArcCos[c*x])^2, x]
```

Maple [A] time = 0.228, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\arccos\left(cx\right)\right)^{2}}\left(dx\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(a+b*arccos(c*x))^2,x)
```

```
[Out] int((d*x)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{cx+1}\sqrt{-cx+1}d^{\frac{3}{2}}x^{\frac{3}{2}}-\frac{1}{2}\left(b^{2}c\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)+abc\right)\sqrt{d}\int\frac{\left(5\,c^{2}dx^{2}-3\,d\right)\sqrt{cx+1}\sqrt{-cx+1}\sqrt{x}}{abc^{3}x^{2}-abc+\left(b^{2}c^{3}x^{2}-b^{2}c\right)\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)}\,dx}{b^{2}c\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)+abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

```
[Out] (sqrt(c*x + 1)*sqrt(-c*x + 1)*d^(3/2)*x^(3/2) - (b^2*c*arctan2(sqrt(c*x + 1
)*sqrt(-c*x + 1), c*x) + a*b*c)*sqrt(d)*integrate(1/2*(5*c^2*d*x^2 - 3*d)*s
qrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(x)/(a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b
^2*c)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)), x))/(b^2*c*arctan2(sqrt(
c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dx}dx}{b^{2}\arccos\left(cx\right)^{2}+2\,ab\arccos\left(cx\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*d*x/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{3}{2}}}{(a + b\operatorname{acos}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(a+b*acos(c*x))**2,x)
```

```
[Out] Integral((d*x)**(3/2)/(a + b*acos(c*x))**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{3}{2}}}{(b\arccos(cx) + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)/(b*arccos(c*x) + a)^2, x)
```

3.225 $\int \frac{\sqrt{dx}}{\left(a+b\cos^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{\sqrt{dx}}{\left(a+b\cos^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[Sqrt[d*x]/(a + b*ArcCos[c*x])^2, x]
```

Rubi [A] time = 0.0233537, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{dx}}{\left(a+b\cos^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[d*x]/(a + b*ArcCos[c*x])^2,x]

[Out] Defer[Int][Sqrt[d*x]/(a + b*ArcCos[c*x])^2, x]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{\left(a+b\cos^{-1}(cx)\right)^2}\,dx=\int \frac{\sqrt{dx}}{\left(a+b\cos^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 4.95259, size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx}}{\left(a+b\cos^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[d*x]/(a + b*ArcCos[c*x])^2,x]
```

```
[Out] Integrate[Sqrt[d*x]/(a + b*ArcCos[c*x])^2, x]
```

Maple [A] time = 0.246, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\arccos\left(cx\right)\right)^{2}}\sqrt{dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(a+b*arccos(c*x))^2,x)
```

```
[Out] int((d*x)^(1/2)/(a+b*arccos(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\frac{1}{2}\left(b^{2}c\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)+abc\right)\sqrt{d}\int\frac{\left(3c^{2}x^{2}-1\right)\sqrt{cx+1}\sqrt{-cx+1}\sqrt{x}}{abc^{3}x^{3}-abcx+\left(b^{2}c^{3}x^{3}-b^{2}cx\right)\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)}\,dx-\sqrt{cx+1}\sqrt{-cx+1}\sqrt{\ }}{b^{2}c\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)+abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)*sqrt(d)*integr
ate(1/2*(3*c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(x)/(a*b*c^3*x^3 -
 a*b*c*x + (b^2*c^3*x^3 - b^2*c*x)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*
x)), x) - sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(d)*sqrt(x))/(b^2*c*arctan2(sqrt
(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dx}}{b^{2}\arccos\left(cx\right)^{2}+2\,ab\arccos\left(cx\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx}}{(a + b\operatorname{acos}(cx))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(a+b*acos(c*x))**2,x)
```

```
[Out] Integral(sqrt(d*x)/(a + b*acos(c*x))**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx}}{(b\arccos(cx) + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)/(b*arccos(c*x) + a)^2, x)
```

3.226 $\int \frac{1}{\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/(Sqrt[d*x]*(a + b*ArcCos[c*x])^2), x]
```

Rubi [A] time = 0.0240853, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sqrt[d*x]*(a + b*ArcCos[c*x])^2),x]
```

```
[Out] Defer[Int][1/(Sqrt[d*x]*(a + b*ArcCos[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 9.58086, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx}\left(a+b\cos^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[d*x]*(a + b*ArcCos[c*x])^2),x]
```

```
[Out] Integrate[1/(Sqrt[d*x]*(a + b*ArcCos[c*x])^2), x]
```

Maple [A] time = 0.244, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\arccos\left(cx\right)\right)^{2}}\frac{1}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(c*x))^2/(d*x)^(1/2),x)
```

```
[Out] int(1/(a+b*arccos(c*x))^2/(d*x)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\frac{1}{2}\left(b^{2}cdx\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)+abcdx\right)\sqrt{d}\int\frac{\left(c^{2}x^{2}+1\right)\sqrt{cx+1}\sqrt{-cx+1}\sqrt{x}}{abc^{3}dx^{4}-abcdx^{2}+\left(b^{2}c^{3}dx^{4}-b^{2}cdx^{2}\right)\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)}\,dx-\sqrt{cx+1}\sqrt{-}}{b^{2}cdx\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)+abcdx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(c*x))^2/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -((b^2*c*d*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*d*x)*sqrt(d
)*integrate(1/2*(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(x)/(a*b*c^3
*d*x^4 - a*b*c*d*x^2 + (b^2*c^3*d*x^4 - b^2*c*d*x^2)*arctan2(sqrt(c*x + 1)*
sqrt(-c*x + 1), c*x)), x) - sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(d)*sqrt(x))/(
b^2*c*d*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*d*x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{dx}}{b^{2}dx\arccos\left(cx\right)^{2}+2\,abdx\arccos\left(cx\right)+a^{2}dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(c*x))^2/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)/(b^2*d*x*arccos(c*x)^2 + 2*a*b*d*x*arccos(c*x) + a^2*d*x
), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx}\,(a+b\operatorname{acos}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(c*x))**2/(d*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*(a + b*acos(c*x))**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx}(b\arccos(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(c*x))^2/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x)*(b*arccos(c*x) + a)^2), x)
```

3.227 $\int \frac{1}{(dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{(dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/((d*x)^(3/2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi [A] time = 0.0285917, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d*x)^(3/2)*(a + b*ArcCos[c*x])^2),x]
```

```
[Out] Defer[Int][1/((d*x)^(3/2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)^2}\,dx=\int \frac{1}{(dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 13.9247, size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{3/2}\left(a+b\cos^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d*x)^(3/2)*(a + b*ArcCos[c*x])^2),x]
```

```
[Out] Integrate[1/((d*x)^(3/2)*(a + b*ArcCos[c*x])^2), x]
```

Maple [A] time = 0.233, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\arccos\left(cx\right)\right)^{2}}\left(dx\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(a+b*arccos(c*x))^2,x)
```

```
[Out] int(1/(d*x)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\frac{1}{2}\left(b^{2}cd^{2}x^{2}\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)+abcd^{2}x^{2}\right)\sqrt{d}\int\frac{\left(c^{2}x^{2}-3\right)\sqrt{cx+1}\sqrt{-cx+1}\sqrt{x}}{abc^{3}d^{2}x^{5}-abcd^{2}x^{3}+\left(b^{2}c^{3}d^{2}x^{5}-b^{2}cd^{2}x^{3}\right)\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)}\,dx+\sqrt{c}}{b^{2}cd^{2}x^{2}\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)+abcd^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

```
[Out] ((b^2*c*d^2*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*d^2*x^2)
*sqrt(d)*integrate(1/2*(c^2*x^2 - 3)*sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(x)/(
a*b*c^3*d^2*x^5 - a*b*c*d^2*x^3 + (b^2*c^3*d^2*x^5 - b^2*c*d^2*x^3)*arctan2
(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)), x) + sqrt(c*x + 1)*sqrt(-c*x + 1)*sqr
t(d)*sqrt(x))/(b^2*c*d^2*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a
*b*c*d^2*x^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dx}}{b^{2}d^{2}x^{2}\arccos\left(cx\right)^{2}+2\,abd^{2}x^{2}\arccos\left(cx\right)+a^{2}d^{2}x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)/(b^2*d^2*x^2*arccos(c*x)^2 + 2*a*b*d^2*x^2*arccos(c*x) +
 a^2*d^2*x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{3}{2}} (a + b\,\mathrm{acos}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(a+b*acos(c*x))**2,x)
```

```
[Out] Integral(1/((d*x)**(3/2)*(a + b*acos(c*x))**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{3}{2}} (b \arccos\,(cx) + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x)^(3/2)*(b*arccos(c*x) + a)^2), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
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        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType, Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: # isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```python


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```